ROBUST TRANSMIT BEAMPATTERN DESIGN FOR UNIFORM LINEAR ARRAYS USING CORRELATED LFM WAVEFORMS

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ABSTRACT

This paper presents a robust design of the transmit beampattern for uniform linear antenna arrays. Existing designs are usually completed at the stage of achieving an optimal transmit covariance matrix from identifying a weighting matrix with the assumption of ideally orthogonal waveforms. However, we propose a compensation technique to achieve the optimal covariance matrix without the requirement of orthogonality. The corresponding solutions identify a set of weighting matrices that are robust against the imperfection of the waveforms. As a result, a set of easy-to-generate partially correlated linear frequency modulated (LFM) waveforms can be used to achieve identical transmit beampatterns which could be synthesized by ideally orthogonal multiple-input multiple-output (MIMO) radar waveforms. The proposed robust design is evaluated via numerical examples.

1. INTRODUCTION

The design of transmit beampattern for a uniform linear antenna array is not new. Conventionally, phased-array radar systems achieve the design by identifying a weighting vector for a set of coherent waveforms. This problem has been equivalently considered as the mapping from a finite impulse response (FIR) filter design problem [1]. Alternatively, a set of orthogonal waveforms can be used to synthesize the transmit beampatterns, which takes the advantage of waveform diversity to improve the estimation performance at the receiver, such as parameter identifiability [2]. Transmit beampattern design using non-coherent waveforms has drawn much attention recently as discussed in [3–6], especially for multiple-input multiple-output (MIMO) radar systems.

Since the transmit beampattern is characterized by the covariance matrix of the transmitted weighted waveforms, the design is split into two procedures: i) the design of the transmit covariance matrix, and ii) the signaling strategy to achieve the covariance matrix obtained in i). The definition of the covariance matrix of the weighted waveforms can be approximated as

$$\mathbf{R} \triangleq \mathbf{W}\left(\lim_{N\to\infty}\frac{1}{N}\mathbf{S}\mathbf{S}^H\right)\mathbf{W}^H \approx \mathbf{W}\mathbf{S}\mathbf{S}^H\mathbf{W}^H \approx \mathbf{W}\mathbf{W}^H, \tag{1}$$

where N is the sample length of the waveforms, and $\{\cdot\}^H$ is the complex conjugate operator. The dimensions of the above matrices are $\mathbf{R} \in \mathbb{C}^{N_\text{T}\times N_\text{T}}$, weighting matrix $\mathbf{W} \in \mathbb{C}^{N_\text{T}\times K}$, and waveform matrix $\mathbf{S} \in \mathbb{C}^{K\times N}$, where N_T is the number of transmit antennas and K is the number of orthogonal waveforms, $K \leq N_\text{T}$. It is assumed in (1) that the covariance matrix could be approximated by a finite number of samples, the waveforms have unit norm, and the waveforms are perfectly orthogonal, i.e., $\mathbf{S}\mathbf{S}^H = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix with appropriate dimension.

Let the optimal transmit covariance matrix obtained from i) be $\hat{\mathbf{R}}$, then according to [5] and [6], the weighting matrix $\mathbf{W}$ is obtained by solving $\mathbf{W}\mathbf{W}^H = \hat{\mathbf{R}}$. Throughout this paper, $\hat{\mathbf{R}}$ is considered available, obtained via existing methods as in [5] and [6]. Eigenvalue decomposition of $\hat{\mathbf{R}}$ is an efficient way to obtain $\mathbf{W}$. Let $\hat{\mathbf{R}} = \hat{\mathbf{Q}}\hat{\mathbf{\Lambda}}\hat{\mathbf{Q}}^H$, where $\hat{\mathbf{Q}} \in \mathbb{C}^{N_\text{T}\times K}$, and $\hat{\mathbf{\Lambda}} \in \mathbb{C}^{K\times K}$, then the solution can be obtained as

$$\hat{\mathbf{W}} = \hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}_0, \tag{2}$$

where $\mathbf{U}_0 \in \mathbb{C}^{K\times K}$ is an arbitrary unitary matrix. Equations (1)-(2) provide a commonly agreed solution which has theoretical merits in terms of simplicity and efficiency when used with well designed orthogonal waveforms as noted in [7, 8], and the references therein. However, the practical issues such as implementation difficulty of the orthogonal waveforms and the imperfection of the orthogonality, have not received sufficient attention.

The novelty of the reported work is as follows. We propose a robust design of the transmit beampattern using a set of easy-to-generate partially correlated linear frequency modulated (LFM) waveforms. Instead of using (1)-(2), we propose a robust solution with the formulation using a compensation technique. The proposed work is an extension to our work in [6]. In there it was assumed that perfectly orthogonal waveforms are available. However, as shown in literature (e.g. [9]), obtaining a large set of perfectly orthogonal waveforms is difficult. The robust design proposed here generally exhibits the advantage of easing the burden on orthogonal waveform design. Via the obtained robust weighting matrix, non-orthogonal waveforms can be used to achieve identical beampatterns that could be obtained by ideal waveforms. We also provide a quantitative assessment of the impact of the waveform non-orthogonality on the design. In addition, the robust design is not limited to LFM signals, and can also be used with other radar waveforms. In summary, the proposed method is more general than either approaches in [4] and [5] and achieves optimum performance to that shown in [6].

2. THE ROBUST DESIGN

The robust design assumes no perfect orthogonality among the transmitted waveforms. Due to this, certain degradation occurs in the covariance matrix of the un-weighted waveforms, and (1) is rewritten as

$$\mathbf{R} = \mathbf{W}\mathbf{S}\mathbf{S}^H\mathbf{W}^H. \tag{3}$$

Although perfect orthogonality is not used in the above equation, the free variable matrix $\mathbf{W}$ can be designed to compensate such imperfection resulting from non-orthogonality. Here a compensation technique is formulated by setting $\mathbf{R} = \hat{\mathbf{R}}$, i.e., $\mathbf{W}\mathbf{S}\mathbf{S}^H\mathbf{W}^H = \hat{\mathbf{R}}$. Let $\tilde{\mathbf{R}} = \mathbf{S}\mathbf{S}^H$, then the following derivations provide the corresponding solution.

$$\mathbf{W}\tilde{\mathbf{R}}\mathbf{W}^H = \hat{\mathbf{R}} \tag{4}$$

$$\Longleftrightarrow \mathbf{W}\tilde{\mathbf{Q}}\tilde{\mathbf{\Lambda}}\tilde{\mathbf{Q}}^H\mathbf{W}^H = \hat{\mathbf{Q}}\hat{\mathbf{\Lambda}}\hat{\mathbf{Q}}^H \tag{5}$$

$$\Longleftrightarrow \mathbf{W}\tilde{\mathbf{Q}}\sqrt{\tilde{\mathbf{\Lambda}}} = \hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U} \tag{6}$$

$$\Longleftrightarrow \hat{\mathbf{W}} = \hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\left(\sqrt{\tilde{\mathbf{\Lambda}}}\right)^{-1}\tilde{\mathbf{Q}}^H, \tag{7}$$

where $\tilde{\mathbf{Q}}, \tilde{\mathbf{\Lambda}} \in \mathbb{C}^{K\times K}$ are from the eigenvalue decomposition of $\tilde{\mathbf{R}}$, and $\mathbf{U} \in \mathbb{C}^{K\times K}$ is an arbitrary unitary matrix. It can be seen that (2) is a special case of (7) when $\tilde{\mathbf{\Lambda}} = \mathbf{I}$, i.e., the waveforms are perfectly orthogonal. It is also noted that the existence of the robust solution only requires $\tilde{\mathbf{\Lambda}}$ to be invertible, i.e., $\mathbf{S}$ is full row rank. This is a significant relaxation on the waveform requirements. Note that in (7), the portion $\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}$ is fixed because it is arising from $\hat{\mathbf{R}}$, whereas $\mathbf{U}$ and $\tilde{\mathbf{\Lambda}}$ are tunable. The general steps to achieve a robust design of transmit beampattern are thus summarized as follows.

1. Obtain $\hat{\mathbf{R}}$, the optimal transmit covariance matrix based on the required specifications.
2. Choose non-coherent easy-to-generate waveforms to satisfy hardware requirements. (The orthogonality of waveforms determines $\tilde{\mathbf{\Lambda}}$, which is discussed later.)
3. Select a suitable matrix $\mathbf{U}$ and obtain $\hat{\mathbf{W}}$ using (7).

In the following content, we provide further discussions on the selection of $\mathbf{U}$ and $\tilde{\mathbf{\Lambda}}$ to obtain a suitable solution.

2.1. Quantization Error and the Unitary Matrix U

It is indicated in [6] that the design of the transmit beampattern is equivalent to the design of a multiple-input single-output (MISO) FIR filter. We note that the finite-word-length effects are inherent in practical implementations of digital filters [10]. As such we will use the finite word-length effects as an optimality criterion in the proposed robust design [11]. Let the word rounding step-size of the waveforms and the multiplier output be Q_1 and Q_2 respectively. Let the quantization noise of the waveforms be $e_{\mathrm{S}}^k(n), k \in \{0, 1, \cdots, K-1\}$. Let the quantization noise at the multiplier output be $e_{\mathrm{M}}^{l,k}(n), l \in \{0, 1, \cdots, N_{\mathrm{T}}-1\}$. The quantization noises are i.d.d. random variables with uniform distribution over the quantization step-size, i.e., $e_{\mathrm{S}}^k(n) \sim \mathcal{U}(-0.5Q_1, 0.5Q_1)$, $e_{\mathrm{M}}^{l,k}(n) \sim \mathcal{U}(-0.5Q_2, 0.5Q_2)$. It then follows that the corresponding mean and variance values are $\eta_{\mathrm{S}} = \eta_{\mathrm{M}} = 0$, $\sigma_{\mathrm{S}}^2 = Q_1^2/12$, and $\sigma_{\mathrm{M}}^2 = Q_2^2/12$. Denote

$$\hat{W}_k(\theta) = \sum_{l=0}^{N_{\mathrm{T}}-1} w_{l,k} e^{-j\pi l \cos\theta}, \tag{8}$$

which is the spatial domain response of the kth branch of the MISO filter. The overall output noise power σ_{MISO}^2 due to the finite-word-length effects is then given by

$$\begin{aligned}\sigma_{\mathrm{MISO}}^2 &= \underbrace{\sum_{k=0}^{K-1}\sigma_{\mathrm{S}}^2\int_\theta \left|\hat{W}_k(\theta)\right|^2 d\theta}_{\text{Waveform Quantization}} + \underbrace{\sum_{k=0}^{K-1}\sum_{l=0}^{N_{\mathrm{T}}-1}\sigma_{\mathrm{M}}^2}_{\text{Multiplier Output Quantization}} \\ &= \frac{K}{12}\left(Q_1^2\|\hat{\mathbf{W}}\|^2 + N_{\mathrm{T}}Q_2^2\right),\end{aligned} \tag{9}$$

where the Parseval's theorem [10] is used to evaluate the integration, and $\|\cdot\|$ denotes the Frobenius norm. It is seen from (9) that the output quantization noise power is proportional to $\|\hat{\mathbf{W}}\|^2$. Hence the optimization problem to obtain $\mathbf{U}$ to minimize the output noise power σ_{MISO}^2 is expressed as

$$\begin{aligned}\min_{\mathbf{U}} \quad & \|\hat{\mathbf{W}}\|^2 \\ \text{s.t.} \quad & \mathbf{U}\mathbf{U}^H = \mathbf{I}.\end{aligned} \tag{10}$$

Note that optimization under similar constraints have been used in other areas of array processing too, although in different applications [12]. According to (7), we have

$$\begin{aligned}\|\hat{\mathbf{W}}\|^2 &= \mathrm{tr}\{\hat{\mathbf{W}}\hat{\mathbf{W}}^H\} \\ &= \mathrm{tr}\left\{\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\sqrt{\tilde{\mathbf{\Lambda}}^{-1}}\tilde{\mathbf{Q}}^H\tilde{\mathbf{Q}}\sqrt{\tilde{\mathbf{\Lambda}}^{-1}}\mathbf{U}^H\sqrt{\hat{\mathbf{\Lambda}}}\hat{\mathbf{Q}}^H\right\} \\ &= \mathrm{tr}\left\{\sqrt{\hat{\mathbf{\Lambda}}}\hat{\mathbf{Q}}^H\hat{\mathbf{Q}}\sqrt{\hat{\mathbf{\Lambda}}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\right\} \\ &= \mathrm{tr}\{\hat{\mathbf{\Lambda}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\}.\end{aligned} \tag{11}$$

Hence (10) can be rewritten as

$$\begin{aligned}\min_{\mathbf{U}} \quad & \mathrm{tr}\{\hat{\mathbf{\Lambda}}\mathbf{U}\tilde{\mathbf{\Lambda}}^{-1}\mathbf{U}^H\} \\ \text{s.t.} \quad & \mathbf{U}\mathbf{U}^H = \mathbf{I}.\end{aligned} \tag{12}$$

Note that the solution (2) under perfect waveform correlation assumption has a fixed norm for the weighting matrix, and thus the selection of the unitary matrix $\mathbf{U}_0$ is trivial. Next, we investigate the eigenvalue spread of $\tilde{\mathbf{R}}$.

2.2. The Eigenvalue Spread of $\tilde{\mathbf{R}}$

The eigenvalue spread of $\tilde{\mathbf{R}}$ (equivalently of $\tilde{\mathbf{\Lambda}}$) is determined by the correlation property of the waveforms. Let the diagonal elements of $\tilde{\mathbf{\Lambda}}$ be in descending order, i.e., $\{\tilde{\lambda}_{\text{max}}, \cdots, \tilde{\lambda}_{\text{min}}\}$, then the eigenvalue spread, denoted as ρ, is defined as

$$\rho = \frac{\tilde{\lambda}_{\text{max}}}{\tilde{\lambda}_{\text{min}}}. \tag{13}$$

In this paper, we consider the most commonly used and easy-to-generate radar waveforms–the linear frequency modulated (LFM) waveforms (chirps) as an example to study the eigenvalue spread of $\tilde{\mathbf{R}}$. The use of a set of LFM waveforms for MIMO radar systems is presented in [13]. Here, we use a similar approach to generate a set of LFM waveforms and quantitatively study the relationship between the eigenvalue spread of $\tilde{\mathbf{R}}$ and the waveform parameters. Let the bandwidth of the baseband waveforms be B. Let the duration of the single pulse transmission be T. Let the (initial) frequency step-size be f_0. Let the chirp rate be κ. The set of K non-orthogonal LFM waveforms are then expressed as

$$s_k(t) = \frac{1}{\sqrt{T}} \exp\left\{ j2\pi \left(kf_0 t + \frac{1}{2}\kappa t^2 \right) \right\}, \tag{14}$$

where $k \in \{0, 1, \cdots, K-1\}$. The parameters f_0 and κ are confined by B, T, and K: the instantaneous frequency of $s_{K-1}(t)$ at time T should be no greater than B, i.e.,

$$\begin{aligned} \left. \frac{d\left((K-1)f_0 t + \frac{1}{2}\kappa t^2\right)}{dt} \right|_{t=T} &\leq B \\ \Longleftrightarrow (K-1)f_0 + \kappa T &\leq B \\ \Longleftrightarrow \kappa &\leq \frac{(B-(K-1)f_0)}{T}. \end{aligned} \tag{15}$$

Substituting (15) into (14), one can easily obtain the expressions of the K LFM waveforms. The correlation between $s_k(t)$ and $s_{k+\Delta k}(t)$, $\Delta k \in \{1, 2, \cdots, K-1\}$, at zero lag, which is the $(k, k+\Delta k)$th element of $\tilde{\mathbf{R}}$, is given by

$$\begin{aligned} |R_{k,k+\Delta k}(f_0)| &= \left| \int_0^T s_k(t) s^*_{k+\Delta k}(t) dt \right| \\ &= \frac{1}{T} \left| \int_0^T e^{j2\pi\left\{kf_0 t + \frac{1}{2}\kappa t^2 - (k+\Delta k) f_0 t - \frac{1}{2}\kappa t^2\right\}} dt \right| \\ &= \left| \operatorname{sinc}\left\{ \pi \Delta k f_0 T \right\} \right|. \end{aligned} \tag{16}$$

It is observed that if $\Delta k = 0$, then (16) becomes the autocorrelation of $s_k(t)$ at zero lag, which is unity. An intuitive way to select f_0 in (16) is by setting $f_0 T = 1$, thus $f_0 = 1/T$. Then $\forall \Delta k$, $|R_{k,k+\Delta k}(f_0)| = 0$. Therefore $\tilde{\lambda}_{\text{max}} = \tilde{\lambda}_{\text{min}}$. Due to the sampling effects, small values may exist in off-diagonal elements of $\tilde{\mathbf{R}}$. This is the best possible way to obtain a decorrelated set of LFM waveforms.

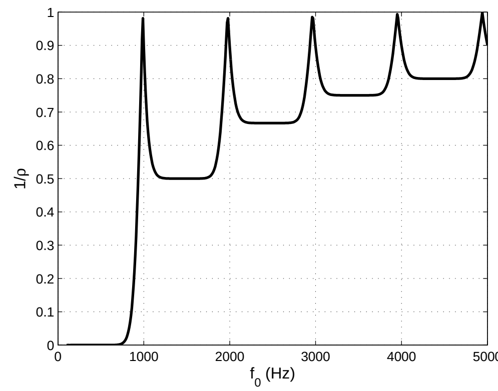


Fig. 1. $1/\rho$ versus f_0, where $B = 100$ kHz, and $T = 1$ ms.

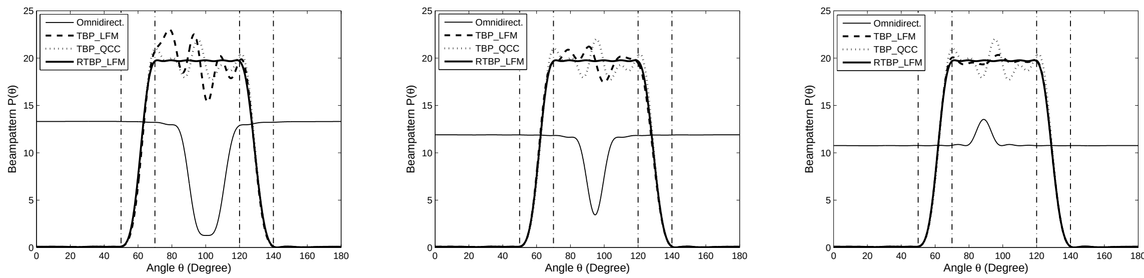
However, T is usually chosen very small to preserve the range ability of the radar system. In this situation, large value of f_0 reduces the bandwidth efficiency. Since the robust design allows waveforms without good correlation properties, it is possible to select $f_0 < 1/T$. However if f_0 is chosen too small such that the LFM waveforms are highly correlated (as can be seen from Fig. 1), then large values will appear in the diagonal elements of $\tilde{\mathbf{\Lambda}}^{-1}$, which results in a large value of $\|\hat{\mathbf{W}}\|^2$ in (11) and amplifies the quantization error. Hence, with the incorporation of (12), the overall optimization problem for robust transmit beampattern design using LFM waveforms is formulated as

$$\begin{aligned} \min_{\mathbf{U}, f_0} \quad & |R_{k,k+\Delta k}(f_0)| + |f_0| \\ \text{s.t.} \quad & \left| \operatorname{tr}\{\hat{\mathbf{\Lambda}} \mathbf{U} \tilde{\mathbf{\Lambda}}^{-1} \mathbf{U}^H\} - N_{\text{T}} \right| < \zeta, \\ & \mathbf{U}\mathbf{U}^H = \mathbf{I}, \end{aligned} \tag{17}$$

where ζ is a small positive real number. The inequality constraint ensures that $\|\hat{\mathbf{W}}\|^2$ resulting from the robust design is close to, if no less than, that resulting from the design under ideal assumptions. The solution to (17) identifies $\mathbf{U}$ for a selected $\tilde{\mathbf{\Lambda}}$, which are then substituted into (7) to obtain the weighting matrix. In the following section, we provide several empirical solutions to (17).

3. NUMERICAL EXAMPLES

In this section, we present several examples to illustrate the advantages of the proposed robust design. We use the feasibility problem (FP) based algorithm [6] to obtain $\hat{\mathbf{R}}$ with minimum number of antennas. The free field is modeled as a 2 dimensional space with azimuth angle from 0° to 180° , where 90° corresponds to the broadside. The desired transmit beampattern is specified as follows. The passband is $[70°, 120°]$, the transition band is 20°, passband ripple bound is 0.1, and



(a) $f_0 = 800$ Hz, $\|\hat{\mathbf{W}}\|^2 = 42.9635$. (b) $f_0 = 900$ Hz, $\|\hat{\mathbf{W}}\|^2 = 11.9692$. (c) $f_0 = 1$ kHz, $\|\hat{\mathbf{W}}\|^2 = 11.1096$.

Fig. 2. Designed transmit beampatterns using standard and robust techniques, where $\hat{N}_{\mathrm{T}} = 11$, and $\mathbf{U} = \mathbf{U}_0 = \mathbf{I}$.

stopband ripple bound is 0.1. The solution of the minimum required number of transmit antennas is $\hat{N}_{\mathrm{T}} = 11$.

The transmit beampatterns obtained from the robust designs are shown in Fig. 2, where the beampatterns of omnidirectional transmission, and the standard transmit beamspace processing (TBP) designs using quadratic congruence coded (QCC) [9] and LFM waveforms are also provided for comparison. QCC waveforms are generally better than LFM waveforms for improved orthogonality, but they are more difficult to generate. The bandwidth and time duration of the LFM waveforms are set as $B = 100$ kHz, and $T = 1$ ms. Because (4)-(7) indicate that the transmit beampattern is independent of $\mathbf{U}$, we set $\mathbf{U} = \mathbf{U}_0 = \mathbf{I}$. Note that $\mathbf{U}$ only affects $\|\hat{\mathbf{W}}\|^2$ as indicated in (11). It is seen from Fig. 2 that the robust design can compensate the imperfection of the waveform correlations and recovers the desired beampatterns, i.e., the theoretical one resulting from $\hat{\mathbf{R}}$. It is also seen that reducing the correlations among the LFM waveforms results the TBP based design approaching the robust TBP design. However, even at the least correlation point $f_0 = 1$ kHz, there still exist mismatches between the two designs. The advantage of the robust design is therefore demonstrated. Next, we illustrate the tuning of f_0 and $\mathbf{U}$ to change waveform correlations, bandwidth usage, and reduce $\|\hat{\mathbf{W}}\|^2$.

For efficient bandwidth usage, it is preferable that $f_0 < 1/T$. However, reducing f_0 increases the correlations of the LFM waveforms, which results in large values appearing in $\tilde{\mathbf{\Lambda}}^{-1}$ and thus $\|\hat{\mathbf{W}}\|^2$ becomes very sensitive to $\mathbf{U}$ due to (11). In this situation, $\mathbf{U}$ need to be carefully selected to limit $\|\hat{\mathbf{W}}\|^2$. An empirical approach to study the impact of $\mathbf{U}$ on $\|\hat{\mathbf{W}}\|^2$ is provided through Fig. 3, where the specifications of the desired beampattern is the same as those used in Fig. 2, and f_0 is chosen along the rising edge within $0 < f_0 < 1$ kHz. Among 1000 independent realizations of $\mathbf{U}$ for 3 different values of f_0 respectively, one can obtain the $\mathbf{U}$'s that minimize $\|\hat{\mathbf{W}}\|^2$. The minimum values of $\|\hat{\mathbf{W}}\|^2$ are 14.9353, 12.1387, and 10.6882 for $f_0 = 800$, 850, and 1 kHz, respectively. For example when $f_0 = 800$ Hz, a suitable choice of $\mathbf{U}$ is able to reduce $\|\hat{\mathbf{W}}\|^2$ from 42.9635 in Fig. 2 (a) to 14.9353, which is a significant improvement.

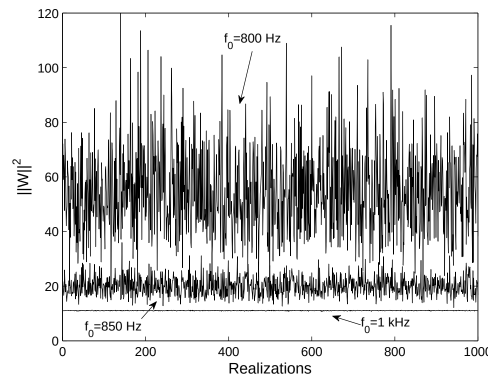


Fig. 3. $\|\hat{\mathbf{W}}\|^2$ versus random realizations of $\mathbf{U}$, where $\hat{N}_{\mathrm{T}} = 11$, and the minimum values of $\|\hat{\mathbf{W}}\|^2$ are 14.9353, 12.1387, and 10.6882 for $f_0 = 800$, 850, and 1 kHz, respectively.

4. CONCLUSIONS

This paper has presented a robust design of the transmit beampatterns for active uniform linear antenna arrays. A compensation technique is formulated and investigated. Instead of imposing perfect correlations conditions for the waveforms, or using waveforms with very good correlation properties as seen in existing literature, the robust design utilizes non-orthogonal and easy-to-generate waveforms. The only constraint on the waveforms is that they should not be fully coherent. We use a set of LFM waveforms as an example to illustrate the design. An overall optimization problem is formulated to identify the LFM waveform parameter f_0 and the unitary matrix $\mathbf{U}$. Empirical solutions are presented to demonstrate the advantages. The resultant transmit beampatterns are identical to those designed under ideal orthogonality assumption. Generally, the proposed method is applicable to any arbitrarily selected waveforms.

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