MULTICAST BEAMFORMING WITH ANTENNA SELECTION USING EXACT PENALTY APPROACH

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ABSTRACT

In this paper, multi-group multicast beamforming is considered with antenna selection. Nonconvex joint problem is converted to an equivalent biconvex problem by using exact penalty approach. The equivalent problem is solved iteratively using alternating maximization where a convex problem is solved at each step. Antenna selection reduces the total transmitted power significantly compared to the scenario with fixed antennas. Proposed method is computationally efficient and presents significant improvements in total transmitted power.

Index Terms— Antenna selection, multicast beamforming, convex optimization, alternating maximization

1. INTRODUCTION

In this paper, multi-group multicast transmit beamforming is considered where a transmitter with multiple antennas transmits different signals to groups of users by using different beamformer weight vector for each group. Multicast beamforming employs channel state information (CSI) to effectively steer power towards multicast groups while minimizing the interference between them. Multicast beamforming is included in LTE standard [1].

Using multiple antennas at the base station is an efficient approach to overcome signal fading and increase the capacity. However, the cost and the complexity of multichannel structures increase as the number of antennas increases [2]. As the antenna technology and fabrication techniques develop, antennas become cheaper and it is possible to switch large number of antennas with a limited number of RF chains [3]. It is known that antenna selection can improve power efficiency compared to a fixed antenna structure. It is a good low-cost alternative to increase spatial diversity and improve channel capacity [4]. Antenna selection is used in different applications [5], [6] and proposed for multicast beamforming in [3], [7].

The optimization problem for multicast beamforming with antenna selection is nonconvex and NP hard. Previous approaches employ semidefinite relaxation and randomization together with suboptimal antenna selection schemes [3]. Nonconvex rank condition together with the antenna selection scheme result a hard to solve problem. While most of the previous works drop the rank condition and solve a relaxed version of the optimization problem, an alternative and more complete form is presented in our case.

In this paper, nonconvex rank and integer constraints are converted to bilinear constraints which allow the use of exact penalty approach. This conversion is then exploited to implement alternating maximization where the resulting biconvex problem is solved iteratively. These iterations are guaranteed to converge. This is due to the fact that a convex problem is solved at each iteration. The worst case complexity of the semidefinite programming (SDP) problem which is solved at each iteration is less than the alternatives [3]. Simulations show that small number of iterations is needed for the proposed algorithm. The proposed solution performs well, significantly improving the total transmitted power compared to the alternative techniques.

2. SYSTEM MODEL

Consider a wireless scenario comprising a base station equipped with M transmit antennas and N receivers, each having a single antenna. Assume that there are G multicast groups, $\{\mathcal{G}_1, ..., \mathcal{G}_G\}$, where $\mathcal{G}_k$ denotes the k^{th} multicast group of users. Each receiver listens to a single multicast, i.e., $\mathcal{G}_k \bigcap \mathcal{G}_l = \emptyset$. The signal transmitted from the antenna array is $\mathbf{x}(t) = \sum_{k=1}^{G} \mathbf{w}_k^* s_k(t)$ where $s_k(t)$ is the information signal for the users in $\mathcal{G}_k$ and $\mathbf{w_k^*}$ is the related $M \times 1$ complex beamformer weight vector. It is assumed that information signals $\{s_k(t)\}_{k=1}^{G}$ are mutually uncorrelated each with zero mean and unit variance, $\sigma_{s_k}^2 = 1$. In this case, the total transmitted power is $\sum_{k=1}^{G} \mathbf{w}_k^H \mathbf{w}_k$. The received signal at the i^{th} receiver is given as, $y_i(t) = \mathbf{h}_i^T \mathbf{x}(t) + n_i(t),\ i = 1, ..., N$, where $\mathbf{h}_i$ is the $M \times 1$ known complex channel vector for the i^{th} receiver and n_i is the additive noise uncorrelated with the source signals, whose variance is σ_i^2. Assuming that i^{th} receiver is in the k^{th} multicast group, $\mathcal{G}_k$, signal-to-interference-plus-noise ratio (SINR) for the i^{th} receiver is,

$$SINR_i = \frac{|\mathbf{w_k}^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w_l}^H \mathbf{h}_i|^2 + \sigma_i^2} \tag{1}$$

Quality of service (QoS) multicast beamforming problem is to minimize the total transmitted power subject to receive-SINR constraint for each user, i.e.,

$$\min_{\{\mathbf{w}_k \in \mathbb{C}^M\}_{k=1}^{G}} \sum_{k=1}^{G} \mathbf{w}_k^H \mathbf{w}_k \tag{2.a}$$

$$s.t. \quad \frac{\mathbf{w_k}^H \mathbf{R}_i \mathbf{w}_k}{\sum_{l \neq k} \mathbf{w_l}^H \mathbf{R}_i \mathbf{w}_l + \sigma_i^2} \geq \gamma_i, \tag{2.b}$$

$$\forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\}$$

where γ_i is the SINR threshold for the i^{th} receiver and $\mathbf{R}_i = \mathbf{h}_i \mathbf{h}_i^H$. The problem in (2) is not convex [1]. Let us define $\mathbf{w} = [\ \mathbf{w}_1^T\ \mathbf{w}_2^T\ ...\ \mathbf{w}_G^T\ ]^T$ and $\mathbf{W} = \mathbf{w}\mathbf{w}^H$. $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ shows the $(k, k)^{th}$ block of $\mathbf{W}$. The problem in (2) can be written as,

$$\min_{\mathbf{W} \in \mathbb{C}^{GM \times GM}} Tr\{\mathbf{W}\} \tag{3.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i \mathbf{W}_k\} - \gamma_i \sum_{l \neq k} Tr\{\mathbf{R}_i \mathbf{W}_l\} \geq \gamma_i \sigma_i^2, \tag{3.b}$$

$$\forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\}$$

$$\mathbf{W} \succeq 0 \tag{3.c}$$

$$rank(\mathbf{W}) = 1 \tag{3.d}$$

In [1] and [8], the above problem is solved by convex optimization with semidefinite relaxation where the rank condition is dropped and randomization is used to obtain a feasible suboptimal solution. In this paper, the problem is exploited to generate a biconvex structure which can be solved more effectively than the relaxation and randomization approaches.

3. ANTENNA SELECTION

Assume that L RF transmission chains are available, while there are M antennas. The problem is to select the best L out of M antennas and find the corresponding beamforming weight vectors to minimize the total transmitted power. Let us define a $M \times 1$ vector, $\mathbf{b}$, whose elements are either 0 or 1. The m^{th} element of $\mathbf{b}$, b_m, is the antenna selection coefficient for the m^{th} antenna. Hence, $b_m = 1$ if the m^{th} antenna is selected, and it is zero otherwise. We can use big-M approach to formulate antenna selection problem with the aid of binary variables, b_m [9]. The joint problem can be written as,

$$\min_{\mathbf{W}\in\mathbb{C}^{GM\times GM},\mathbf{b}} \quad Tr\{\mathbf{W}\} \tag{4.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i\mathbf{W}_k\} - \gamma_i \sum_{l\neq k} Tr\{\mathbf{R}_i\mathbf{W}_l\} \geq \gamma_i\sigma_i^2, \tag{4.b}$$

$$\forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\}$$

$$\mathbf{W} \succeq 0 \tag{4.c}$$

$$rank(\mathbf{W}) = 1 \tag{4.d}$$

$$\sum_{k=1}^{G} W_{k_{m,m}} \leq A b_m, \;\; m = 1, ..., M \tag{4.e}$$

$$\sum_{m=1}^{M} b_m = L \tag{4.f}$$

$$b_m \in \{0, 1\}, \;\; m = 1, ..., M \tag{4.g}$$

where A is big enough such that it does not restrict the power of selected antennas [9]. $\sum_{k=1}^{G} W_{k_{m,m}}$ stands for the power transmitted from the m^{th} antenna. In the following part, a condition for an appropriate value for A is found. In the above problem, (4.d) and (4.g) are the only nonconvex constraints. In this paper, (4) is converted into an equivalent form which admits more flexible and manageable solutions. In the following lemma, an equivalent expression for (4.g) is found.

Lemma 1: The inequalities in (5.a-b) with (4.f) imply (4.g), i.e., b_m's are binary variables.

$$\sum_{m=1}^{M} b_m^2 = L \tag{5.a}$$

$$0 \leq b_m \leq 1, \;\; m = 1, ..., M \tag{5.b}$$

Proof: Consider the difference of (5.a) and (4.f), i.e.,

$$\sum_{m=1}^{M} b_m^2 - \sum_{m=1}^{M} b_m = \sum_{m=1}^{M} b_m(b_m - 1) = 0 \tag{6}$$

Each term in the summation in (6) is nonpositive from (5.b). The only case for (6) to be valid is that all terms in the summation are equal to zero, i.e., $b_m \in \{0, 1\}$. Hence, (4.g) can be replaced by (5.a) and (5.b).

The following theorem is used to obtain an equivalent formulation for (4).

Theorem 1: For any $P > 0$, (4) is equivalent to the following problem in (7) up to a scale factor in the sense that their optimum solutions differ only by a real positive scalar.

$$\max_{\mathbf{W}\in\mathbb{C}^{GM\times GM},\mathbf{b}} \quad t \tag{7.a}$$

$$s.t. \quad Tr\{\mathbf{R}_i\mathbf{W}_k\} - \gamma_i \sum_{l\neq k} Tr\{\mathbf{R}_i\mathbf{W}_l\} \geq t\gamma_i\sigma_i^2, \tag{7.b}$$

$$\forall i \in \mathcal{G}_k, \forall k, l \in \{1, ..., G\}$$

$$\mathbf{W} \succeq 0 \tag{7.c}$$

$$rank(\mathbf{W}) = 1 \tag{7.d}$$

$$\sum_{k=1}^{G} W_{k_{m,m}} \leq A b_m, \;\; m = 1, ..., M \tag{7.e}$$

$$\sum_{m=1}^{M} b_m = L \tag{7.f}$$

$$\sum_{m=1}^{M} b_m^2 = L \tag{7.g}$$

$$0 \leq b_m \leq 1, \;\; m = 1, ..., M \tag{7.h}$$

$$Tr\{\mathbf{W}\} = P \tag{7.i}$$

$$t > 0 \tag{7.j}$$

Proof: Assume that (4) is feasible, its optimum solution is $\mathbf{W}_{opt_1}$ and the associated total transmitted power is P_1. It is easy to see that at least one of the SINR constraints in (4.b) should be met with equality. Otherwise $\mathbf{W}_{opt_1}$ could be scaled down, thereby improving the objective function. Then $(P/P_1)\mathbf{W}_{opt_1}$ satisfies the constraints of (7) with the associated variable $t_1 = (P/P_1) > 0$. Hence (7) is feasible if (4) is feasible. Let $\{\mathbf{W}_{opt_2}, t_2\}$ be the optimum solution of (7). t_2 can only be greater than or equal to t_1. If t_2 is strictly greater than t_1, then it is possible to satisfy the constraints of (4) using $\mathbf{W}_{opt_2}/t_2$ whose total transmitted power is $P_2 = (t_1/t_2)P_1 < P_1$, which is a contradiction. Therefore $t_2 = t_1$ and $\mathbf{W}_{opt_2} = (P/P_1)\mathbf{W}_{opt_1}$ is the optimum solution. At this point, we have shown that whenever (4) is feasible, the optimum solutions of both problems are the same up to a scale factor. If (4) is not feasible, the constraints in (4) will not be satisfied. Therefore, for the same problem setting (SINR threshold values, γ_i, and noise variance, σ_i^2) no solution can be found for (7), otherwise a feasible solution can be found for (4) by scaling up or down the solution of (7). ■

Note that $\sum_{k=1}^{G} W_{k_{m,m}} \leq P$ for each m. Big-M parameter A can be selected as $A = P$, without restricting the power of selected antennas.

In the following parts, the problem in (7) is converted into a biconvex structure suitable for alternating maximization. For this purpose, nonconvex constraints (7.d) and (7.g) are expressed in bilinear equivalent forms in order to employ exact penalty approach. Hence, the final form of the optimization problem is obtained after a number of equivalent transformations. The following theorem is used to express rank constraint in a more suitable way.

Theorem 2: For $GM \times GM$ Hermitian symmetric, positive semidefinite matrices $\mathbf{W^I}$ and $\mathbf{W^{II}}$, $Tr\{\mathbf{W^I}\mathbf{W^{II}}\}$ is upper bounded by $Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\}$, i.e. $Tr\{\mathbf{W^I}\mathbf{W^{II}}\} \leq Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\}$. This upper bound is reached if and only if $\mathbf{W^I}$ and $\mathbf{W^{II}}$ are rank one matrices and $\mathbf{W^{II}} = \alpha\mathbf{W^I}$ where α is a positive scalar.

Proof: The proof of this theorem can be found in [10]. ■

In the following part, rank condition in (7.d) is embedded into the problem in terms of continuous bilinear function of parameter matrices suitable for alternating maximization.

Corollary 1: For two Hermitian symmetric, positive semidefinite matrices $\mathbf{W^I}$ and $\mathbf{W^{II}}$, $Tr\{\mathbf{W^I}\}Tr\{\mathbf{W^{II}}\}-Tr\{\mathbf{W^I W^{II}}\}=0$ condition implies rank one matrices, i.e., $\mathbf{W^{II}}=\frac{\lambda_1(\mathbf{W^{II}})}{\lambda_1(\mathbf{W^I})}\mathbf{W^I}$ where $\lambda_1(.)$ is the maximum eigenvalue.

The following theorem is presented to obtain an intermediate problem structure before the final form.

Theorem 3: The optimum solution of (7) and the following optimization problem in (8) are the same, namely $\mathbf{W^I_{opt}}=\mathbf{W^{II}_{opt}}=\mathbf{W_{opt}}$, $\mathbf{b^I_{opt}}=\mathbf{b^{II}_{opt}}=\mathbf{b_{opt}}$ where $\{\mathbf{W_{opt}},\mathbf{b_{opt}}\}$ is the optimum solution of (7):

$$\max_{\mathbf{W^I},\mathbf{W^{II}}\in\mathbb{C}^{GM\times GM},\mathbf{b^I},\mathbf{b^{II}}} \quad t^I+t^{II} \tag{8.a}$$

$$s.t.\quad Tr\{\mathbf{R}_i\mathbf{W^I}_k\}-\gamma_i\sum_{l\neq k}Tr\{\mathbf{R}_i\mathbf{W^I}_l\}\geq t^I\gamma_i\sigma_i^2, \tag{8.b}$$

$$Tr\{\mathbf{R}_i\mathbf{W^{II}}_k\}-\gamma_i\sum_{l\neq k}Tr\{\mathbf{R}_i\mathbf{W^{II}}_l\}\geq t^{II}\gamma_i\sigma_i^2, \tag{8.c}$$

$$\forall i\in\mathcal{G}_k,\forall k,l\in\{1,...,G\}$$

$$\mathbf{W^I}\succeq 0,\ \mathbf{W^{II}}\succeq 0 \tag{8.d}$$

$$\sum_{k=1}^{G}W^I_{k_{m,m}}\leq Pb^I_m,\ \ m=1,...,M \tag{8.e}$$

$$\sum_{k=1}^{G}W^{II}_{k_{m,m}}\leq Pb^{II}_m,\ \ m=1,...,M \tag{8.f}$$

$$\sum_{m=1}^{M}b^I_m=\sum_{m=1}^{M}b^{II}_m=L \tag{8.g}$$

$$\sum_{m=1}^{M}b^I_m b^{II}_m=L \tag{8.h}$$

$$0\leq b^I_m,b^{II}_m\leq 1,\ \ m=1,...,M \tag{8.i}$$

$$Tr\{\mathbf{W^I}\}=Tr\{\mathbf{W^{II}}\}=P \tag{8.j}$$

$$P^2-Tr\{\mathbf{W^I W^{II}}\}=0 \tag{8.k}$$

$$t^I,t^{II}>0 \tag{8.l}$$

Proof: $\mathbf{W^I_{opt}}$ and $\mathbf{W^{II}_{opt}}$ are rank one matrices due to $P^2-Tr\{\mathbf{W^I W^{II}}\}=0$, which is the condition in Corollary 1. Hence $\mathbf{W^I_{opt}}=\mathbf{W^{II}_{opt}}$ since $\lambda_1(\mathbf{W^I_{opt}})=\lambda_1(\mathbf{W^{II}_{opt}})=P$ by (8.j) and Corollary 1. The conditions in (8.h) and (8.i) imply that $b^I_{opt_m},b^{II}_{opt_m}\in\{0,1\}$ and $\mathbf{b^I_{opt}}=\mathbf{b^{II}_{opt}}$. Since $\{\mathbf{W^I_{opt}},\mathbf{b^I_{opt}}\}$ and $\{\mathbf{W^{II}_{opt}},\mathbf{b^{II}_{opt}}\}$ independently solve the same problem, $\mathbf{W^I_{opt}}=\mathbf{W^{II}_{opt}}=\mathbf{W_{opt}}$ and $\mathbf{b^I_{opt}}=\mathbf{b^{II}_{opt}}=\mathbf{b_{opt}}$. ■

In the above problem, (8.h) and (8.k) are still nonconvex constraints. Fortunately these constraints can be moved into the objective function using exact penalty approach [11], [12], [13]. This modification does not change the optimum solution of the problem. In the following theorem, the equivalency of the new form and (8) are established.

Theorem 4: The problem in (8) is equivalent to the problem in (9) for $\mu_1,\mu_2>\mu_0$ with μ_0 being a finite positive value in the sense that any local maximum of the problem in (9) is also a local maximum of the problem in (8).

$$\max_{\mathbf{W^I},\mathbf{W^{II}}\in\mathbb{C}^{GM\times GM},\mathbf{b^I},\mathbf{b^{II}}} \quad t^I+t^{II}-\mu_1|P^2-Tr\{\mathbf{W^I W^{II}}\}|$$
$$-\mu_2|L-\mathbf{b^I}^T\mathbf{b^{II}}| \tag{9.a}$$
$$s.t.\quad \text{(8.b), (8.c), (8.d), (8.e), (8.f), (8.g), (8.i), (8.j), (8.l)}$$

Proof: Constraints in (9) are all continuous functions. The feasible sets of (8) and (9) are both closed and bounded and hence they are compact due to the finite dimensional space [14]. Therefore $\mu_1|P^2-Tr\{\mathbf{W^I W^{II}}\}|+\mu_2|L-\mathbf{b^I}^T\mathbf{b^{II}}|$ corresponds to an l_1 exact penalty function [12], [13]. Theorem 4 is valid by definition [13] and due to [12] (page 408). ■

Note that $|P^2-Tr\{\mathbf{W^I W^{II}}\}|=P^2-Tr\{\mathbf{W^I W^{II}}\}$ from Theorem 2. $|L-\mathbf{b^I}^T\mathbf{b^{II}}|=L-\mathbf{b^I}^T\mathbf{b^{II}}$ since $\mathbf{b^I}^T\mathbf{b^{II}}=\sum_{m=1}^{M}b^I_m b^{II}_m\leq\sum_{m=1}^{M}b^I_m=L$.

The final form of the optimization problem can be given as,

$$\max_{\mathbf{W^I},\mathbf{W^{II}}\in\mathbb{C}^{GM\times GM},\mathbf{b^I},\mathbf{b^{II}}} \quad t^I+t^{II}+\mu_1 Tr\{\mathbf{W^I W^{II}}\}$$
$$+\mu_2\mathbf{b^I}^T\mathbf{b^{II}} \tag{10.a}$$
$$s.t.\quad \text{(8.b), (8.c), (8.d), (8.e), (8.f), (8.g), (8.i), (8.j), (8.l)}$$

The problem in (10) is a biconvex problem and alternating maximization can be used to solve it [15], [16]. Alternating maximization is implemented by using iterations where $\{\mathbf{W^{I,r}},\mathbf{W^{II,r}},\mathbf{b^{I,r}},\mathbf{b^{II,r}}\}$ are the terms at the r^{th} iteration. At the r^{th} iteration, $\{\mathbf{W^{II,r}},\mathbf{b^{II,r}}\}$ are obtained by considering $\{\mathbf{W^{I,r-1}},\mathbf{b^{I,r-1}}\}$ as fixed terms. Then the fixed variables are alternated and $\{\mathbf{W^{I,r}},\mathbf{b^{I,r}}\}$ are obtained from (10) while $\{\mathbf{W^{II,r}},\mathbf{b^{II,r}}\}$ are kept as fixed.

The objective function in (10) is upper bounded by $2P\max_i\frac{Tr\{\mathbf{R}_i\}}{\gamma_i\sigma_i^2}+\mu_1P^2+\mu_2L$ which can be found similar to [10]. Since a convex problem is solved at each iteration, the objective function improves at each iteration and the iterative approach is guaranteed to converge [16].

4. ALTERNATING MAXIMIZATION ALGORITHM

In the previous parts, the problems in (7) and (10) are shown to be equivalent in the sense that they have the same optimum solutions. Furthermore, it is shown that (10) can be solved with alternating maximization. The convergence of this approach is guaranteed. However, there is no guarantee for the optimum solution after the convergence. The steps for the proposed approach can be presented as follows,

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Multicast Beamforming with Antenna Selection (MBAS)

Let $\lambda_1(\mathbf{W})$ be the maximum eigenvalue of the matrix $\mathbf{W}$.

Initialization: $r=0$,

Set proper μ_1, μ_2 and solve the relaxed version of (7) by removing (7.d) and (7.g). Let $\{\hat{\mathbf{W}},\hat{\mathbf{b}}\}$ denote the solution. The singular value decomposition of each $\hat{\mathbf{W}}_k$ is calculated as $\hat{\mathbf{W}}_k=\mathbf{U}_k\mathbf{\Sigma}_k\mathbf{U}_k^H$. The following initializations are done for simplicity, i.e., $\mathbf{b^{I,0}}=\hat{\mathbf{b}}$ and $\mathbf{W^{I,0}}_{k,l}=\mathbf{U}_k\sqrt{\mathbf{\Sigma}_k\mathbf{\Sigma}_l}\mathbf{U}_l$. Solve the problem in (10) for $\{\mathbf{W^{II,0}},\mathbf{b^{II,0}}\}$.

Phase 1: Iterations (r → r+1)

1) Solve (10) for $\{\mathbf{W^{I,r}},\mathbf{b^{I,r}}\}$ while fixing $\{\mathbf{W^{II}},\mathbf{b^{II}}\}$ as $\{\mathbf{W^{II,r-1}},\mathbf{b^{II,r-1}}\}$.

2) If $rank(\mathbf{W^{I,r}})=1$ or $\lambda_1(\mathbf{W^{I,r}})\geq\lambda_1(\mathbf{W^{II,r-1}})+\beta_1$ (improved solution), where β_1 is a proper positive threshold value (Ex: $P/20$), keep the value of μ_1 same. Otherwise, increase μ_1 (Ex: $\mu_1\rightarrow 2\mu_1$)

3) If $\mathbf{b^{I,r}}^T\mathbf{b^{I,r}}=L$ or $\mathbf{b^{I,r}}^T\mathbf{b^{I,r}}\geq\mathbf{b^{II,r-1}}^T\mathbf{b^{II,r-1}}+\beta_2$, (improved solution), where β_2 is a proper positive threshold value (Ex: $L/20$), keep the value of μ_2 same. Otherwise, increase μ_2 (Ex: $\mu_2\rightarrow 2\mu_2$)

4) Solve (10) for $\{\mathbf{W^{II,r}},\mathbf{b^{II,r}}\}$ while fixing $\{\mathbf{W^I},\mathbf{b^I}\}$ as $\{\mathbf{W^{I,r}},\mathbf{b^{I,r}}\}$.

5) If $rank(\mathbf{W}^{\mathbf{II},\mathbf{r}}) = 1$ or $\lambda_1(\mathbf{W}^{\mathbf{II},\mathbf{r}}) \geq \lambda_1(\mathbf{W}^{\mathbf{I},\mathbf{r}}) + \beta_1$ keep the value of μ_1 same. Otherwise, increase μ_1.
6) If ${\mathbf{b}^{\mathbf{II},\mathbf{r}}}^T \mathbf{b}^{\mathbf{II},\mathbf{r}} = L$ or ${\mathbf{b}^{\mathbf{II},\mathbf{r}}}^T \mathbf{b}^{\mathbf{II},\mathbf{r}} \geq {\mathbf{b}^{\mathbf{I},\mathbf{r}}}^T \mathbf{b}^{\mathbf{I},\mathbf{r}} + \beta_2$, keep the value of μ_2 same. Otherwise, increase μ_2.
7) If $r = r_1$ where r_1 is a certain maximum number of iterations for Phase 1, go to the Phase 2.
Phase 2:
8) Set $\mathbf{b}^{\mathbf{I}} = \mathbf{b}^{\mathbf{II}}$ as binary vectors obtained from $\mathbf{b}^{\mathbf{II},\mathbf{r_1}}$ by quantizing it such that the largest L values are set to one whereas the rest are zero. Repeat Phase 1 without antenna selection where $\mathbf{b}^{\mathbf{I}}$ and $\mathbf{b}^{\mathbf{II}}$ are now fixed.
9) Terminate if the maximum iteration number, r_2, is reached or rank one solution is obtained. Take the beamforming weight vector as the principal eigenvector of $\mathbf{W}^{\mathbf{I}}$ or $\mathbf{W}^{\mathbf{II}}$ depending on the termination.

It can be easily shown that the worst case complexity of MBAS at each iteration using interior point methods is $O(\sqrt{GM + M + 1}\, log(1/\epsilon))$ iterations where ϵ is the accuracy of the solution at termination [1]. Simulation results show that the average number of total semidefinite programming (SDP) problems for MBAS algorithm is less than 15. Hence the complexity of the proposed algorithm is less than the alternatives [3].

5. SIMULATION RESULTS

In this part, proposed method, MBAS, is evaluated and its performance is compared with the method in [3]. Both methods are implemented with a convex programming solver CVX [17]. Rayleigh fading channels with unit variances are considered. The total number of antennas is $M = 16$. SINR threshold and noise variance for each user are the same and taken as $\gamma_i = 1$ and $\sigma_i^2 = 1$ respectively in accordance with [3]. The average of 100 random channel realizations is presented for each experiment. The parameters of the algorithm, MBAS, are selected as $P = 100$, $\beta_1 = P/20 = 5$, $\beta_2 = L/20$, $r_1 = 2$, $r_2 = 50$. Initial values of μ_1 and μ_2 are taken as $\mu_1 = \mu_2 = 0.001$. Proposed method returned rank=1 solution for all the experiments even though there is no guarantee for such an outcome.

In the first experiment, single group multicasting scenario is considered where there are $N = 20$ users. L out of $M = 16$ antennas are selected. In fixed antenna case, $M = L$ antennas are used. As it is seen in Fig. 1, proposed method results significantly less power compared to [3]. The difference increases by the number of selected antennas and approaches almost 2.5 dB.

In the second experiment, two-group multicasting scenario is considered where there are 10 users in each multicast group. Fig. 2 shows the total transmitted power for different cases and methods. Proposed method performs significantly better than [3]. The difference between two methods is approximately 3 dB. Proposed method for fixed antenna case performs better than [3] even when it uses the antenna selection scheme. Antenna selection is effective especially when the number of selected antennas is relatively small.

In Fig.3, average number of SDP problems for the proposed algorithm is presented for different number of users and selected antennas, L, by averaging 100 different channel realizations. It can be seen that the average number of SDP problems is almost independent of L. When there are $N = 40$ users, there is a slight increase in the number of SDP's. For all scenarios, the average number of SDP's is less than 15, which shows that the algorithm terminates in a small number of iterations.

6. CONCLUSION

In this paper, joint multicast beamforming with antenna selection is considered. An equivalent biconvex formulation is obtained by embedding the rank condition and antenna selection such that exact penalty approach is employed to obtain an effective solution. The equivalent problem is solved iteratively by using alternating maximization where a convex problem is solved at each iteration. This approach is guaranteed to converge and has been shown to perform significantly better than the alternatives.

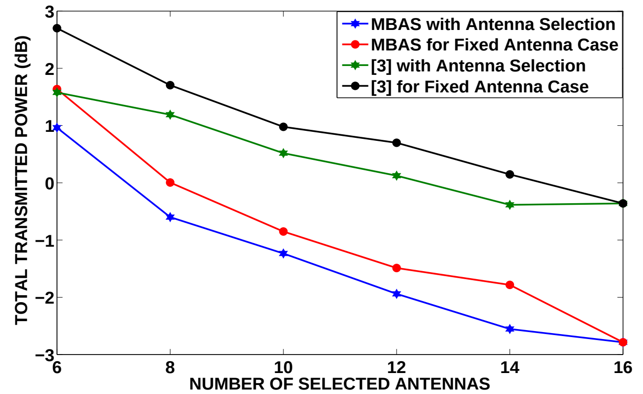

Fig. 1. Total transmitted power versus number of selected antennas for single group multicasting scenario.

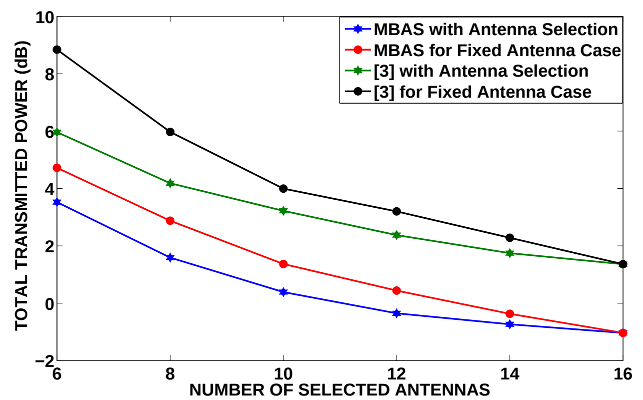

Fig. 2. Total transmitted power versus number of selected antennas for two-group multicasting scenario.

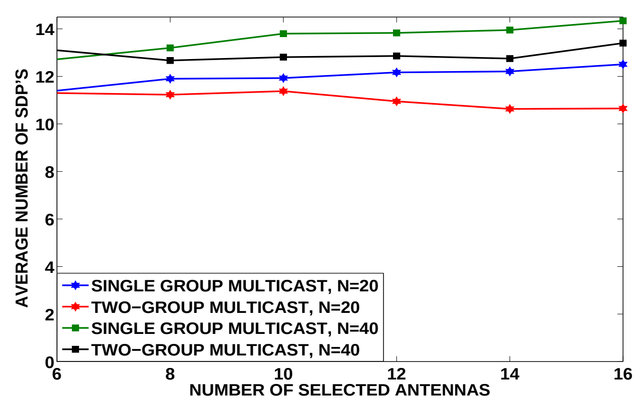

Fig. 3. Average number of SDP's versus number of selected antennas.

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