Generalised Array Reconfiguration for Adaptive Beamforming by Antenna Selection

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Abstract—In this paper, we consider antenna selection and array reconfiguration in the presence of multiple interferences based on the spatial correlation coefficient (SCC) which characterizes the spatial separation between the desired signal and interference subspace. Minimizing the SCC increases the separation between these two subspaces and leads to enhanced beamforming performance. We formulate this problem as a difference of two concave functions, which we solve through the convex-concave procedure (CCP). We derive the lower bound of the SCC as a function of the number of selected antennas which permits us to determine the required number for achieving the desired performance. We suggest two algorithms for implementing the antenna selection and present simulation results to validate the effectiveness of the proposed strategy.

Index Terms—Antenna selection, Difference of convex functions, Correlation Measurements, Difference of convex sets.

I. INTRODUCTION

Multiple antenna receivers are effective tools for interference nulling, as they are capable of spatial filtering, making it possible to receive a desired signal from a particular direction while simultaneously blocking interferences from other directions [1], [2]. The performance of adaptive beamforming is not only dependent on the implemented algorithm, but also the array geometry [3]. Since each antenna requires a separate receiver, the overall cost of a large array is dominated by the cost of active elements and may become prohibitively expensive. It is therefore important to maximize the beamforming performance for a given number of antennas (that is cost) by adapting the array geometry. In order to achieve practical array reconfigurability, we propose in this work a strategy of selecting a subarray over a full layout using Radio Frequency (RF) switches.

The problem of antenna selection for adaptive beamforming was considered in [4], [5], but that work dealt only with the single interference case, which significantly limits its range of practical applications. In this paper, we consider antenna selection and array reconfiguration in the general case of multiple interferences. We derive the spatial correlation coefficient (SCC), which characterizes the spatial separation between the desired signal and interference subspace. Under the assumption of strong interferences, the output signal to interference plus noise ratio (SINR_{out}) is determined by the

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squared SCC value with a fixed number of antennas. In order to make the antenna selection based on the SCC in the multiinterference case tractable, we express the SCC as a fraction of two matrix determinants. Then the objective function in terms of the SCC becomes a difference of two concave functions, which is non-convex optimization. We solve this problem by utilising the convex-concave procedure (CCP) [6]. We first obtain the lower bound of the optimum SCC value and use it to determine the number of selected antennas that gives the desired trade-off between the performance and cost.

Having calculated the suitable number of antennas to be selected, we need to develop a polynomial-time algorithm for solving antenna selection problem. Usually, the binary constraint, $\mathbf{x} \in \{0, 1\}^N$, is replaced by a convex interval, $\mathbf{x} \in [0, 1]^N$, and various strategies are applied to obtain approximate binary solutions. In [7] for instance, the largest K entries are set to one and a local search is employed to find the global optimum. In [8], a reweighted l_1 -norm is utilised to promote the solution sparseness. Other centralized and distributed methods have also been proposed in [9] to solve this binary constrained problem. Although the resulting solutions are sparse, these methods do not guarantee binary entries. In this paper, we adapt the Difference of Convex Sets (DCS) and Correlation Measurement (CM) methods of [4] to obtain solutions for the multi-interference case.

The remainder of the paper is organized as follows: In section 2, we derive generalized expressions of the SCC, with the second incorporating antenna selection. In section 3, we formulate the lower bound of the SCC and suggest two optimization algorithms for solving selection problem. In section 4 we present simulation results, while the last section gives some concluding remarks.

II. GENERALIZED SPATIAL CORRELATION COEFFICIENT

In addition to single interference cases [10], we formulate the SCC to deal with multiple interferences in what follows.

A. Matrix-Vector Expression

Let the direction of arrival (DOA) be specified by (θ, ϕ) with θ and ϕ being the elevation and azimuth angles respectively. Then the **u**-space DOA parameter corresponding to (θ, ϕ) is defined as $\mathbf{u} = [\cos \theta \cos \phi \ \cos \theta \sin \phi]^T$, where T is the transpose operation. Assume the number of antennas is N and that the matrix $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_N]^T \in \mathbb{R}^{N \times 2}$ contains the coordinates $\mathbf{p}_n = [x_n, y_n]^T$ of the antenna elements for $n = 1, \dots, N$. Suppose the desired signal has a DOA \mathbf{u}_s and that a total of L interferences have DOAs $\mathbf{u}_j, j = 1, \dots, L$ respectively. Then the spatial steering vectors are

$$\mathbf{s} = e^{jk_0 \mathbf{P} \mathbf{u}_s}, \quad \mathbf{v}_j = e^{jk_0 \mathbf{P} \mathbf{u}_j}, j = 1, \cdots, L, \tag{1}$$

where $k_0 = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. Under the assumption that the interferences are uncorrelated with one another and with the noise, the covariance matrix of the interference plus noise becomes,

$$\mathbf{R}_n = \sigma^2 \mathbf{I} + \sum_{j=1}^L \sigma_j^2 \mathbf{v}_j \mathbf{v}_j^H, \qquad (2)$$

where $\mathbf{I} \in \mathbb{R}^{N \times N}$ is the identity matrix and σ^2 the thermal noise power and $\sigma_j^2, j = 1, \dots, L$ the power of the *j*th interference. Now we arrange the desired signal and *L* interference steering vectors into the matrices,

$$\mathbf{V}_I = [\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L], \ \mathbf{V}_s = [\mathbf{s}, \mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_L].$$
(3)

Putting $\Sigma_I = \text{diag}[\sigma_1^2, \sigma_2^2, \cdots, \sigma_L^2]$, we can write the interference plus noise covariance matrix into a more concise form,

$$\mathbf{R}_n = \sigma^2 \mathbf{I} + \mathbf{V}_I \boldsymbol{\Sigma}_I \mathbf{V}_I^H. \tag{4}$$

Applying the Woodbury matrix identity to the inverse noise covariance matrix, \mathbf{R}_n , yields,

$$\mathbf{R}_{n}^{-1} = \frac{1}{\sigma^{2}} (\mathbf{I} - \frac{1}{\sigma^{2}} \mathbf{V}_{I} (\boldsymbol{\Sigma}_{I}^{-1} + \frac{1}{\sigma^{2}} \mathbf{V}_{I}^{H} \mathbf{V}_{I})^{-1} \mathbf{V}_{I}^{H}).$$
(5)

Now let us assume that the interferences are much stronger than white noise, i.e. $\sigma_j^2 \gg \sigma^2, \forall j$, then Eq. (5) can be further simplified as

$$\mathbf{R}_n^{-1} \approx \frac{1}{\sigma^2} (\mathbf{I} - \mathbf{V}_I (\mathbf{V}_I^H \mathbf{V}_I)^{-1} \mathbf{V}_I^H).$$
(6)

We see that, when the interference to noise ratio (INR) is high, \mathbf{R}_n^{-1} approximates the interference nullspace. Accordingly, the optimum minimum variance distortionless response (MVDR) adaptive beamforming filter, [11],

$$\mathbf{w}_{\text{opt}} = \gamma \mathbf{R}_n^{-1} \mathbf{s},\tag{7}$$

becomes the interference eigencanceler proposed in [12]. Here γ is a constant that does not affect the SINR_{out}. The relationship between the optimum beamforming filter \mathbf{w}_{opt} , the interference subspace \mathbf{V}_I and the interference nullspace \mathbf{R}_n^{-1} is shown in Fig. 1. The steering vector of the desired signal \mathbf{s} can be decomposed into two orthogonal components: the interference subspace component \mathbf{s}_v and the nullspace component \mathbf{s}_n , i.e. $\mathbf{s} = \mathbf{s}_v + \mathbf{s}_n$ with $\mathbf{s}_v = (\mathbf{V}_I (\mathbf{V}_I^H \mathbf{V}_I)^{-1} \mathbf{V}_I^H) \mathbf{s}$ and $\mathbf{s}_n = (\mathbf{I} - \mathbf{V}_I (\mathbf{V}_I^H \mathbf{V}_I)^{-1} \mathbf{V}_I^H) \mathbf{s}$ respectively. The optimum beamforming filter, i.e. eigencancelor weight vector \mathbf{w}_{opt} , is along the interference nullspace direction \mathbf{s}_n . We define the SCC as the absolute value of the cosine of the angle between the desired signal \mathbf{s} and interference subspace component \mathbf{s}_v ,

$$|\alpha| = |\cos\vartheta| = \left|\frac{\mathbf{s}^H \mathbf{s}_v}{\|\mathbf{s}\|_2 \|\mathbf{s}_v\|_2}\right|.$$
(8)



Fig. 1. The relationship between the optimum beamforming filter \mathbf{w}_{opt} , the interference subspace \mathbf{V}_I and the nullspace \mathbf{R}_n^{-1} .

Here the length of **s** is $\|\mathbf{s}\|_2 = \sqrt{N}$ under the assumption of isotropic antennas. Since the SINR_{out} is directly related to the squared value of the SCC, substituting the expression of \mathbf{s}_v into Eq. (8) and taking the squared value yields

$$|\alpha|^{2} = \left|\frac{\mathbf{s}^{H}\mathbf{s}_{v}}{\sqrt{N}\|\mathbf{s}_{v}\|_{2}}\right|^{2} = \frac{\left|\mathbf{s}^{H}\mathbf{V}_{I}(\mathbf{V}_{I}^{H}\mathbf{V}_{I})^{-1}\mathbf{V}_{I}^{H}\mathbf{s}\right|^{2}}{N\|\mathbf{V}_{I}(\mathbf{V}_{I}^{H}\mathbf{V}_{I})^{-1}\mathbf{V}_{I}^{H}\mathbf{s}\|_{2}^{2}},$$
$$= \frac{1}{N}\mathbf{s}^{H}\mathbf{V}_{I}(\mathbf{V}_{I}^{H}\mathbf{V}_{I})^{-1}\mathbf{V}_{I}^{H}\mathbf{s}.$$
(9)

Finally, the SINR_{out} becomes

$$\operatorname{SINR}_{\operatorname{out}} = \sigma_s^2 \mathbf{s}^H \mathbf{R}_n^{-1} \mathbf{s} = \operatorname{SNR} \cdot N(1 - |\alpha|^2), \qquad (10)$$

where σ_s^2 denotes the power of the desired signal and SNR is the signal-to-noise ratio. Eq. (10) shows that the SINR_{out} of the interference eigencancelor depends on two factors: the number of available antennas N and the squared SCC value $|\alpha|^2$. When the number of selected antennas is fixed, the performance can be improved by changing the array configuration to reduce the SCC value. Thus the SCC characterizes the effect of the array geometry on the beamforming performance and is an effective metric for optimum subarray selection.

B. Determinant Expression

The matrix-vector expression of the SCC given in Eq. (9) is not a convenient form for antenna selection. Thus we derive here another compact formula of the SCC in terms of matrix determinants.

Let the interference cross-correlation matrix $\mathbf{D}_I \in \mathbb{C}^{L \times L}$ be

$$\mathbf{D}_{I} = \mathbf{V}_{I}^{H} \mathbf{V}_{I}$$

$$= \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1L} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2L} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{L1} & \rho_{L2} & \cdots & \rho_{LL} \end{bmatrix}, \quad (11)$$

where the entry $\rho_{ij} = \mathbf{v}_i^H \mathbf{v}_j$ for i, j = 1, ..., L. The desired signal plus interferences cross-correlation matrix $\mathbf{D}_s \in \mathbb{C}^{(L+1)\times(L+1)}$ is

$$\mathbf{D}_{s} = \begin{bmatrix} \rho_{ss} & \rho_{s1} & \cdots & \rho_{sL} \\ \rho_{1s} & \rho_{11} & \cdots & \rho_{1L} \\ \rho_{2s} & \rho_{21} & \cdots & \rho_{2L} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{Ls} & \rho_{L1} & \cdots & \rho_{LL} \end{bmatrix}, \qquad (12)$$

where the entry $\rho_{sj} = \mathbf{s}^H \mathbf{v}_j$ for j = 1, ..., L. Let \mathbf{C}_I , the cofactor matrix of \mathbf{D}_I , be

$$\mathbf{C}_{I} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1L} \\ C_{21} & C_{22} & \cdots & C_{2L} \\ \cdots & \cdots & \cdots & \cdots \\ C_{L1} & C_{L2} & \cdots & C_{LL} \end{bmatrix}, \quad (13)$$

where C_{ij} is the cofactor of $\rho_{ij}, i, j = 1, ..., L$. Then the inverse of \mathbf{D}_I can be expressed as

$$\mathbf{D}_{I}^{-1} = \frac{1}{|\mathbf{D}_{I}|} \mathbf{C}_{I}^{T},\tag{14}$$

where $|\mathbf{D}_I|$ means determinant of the matrix \mathbf{D}_I . The determinant of \mathbf{D}_s can be written as

$$\begin{aligned} |\mathbf{D}_{s}| &= \rho_{ss}|\mathbf{D}_{I}| + \sum_{i=1}^{L} \rho_{si}C_{si} \\ &= \rho_{ss}|\mathbf{D}_{I}| - \sum_{i=1}^{L} \sum_{j=1}^{L} \rho_{si}C_{ji}\rho_{js} \\ &= \rho_{ss}|\mathbf{D}_{I}| - \mathbf{s}^{H}\mathbf{V}_{I}\mathbf{C}_{I}^{T}\mathbf{V}_{I}^{H}\mathbf{s}, \end{aligned}$$
(15)

where C_{si} is the cofactor of ρ_{si} for i = 1, ..., L. Here we have also used the fact that $|\mathbf{D}_I|$ is the cofactor of ρ_{ss} . Thus we have that

$$\mathbf{s}^{H}\mathbf{V}_{I}\mathbf{C}_{I}^{T}\mathbf{V}_{I}^{H}\mathbf{s} = \rho_{ss}|\mathbf{D}_{I}| - |\mathbf{D}_{s}| = N|\mathbf{D}_{I}| - |\mathbf{D}_{s}|.$$
(16)

Substituting Eqs. (11) and Eq. (14) into Eq. (9) yields,

$$|\alpha|^{2} = \frac{1}{N} \mathbf{s}^{H} \mathbf{V}_{I} \mathbf{D}_{I}^{-1} \mathbf{V}_{I}^{H} \mathbf{s} = \frac{1}{N|\mathbf{D}_{I}|} \mathbf{s}^{H} \mathbf{V}_{I} \mathbf{C}_{I}^{T} \mathbf{V}_{I}^{H} \mathbf{s}, \quad (17)$$

Proceeding to substitute Eq. (16) into Eq. (17), the expression of the SCC can be rewritten as

$$|\alpha|^2 = \frac{1}{N|\mathbf{D}_I|} \mathbf{s}^H \mathbf{V}_I \mathbf{C}_I^T \mathbf{V}_I^H \mathbf{s} = 1 - \frac{|\mathbf{D}_s|}{N|\mathbf{D}_I|}.$$
 (18)

The SCC expression of the multi-interference case shown in Eq. (9) reduces for L = 1 to that of the single interference case given by Eq. (11) in [4]. Furthermore, when these interferences are mutually orthogonal, i.e., $\mathbf{v}_i^H \mathbf{v}_j = 0, i, j = 1, \dots, L, i \neq j$, we have

$$|\alpha|^{2} = \frac{1}{N^{2}} \mathbf{s}^{H} \mathbf{V}_{I} \mathbf{V}_{I}^{H} \mathbf{s} = \sum_{i=1}^{L} \frac{|\mathbf{s}^{H} \mathbf{v}_{i}|^{2}}{(\mathbf{s}^{H} \mathbf{s})(\mathbf{v}_{i}^{H} \mathbf{v}_{i})} = \sum_{i=1}^{L} |\alpha_{i}|^{2}, \quad (19)$$

due to the fact that $\mathbf{V}_{I}^{H}\mathbf{V}_{I} = N\mathbf{I}$. Here α_{i} is the SCC value between the desired signal and the *i*th interference. Thus the squared SCC value in the multi-interference case is the sum of squared SCC value of each interference in the special scenario.

III. ANTENNA SELECTION FOR ADAPTIVE BEAMFORMING

In this section, we first incorporate antenna selection into the SCC expression and then formulate the lower bound of the SCC in order to determine the number of selected antennas. Finally, we give two algorithms for antenna selection.

A. SCC with antenna selection

We implement antenna selection on the derived SCC parameter. Define the binary selection vector $\mathbf{x} \in \{0, 1\}^N$ with "one" meaning the corresponding antenna is selected and "zero" meaning discarded. Then, the two cross-correlation matrices of the selected subarray can be expressed as

$$\mathbf{D}_{I}(\mathbf{x}) = \mathbf{V}_{I}^{H} \operatorname{diag}(\mathbf{x}) \mathbf{V}_{I}, \ \mathbf{D}_{s}(\mathbf{x}) = \mathbf{V}_{s}^{H} \operatorname{diag}(\mathbf{x}) \mathbf{V}_{s}.$$
 (20)

Thus, the SCC of the selected subarray can be written as

$$|\alpha|^{2} = 1 - \frac{|\mathbf{D}_{s}(\mathbf{x})|}{K|\mathbf{D}_{I}(\mathbf{x})|} = 1 - \frac{|\mathbf{V}_{I}^{H}\operatorname{diag}(\mathbf{x})\mathbf{V}_{I}|}{K|\mathbf{V}_{s}^{H}\operatorname{diag}(\mathbf{x})\mathbf{V}_{s}|}, \quad (21)$$

where K is the number of selected antennas. Thus the antenna selection problem in terms of minimizing the SCC is

min
$$1 - \frac{|\mathbf{D}_s(\mathbf{x})|}{K|\mathbf{D}_I(\mathbf{x})|},$$

s.t. $\mathbf{x} \in \{0, 1\}^N, \ \mathbf{1}^T \mathbf{x} = K.$ (22)

where $\mathbf{1} \in \mathbb{R}^N$ with all entries being one. Since both $\mathbf{D}_I(\mathbf{x})$ and $\mathbf{D}_s(\mathbf{x})$ are positive definite and the logarithm function is monotonically increasing, Eq. (22) is equivalent to the following problem by ignoring the constant term K,

min
$$\log(|\mathbf{D}_I(\mathbf{x})|) - \log(|\mathbf{D}_s(\mathbf{x})|),$$

s.t. $\mathbf{x} \in \{0, 1\}^N, \ \mathbf{1}^T \mathbf{x} = K.$ (23)

The objective function of Eq. (23) is the difference of two concave functions and belongs to D.C. Programming [13].

B. Lower Bound on Optimal SCC

Let us define the feasible set $S = {\mathbf{x} \in {\{0,1\}}^N : \mathbf{1}^T \mathbf{x} = K}$, which comprises the extreme points of the polytope $\mathcal{D} = {\mathbf{x} : 0 \leq \mathbf{x} \leq 1, \mathbf{1}^T \mathbf{x} = K}$. In order to obtain a lower bound of the optimal SCC value, we relax the binary constraints by replacing the feasible set S by the polytope \mathcal{D} , i.e.

min
$$\log(|\mathbf{D}_I(\mathbf{x})|) - \log(|\mathbf{D}_s(\mathbf{x})|),$$

s.t. $\mathbf{x} \in \mathcal{D}.$ (24)

According to [14], [15], the global optimum solution of a D.C. programming is on the edge of the polytope \mathcal{D} , which is sparse but not necessarily binary. A convex-concave procedure (CCP) is adopted here to solve the D.C. Programming [16], which is proven to converge to a KKT solution in [17]. Now, the concave function $f(\mathbf{x}) = \log(|\mathbf{D}_I(\mathbf{x})|)$ is approximated iteratively by its first-order Taylor decomposition as

$$f(\mathbf{x}) \approx \hat{f}(\mathbf{x}) = f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T (\mathbf{x} - \mathbf{x}^k).$$
(25)

The *j*th entry of the gradient $\nabla f(\mathbf{x}^k)$ is

$$\nabla f_j(\mathbf{x}^k) = \operatorname{tr}\left\{\mathbf{D}_I^{-1}(\mathbf{x}^k)(\mathbf{V}_I^j\mathbf{V}_I^{jH})\right\},\tag{26}$$

here the operator tr $\{\bullet\}$ takes the trace of the matrix \bullet and \mathbf{V}_I^j is the *j*th row of \mathbf{V}_I . Thus the lower bound of the optimal SCC

value can be obtained by a sequence of convex optimizations, where the kth iteration is,

min
$$\nabla f(\mathbf{x}^k)^T \mathbf{x} - \log(|\mathbf{D}_s(\mathbf{x})|),$$

s.t. $\mathbf{x} \in \mathcal{D}.$ (27)

The termination condition can be chosen as the distance between two successive solutions, i.e. $\|\mathbf{x}^{k+1} - \mathbf{x}^k\|_2$, less than a predetermined threshold value. After obtaining the lower bound of the SCC, the upper bound of the SINR_{out} can be calculated through Eq. (10). A trade-off curve between the performance and the cost can then be plotted to determine the suitable number K as shown in [4]. Subsequently, both the CM and DCS methods in [4] can be modified and adapted to select an optimum subset of K antennas here. Let us take the DCS as an example, the problem is formulated as,

min
$$\nabla f(\mathbf{x}^k)^T \mathbf{x} - \log(|\mathbf{D}_s(\mathbf{x})|) + \mu(\mathbf{1} - 2\mathbf{x}^k)^T \mathbf{x},$$

s.t. $\mathbf{x} \in \mathcal{D}.$ (28)

Here μ is a trade-off parameter that compromises between the minimization of SCC value and the solution sparseness. For the CM method, all antennas are switched on initially and a backward search is implemented to switch off the antenna that gives the largest SCC value in Eq. (21) in each iteration until K antennas remain.

IV. SIMULATION RESULTS

In the following, we present simulation results to show the advantage of our approach. The desired signal is fixed at $\theta_s = 0.1\pi, \phi_s = 0.2\pi$ radian. We consider two scenarios: In the first, the desired signal is close to the interference subspace, whereas in the second, the two are well separated. In the first scenario, the first and second interferences are arriving from $\theta_1 = 0.15\pi, \phi_1 = 0.25\pi$ radian and $\theta_2 = 0.2\pi, \phi_2 = 0.3\pi$ radian. In the second scenario, the two interferences arrive from $\theta_1 = 0.3\pi, \phi_1 = 0.4\pi$ radian and $\theta_2 = 0.35\pi, \phi_2 = 0.3\pi$ radian. We adopt a 4×4 square array as the full layout and present the trade-off curve between the output SINR and cost in Fig. 2 to illustrate the determination of the number of selected antennas. The number K is changing from 3 to 16 in steps of 1. We calculate the normalised output SINR and computational cost by taking the entire full array as a reference, where the computational cost is of order K^3 . Observe that using an 8-antenna subarray saves 87.5% of computational cost with only 1dB performance degradation in the close scenario and 2.8dB SINR loss in the far scenario. This is a significant saving in computational load for a modest performance loss. Note that this does not take into account the additional hardware saving due to the reduction in the number of front ends, which is equal to the reduction in the number of antennas.

Next, we select 10 antennas from a 20-antenna uniform linear array for enhanced interference nulling. The desired signal arrives from 60° in elevation with SNR being -20dB and four interferences coming from $45^{\circ}, 55^{\circ}, 65^{\circ}, 70^{\circ}$ respectively with INR all being 30dB. We select two optimum subarrays through



Fig. 2. The trade-off curve between the performance and the computational cost for both far and close scenarios.



Fig. 3. The structure of three 10-antenna subarrays.



Fig. 4. MVDR beampatterns of three 10-antenna subarrays as shown in Fig. 3. The number of time snapshots is 100 and 1000 Monte Carlo runs are used.

minimizing the proposed subspace based SCC and the sum of SCC values in Eq. (19) ignoring the fact that the interferences are not orthogonal with each other as shown in Fig. 3. We also select a third subarray with two antennas fixed at two ends and other eight randomly spaced in between for comparison. The MVDR beampatterns of the three arrays are shown in Fig. 4 by averaging 1000 Monte-Carlo simulations. We can see that the first subarray, denoted as "sub₁", produces deeper nulls than the other two subarrays, but exhibits nearly same mainlobe width and peak sidelobe level.

V. CONCLUSION

In this paper, we studied reconfigurable adaptive antenna arrays by antenna selection in multi-interference cases. We generalized the SCC and formulated the problem as a difference of two concave functions. We then employed the CCP to solve the problem and suggested two combinatorial optimization algorithms to select optimum antennas. Simulation results validate the effectiveness of the proposed method.

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