Direction-Finding Based on The Theory of Super-Resolution in Sparse Recovery Algorithms

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***Abstract*— The problem of recovering directions-of-arrival in the sparse signal model with multiple snapshots is considered. Based on the theory of super resolution, multiple snapshots are used to jointly estimate directions-of-arrival in the continuous domain. Instead of uniformly discretizing the search range, interpolation preprocessing on the estimated super-resolution directions is suggested leading to a sparse convex optimization formulation. Moreover, a first order iterative algorithm is employed to reduce the computational time. A good selection of regularization parameter is guaranteed via the modified generalized cross validation (GCV). Numerical results demonstrate the performance of the proposed methods.**

***Index Terms*— Directions of Arrival, Super Resolution, Sparsity, Multiple Measurement Vectors, Generalized Cross Validation**

I. Introduction

A common goal of array signal processing for Directions-of-arrival (DoA) estimation is to locate closely-spaced signals in the presence of high-variance noise and low number of snapshots. Two well-known examples of by now classical high-resolution DoA estimators are the multiple signal classification (MUSIC) [1], and the minimum variance distortionless response (MVDR) [2] methods.

Compressed sensing and sparse representation of signals have inspired sparse recovery algorithms for source localization based on single observation vector [3]. Moreover, multiple measurement vectors (MMV) [4] with common sparsity pattern are exploited to obtain solutions in sparse models. In these algorithms, the search range of directions is discretized into a grid in which the actual DoAs most likely will not belong. This leads to the so-called off-grid effect, which might cause their performance to be substantially degraded. In [5], the off-grid error is linearized in a sparse spatial spectral model and solved by an alternating Lasso approach (the references in this paper provide several other approaches for dealing with the off-grid error). In [6], [7], a mathematical theory of super-resolution (SR) is proposed to potentially estimate signals correctly in the continuous domain in which the off-grid effect is neglected to avoid the degradation of estimation in the discrete domain. The above problems can be formulated as a least square function plus a penalty term with a regulariztion parameter. However, regularization parameters are hard to be chosen. There are several automatic ways to select a good regulariztion parameter in the literature. For example, the generalized cross validation (GCV) [8] is a popular technique to select regularization parameters without any prior information, such as the noise variance. In [9], an automatic regularization parameter selector is proposed based on the probabilistic recovery estimator with finite snapshots.

In this work, the super-resolution theory is applied to the DoA estimation problem with multiple measurement vectors (MMV). The multiple source signals with common sparsity pattern of source locations is treated as a summation of weighted superposition of spikes and any of two spikes have to obey the minimum distance defined in [6]. Then, the joint estimation of DoAs is obtained by reformulating the objective function of the optimization problem in semidefinite programming form. Instead of the descretized grid search, interpolation preprocessing with the prior information of minimum distance are utilized to add more likely candidates of directions into the support set of DoA, and then a L_1-norm minimization problem is formulated in matrix form. Furthermore, a modified GCV (MGCV) is proposed for MMV models. In order to reduce the computational complexity of the approach, a fast first order iterative algorithm is employed with an adaptive MGCV-based regularization parameter selector. The peformance of the proposed methods are compared by simulations with MUSIC and Cramer-Rao Lower Bound (CRLB).

Notation: $diag(\mathbf{x})$ represents a diagonal square matrix with the diagonal vector $\mathbf{x}$. $(\mathbf{X})_{i,j} = x_{i,j}$ represents the entry of matrix $\mathbf{X}$ at row i and column j. The L_1-norm of a matrix $\mathbf{X}$ is defined as $\|\mathbf{X}\|_1 = \sum_i \sum_j |x_{i,j}|$. $\|\mathbf{X}\|_F$ means the Frobenius norm. $\sigma_{max}(\mathbf{X})$ is the maximum singular value of matrix $\mathbf{X}$. $trace(\mathbf{X})$ takes the sum of diagonal entries of matrix $\mathbf{X}$. Denote $|\mathcal{T}|$ as the cardinality of the set $\mathcal{T}$. $sgn(x)$ takes the sign of any real number x. $|x|$ takes the absolute value of any real number x. $\mathbb{R}_+$ denotes the set of positive real numbers.

II. Problem Formulation

Consider an array of M sensors and suppose that there are K far-field narrowband sources impinging on the array from angles $\theta_1, \ldots, \theta_K$. The observation vector $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T \in \mathbb{C}^{M\times 1}$ at time t is modeled as

$$\mathbf{y}(t) = \mathbf{G}\mathbf{x}(t) + \mathbf{n}(t), t = 1\ldots, T, \quad (1)$$

where the measurement matrix $\mathbf{G} = [\mathbf{g}(\theta_1), \cdots, \mathbf{g}(\theta_L)] \in \mathbb{C}^{M\times L}$ is composed of the steering vectors $\{\mathbf{g}(\theta_i) = [e^{-j(-(M-1)/2)2\pi\frac{d}{\lambda}sin\theta_i}, \ldots, e^{-j((M-1)/2)2\pi\frac{d}{\lambda}sin\theta_i}]^T\}_{i=1}^{L}$ with wavelength λ, and $\mathbf{n}$ is i.i.d. white Gaussian noise with $\mathcal{N}(0, \sigma^2 I)$. The vector $\mathbf{x}(t) = [x_1(t), \ldots, x_K(t)]^T \in \mathbb{R}_+^{K\times 1}$ represents the arriving stochastic signal vector with covariance matrix $\mathbf{C}_s$. Denote $\mathcal{T}_\theta = \{sin(\theta_k)\}_{k=1}^K \subset \mathbb{T} = [-1,1]$ as the support set for the sines of the angles of arrival. If $T > 1$ multiple snapshots are considered, we can define the following MMV system as

$$\mathbf{Y} = \mathbf{G}\mathbf{X} + \mathbf{N}, \tag{2}$$

where the obervation matrix $\mathbf{Y} = [\mathbf{y}(1), \ldots, \mathbf{y}(T)] \in \mathbb{C}^{M\times T}$, the source signal matrix $\mathbf{X} = [\mathbf{x}(1), \ldots, \mathbf{x}(T)] \in \mathbb{R}_+^{K\times T}$, and the noise matrix $\mathbf{N} = [\mathbf{n}(1), \ldots, \mathbf{n}(T)] \in \mathbb{R}^{M\times T}$. Since multiple snapshots of vectors in $\mathbf{X}$ share the common sparsity pattern of the support set in the domain $\mathbb{T}$, this property can be exploited to jointly estimate the support set $\mathcal{T}_\theta$.

By applying the super resolution mathematical theory of [6] to the MMV system, we model the continuous domain signal $s(\tau)$ as

$$s(\tau) = \sum_{t=1}^{T}\sum_{k=1}^{K} x_{k,t}\delta_{\tau_k}, \tag{3}$$

where $\tau_k \in \mathbb{T} = [-1,1]$ and $x_{k,t} \in \mathbb{C}$ or $\mathbb{R}, \forall k, t$. δ_{τ_k} is a Dirac delta function at time τ_k. It is noted that $x_{k,t}$ equals to the entry $x_k(t)$ of the matrix $\mathbf{X}$. The Fourier transform of $s(\tau)$ is expressed as

$$z(n) = \int_{-1}^{1} e^{-j2\pi nt} s(d\tau) = \sum_{t=1}^{T}\sum_{k=1}^{K} x_{k,t}e^{-j2\pi n\tau_k} \tag{4}$$
$$= \sum_{t=1}^{T} r_t(n), n = -f_c, \ldots, f_c,$$

where $r_t(n) = \sum_{k=1}^{K} x_{k,t}e^{-j2\pi n\tau_k}$. Thus, the Gaussian noise model can be rewritten in the compact form:

$$\mathbf{z} = \mathbf{F}s + \mathbf{e}, \tag{5}$$

where $\mathbf{z} = [z(-f_c), \ldots, z(f_c)]^T \in \mathbb{C}^{M\times 1}$. By letting $\tau_k = sin(\theta_k), \forall k$ and $f_c = \frac{d}{\lambda}(M-1)/2$, it is easy to show that

$$\mathbf{z} = \mathbf{F}s + \mathbf{e} = \mathbf{G}(\sum_{t=1}^{T}\mathbf{x}(t)) + \sum_{t=1}^{T}\mathbf{n}(t) = \sum_{t=1}^{T}\mathbf{y}(t). \tag{6}$$

In this case of MMV, we desire to estimate the data $x_{k,t}$, and directions $\tau_k, \forall k, t$. The *minimum distance* between two directions is defined as [10]

$$\Delta(\boldsymbol{\theta}) = \min_{\forall \theta_i \neq \theta_j} |sin(\theta_i) - sin(\theta_j)|. \tag{7}$$

If $\Delta(\boldsymbol{\theta}) \geq \frac{2}{f_c} = \frac{4\lambda}{(M-1)d}$, then there exists a unique solution for the total variation minimization problem [7]:

$$\min_{s} \|s\|_{TV} \quad \text{s.t. } \|\mathbf{F}s - \mathbf{z}\|_2 \leq \delta, \tag{8}$$

where $\|s\|_{TV}$ is equal to $\|\mathbf{X}\|_1 = \sum_t \sum_k |x_{k,t}|$. Similarly in [7], the dual problem in semidefinite programming (SDP) form is derived as

$$\max_{\mathbf{u},\mathbf{Q}} \ \text{Re}(\sum_{t=1}^{T} < \mathbf{y}(t), \mathbf{u} >) - \delta\|\mathbf{u}\|_2 \tag{9}$$
$$\text{s.t. } \begin{bmatrix} \mathbf{Q} & \mathbf{u} \\ \mathbf{u} & 1 \end{bmatrix} \succeq 1, \sum_{i=1}^{M-j} \mathbf{Q}_{i,i+j} = \begin{cases} 1, & j = 0, \\ 0, & j = 1, 2, \ldots, M-1 \end{cases},$$

where $\mathbf{Q} \in \mathbb{C}^{M\times M}$ is a Hermitian matrix. According to Lemma 3.1 in [7] and by using the root finding procedure, the estimated support set $\mathcal{T}_\theta^{est} = \{sin(\theta_k^{est})\}_{k=1}^K$. is obtained, and then the measurement matrix $\mathbf{G}_{est}$ is reconstructed.

III. Approach

In this section, methods are proposed to estimate the signal $x_{k,t}, \forall k, t$ with its corresponding direction. First, an interpolation preprocessing on the estimated support set $\mathcal{T}_\theta^{est}$ is performed, and Lasso-type optimization problems are presented. Subsequently, reduced-complexity approaches are developed with an adaptive regularization parameter selector based on the generalized cross validation.

A. Interpolation Preprocessing and Lasso-Type Solvers

Because of the numerical issues of the root finding procedure, the cardinality of $\mathcal{T}_\theta^{est}$ is usually greater than the cardinality of $\mathcal{T}_\theta$, but it might be less when SNR is low or minimum distance is not obeyed. In order to overcome this situation, the minimum distance $\Delta(\boldsymbol{\theta})$ is utilized to increase the likely candidates into the estimated support set $\mathcal{T}_\theta^{est}$. In other words, we discretize the domain $\mathbb{T}$ in terms of the prior information $\Delta(\boldsymbol{\theta})$ and the estimated support set $\mathcal{T}_\theta^{est}$. Given a positive constant $\mu < 1$ and initializing the augmented support set $\tilde{\mathcal{T}}_\theta^{est} = \mathcal{T}_\theta^{est}$, we repeat the following steps for each element τ_i^{est} in $\tilde{\mathcal{T}}_\theta^{est}$ sequentially until each element is checked:

1) When $i = 1 \sim (|\tilde{\mathcal{T}}_\theta^{est}| - 1)$: check

$$\tilde{\mathcal{T}}_\theta^{est} = \tilde{\mathcal{T}}_\theta^{est} \bigcup \{\tau_i^{est} + \Delta(\boldsymbol{\theta})\}, \text{ if } \tau_i^{est} + \mu\Delta(\boldsymbol{\theta}) \leq \tau_{i+1}^{est}$$

2) When $i = 1$: check

$$\tilde{\mathcal{T}}_\theta^{est} = \tilde{\mathcal{T}}_\theta^{est} \bigcup \{\tau_i^{est} - \Delta(\boldsymbol{\theta})\}, \text{ if } \tau_i^{est} - \Delta(\boldsymbol{\theta}) \geq -1$$

3) When $i = |\tilde{\mathcal{T}}_\theta^{est}|$: check

$$\tilde{\mathcal{T}}_\theta^{est} = \tilde{\mathcal{T}}_\theta^{est} \bigcup \{\tau_i^{est} + \Delta(\boldsymbol{\theta})\}, \text{ if } \tau_i^{est} + \Delta(\boldsymbol{\theta}) \leq 1$$

Thus, the cardinality of $\tilde{\mathcal{T}}_\theta^{est}$ must be greater than the cadinality of $\mathcal{T}_\theta$, in which leads to a sparse signal reconstruction problem. By using the paramter μ in case 1, ambiguity of candidates in the augmented support set can be avoided, and the augmented mearsurement matrix $\tilde{\mathbf{G}}_{est}$ can be obtained in terms of $\tilde{\mathcal{T}}_\theta^{est}$. Then, two Lasso-type solvers are proposed.

After interpolation preprocessing, we can formulate the following convex optimization problem

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \frac{1}{2}||\mathbf{Y} - \tilde{\mathbf{G}}_{est}\mathbf{X}||_F^2 + \beta||\mathbf{X}||_1. \tag{10}$$

Furthermore, since each column vector of $\mathbf{X}$ has the same sparsity pattern, another form of convex optimization problem by the notion of Group Lasso [11] can be formulated as

$$\tilde{\mathbf{X}} = \arg\min_{\mathbf{X}} \frac{1}{2}||\mathbf{Y} - \tilde{\mathbf{G}}_{est}\mathbf{X}||_F^2 + \gamma||\mathbf{X}||_{2,1}, \tag{11}$$

where $||\mathbf{X}||_{2,1} = \sum_{k=1}^{K} \|\mathbf{X}_{k,:}\|_2$, and $\mathbf{X}_{k,:}$ denotes the k^{th} row of $\mathbf{X}$.

According to $\tilde{\mathbf{X}}$, the most likely directions of signals is determined from the augmented support set $\tilde{\mathcal{T}}_{\theta}^{est}$. It is noted that the regularization parameters, β and γ, are tuned empirically to attain the best performance.

B. Complexity-Reduction Methods

The computational complexity of solving formulation (10) or (11) might become significantly high when the dimension of matrix $\mathbf{X}$ increases. In order to reduce the computing time, *iterative shrinkage-thresholding algorithm* (ISTA) [12] is applied when only the optimization problem (10) is considered. Let $f(\mathbf{X}) = \frac{1}{2}||\mathbf{Y} - \tilde{\mathbf{G}}_{est}\mathbf{X}||_F^2$. By using the quadratic approximation of $f(\mathbf{X})$ at a given matrix $\mathbf{X}^r$:

$$\begin{aligned} f(\mathbf{X}) \cong & f(\mathbf{X}^r) + trace[(\frac{\partial f(\mathbf{X}^r)}{\partial \mathbf{X}})^H(\mathbf{X} - \mathbf{X}^r)] \\ & + \frac{L}{2}\|\mathbf{X} - \mathbf{X}^r\|_F^2, \end{aligned} \tag{12}$$

where $\frac{\partial f(\mathbf{X}^r)}{\partial \mathbf{X}} = -\tilde{\mathbf{G}}_{est}^H\mathbf{Y} + \tilde{\mathbf{G}}_{est}^H\tilde{\mathbf{G}}_{est}\mathbf{X}^r$ and $L = \sigma_{max}(\tilde{\mathbf{G}}_{est}^H\tilde{\mathbf{G}}_{est})$. Then, (10) is equivalent to

$$\mathbf{X}^{r+1} = \arg\min_{\mathbf{X}} \frac{1}{2}||\mathbf{X} - (\mathbf{X}^r - \frac{1}{L}\mathbf{A})||_F^2 + \frac{\beta}{L}||\mathbf{X}||_1, \tag{13}$$

where $\mathbf{A} = \frac{\partial f(\mathbf{X}^r)}{\partial \mathbf{X}}$. The closed form solution of the above problem is $\forall k, t$,

$$(\mathbf{X}^{r+1})_{k,t} = sgn[(\mathbf{X}^r - \frac{1}{L}\mathbf{A})_{k,t}]\max(|(\mathbf{X}^r - \frac{1}{L}\mathbf{A})_{k,t}| - \frac{\beta}{L}, 0). \tag{14}$$

In [13], another iterative shrinkage method, called *fast iterative shrinkage-thresholding algorithm* (FISTA), is proved to have faster convergence than ISTA. A key function employed in FISTA is proximal operator [14]. Let $h : \mathbb{R}^{K\times T} \to \mathbb{R} \cup \{\infty\}$. The proximal operator of h is denoted by

$$\text{prox}_h(\mathbf{V}) = \arg\min_{\mathbf{X}} \ (h(\mathbf{X}) + \frac{1}{2}\|\mathbf{X} - \mathbf{V}\|_F^2), \tag{15}$$

which is the generalization of (13). If $h(\mathbf{X}) = \beta||\mathbf{X}||_1$, the proximal operator is

$$(\text{prox}_h(\mathbf{V}))_{k,t} = sgn[(\mathbf{V})_{k,t}]\max(|(\mathbf{V}_{k,t})| - \beta, 0), \forall k, t, \tag{16}$$

which is similar to (14). For our problem (10), a FISTA-based algorithm can be similarly to the approach of [13], except the main iterative step is replaced by the matrix form $\mathbf{X}^r = \text{prox}_{\frac{1}{L}h}(\mathbf{Z}^r - \frac{1}{L}\frac{\partial f(\mathbf{Z}^r)}{\partial \mathbf{Z}})$, where $\mathbf{Z}^r$ is a dummy matrix.

C. Modified Generalized Cross Validation (MGCV)

Although the convergence of FISTA is fast, a proper regularization parameter must be selected. The generalized cross validation [15] is a popular method and was proposed in a Lasso model to select a good parameter β by minimizing the function

$$GCV(\beta) = \frac{1}{M}\frac{\|(\mathbf{y} - \mathbf{B}(\beta)\mathbf{y})\|_2^2}{(1 - p(\beta)/M)^2}, \tag{17}$$

where $\mathbf{B}(\beta) = \mathbf{H}(\mathbf{H}^H\mathbf{H} + \beta\mathbf{W}^-)^{-1}\mathbf{H}^H$ and $p(\beta) = trace(\mathbf{B}(\beta))$. $\mathbf{W}^-$ is a generalized inverse of matrix $\mathbf{W}$, where $\mathbf{W} = diag(\{|\tilde{x}_i|\}_{i=1}^K)$ in which $\tilde{x}_i$ is the entry of the solution $\tilde{\mathbf{x}} = (\mathbf{H}^H\mathbf{H} + \beta\mathbf{W}^-)^{-1}\mathbf{H}^H\mathbf{y}$ in the ridge regression form. More details can be found in [15].

For our problem formulation (10), a modified GCV (MGCV) is suggested as

$$MGCV(\beta) = \frac{1}{M}\frac{\|(\mathbf{Y} - \mathbf{C}(\beta)\mathbf{Y})\|_F^2}{(1 - p(\beta)/M)^2}, \tag{18}$$

where $\mathbf{C}(\beta) = \tilde{\mathbf{G}}_{est}(\tilde{\mathbf{G}}_{est}^T\tilde{\mathbf{G}}_{est} + \beta\tilde{\mathbf{W}}^-)^{-1}\tilde{\mathbf{G}}_{est}^H$, $p(\beta) = trace(\mathbf{C}(\beta))$, and $\tilde{\mathbf{W}}^- = \frac{1}{T}\sum_{t=1}^T \tilde{\mathbf{W}}_t^-$, where $\tilde{\mathbf{W}}_t^-$ is a generalized inverse of $\tilde{\mathbf{W}}_t = diag(\{|\tilde{x}_{k,t}|\}_{k=1}^K)$. It is noted that $\tilde{x}_{k,t}$ of $\tilde{\mathbf{X}}$ can be obtained by solving (10) or (11) via CVX tools, or $\tilde{x}_{k,t}^r$ of $\mathbf{X}^r$ is computed for $\tilde{\mathbf{W}}_t = diag(\{|\tilde{x}_{k,t}^r|\}_{k=1}^K)$ when performing the FISTA-based algorithm for (10).

We suggest combining FISTA with MGCV. After the function of MGCV is updated based on the matrix update of $\tilde{\mathbf{W}}^-$ in FISTA, the best estimate of β can be selected adaptively by minimizing the function of MGCV at each iteration. The FISTA-MGCV algorithm for (10) is summarized as follows:

FISTA-MGCV Algorithm:

Input: Initialize $\mathbf{X}^0 = \mathbf{0}$, $\beta_0 = 0$, and the Lipschitz constant of $\frac{\partial f(\mathbf{X}^r)}{\partial \mathbf{X}}$ is $L = \sigma_{max}(\tilde{\mathbf{G}}_{est}^H\tilde{\mathbf{G}}_{est})$

Step 0:Take $\mathbf{Z}^1 = \mathbf{X}^0$, $t_1 = 1$, $\beta_1 = \beta_0$

Step $\mathbf{r}$:$(r \geq 1)$ Compute

$$\mathbf{X}^r = \text{prox}_{\frac{1}{L}h}(\mathbf{Z}^r - \frac{1}{L}\frac{\partial f(\mathbf{Z}^r)}{\partial \mathbf{Z}}) \tag{19}$$

Update $\tilde{\mathbf{W}}^-$in terms of $\mathbf{X}^r$ (20)

$$\beta_{r+1} = \arg\min_{\beta} MGCV(\beta) \tag{21}$$

$$t_{r+1} = \frac{1 + \sqrt{1 + 4t_r^2}}{2} \tag{22}$$

$$\mathbf{Z}^{r+1} = \mathbf{X}^r + \frac{t_r - 1}{t_{r+1}}(\mathbf{X}^r - \mathbf{X}^{r-1}) \tag{23}$$

Until some stopping criteria is satisfied.

Note that the proximal operator in the algorithm is $\text{prox}_{\frac{1}{L}h}(\mathbf{V}^r) = sign[\mathbf{V}^r]\max(|\mathbf{V}^r| - \frac{\beta_r}{L}, 0)$ for the input matrix $\mathbf{V}^r$.

IV. Numerical Results

In these simulations, the proposed joint super-resolution DoA estimation by FISTA-MGCV (JSR-FISTA-MGCV) is compared with MUSIC and CRLB. In a MMV system with the super-resolution view, the minimum distance $\Delta(\boldsymbol{\theta})$ is set to 0.1 so that $M = 81$ sensors is needed. The large number of sensors is used for illustration to effect the arbitrarily-selected 0.1 minimum separation required by super-resolution theory. At time t, there are $K = 2$ positive-valued uncorrelated Gaussian source signals which are located at $sin(\boldsymbol{\theta}) = [sin(\theta_1), sin(\theta_2)]$, where two entries are randomly generated

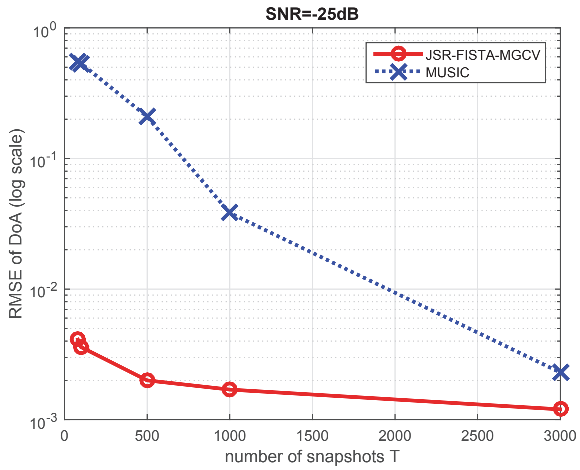

Fig. 1. RMSE of DoA estimation versus Number of snapshots of the proposed methods and MUSIC.

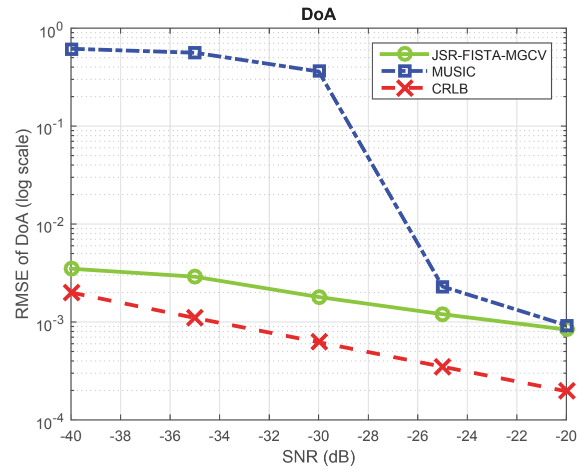

Fig. 2. RMSE of DoA estimation versus SNR performance of the proposed methods, CRLB and MUSIC.

and their difference of distance $= 0.11$. Assume that the number of sources is known. d is set a half of the signal wavelength. For MUSIC, the step-size of uniform search grid is 0.001 in the domain $[-1, 1]$. Least-squares (LS) is used to estimate the amplitude of signals after signal directions are detected by MUSIC. The noise level δ is choosen manually. The parameter μ is set 0.1. One hundred realizations are performed at each SNR.

First, we verify that the performance of joint DoA estimation by the super-resolution theory. In Figure 1, the RMSE of DoA estimation of the proposed method and MUSIC are presented at SNR=-25 dB. The result shows that the proposed method achieves better estimation accuracy than MUSIC no matters how many snapshots are used. Furthermore, both of the performance become better when the number of snapshots T increases, but JSR-FISTA-MGCV converges faster than MUSIC by fewer number of snapshots.

In Figure 2, we present the RMSE of DoA estimation of JSR-FISTA-MGCV, CRLB and MUSIC. The number of snapshots T is set 3000. The estimation accuracy of JSR-FISTA-MGCV outperforms MUSIC at each SNR, except SNR= -25 dB at which they are close to each other. Compared with MUSIC, the performance of JSR-FISTA-MGCV is much closer to CRLB even at low SNR. In Figure 3, the normalized RMSE of signal amplitude estimation of JSR-FISTA-MGCV and MUSIC are presented. The performance of signal amplitude estimation partially depends on the accuracy of reconstructing the measurement matrix $\mathbf{G}$ in terms of the augmented support set. MUSIC with LS have poor performance due to the erroneous reconstructed measurement matrix and the noise effect which is amplified by the LS method. The JSR-FISTA-MGCV achieves better performance than MUSIC.

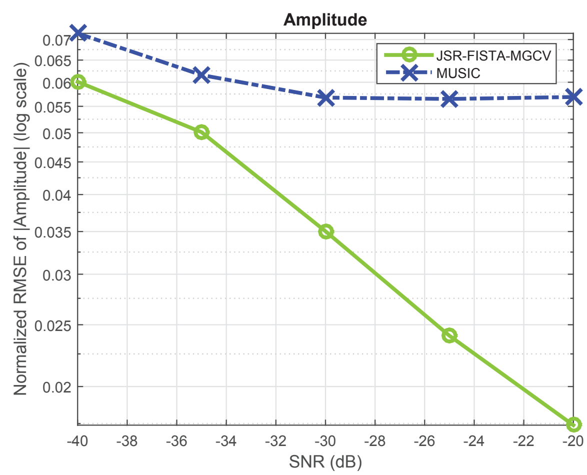

Fig. 3. RMSE of Amplitude estimation versus SNR performance of the proposed methods and MUSIC.

V. Conclusion

Based on the extension of the super-resolution theory to the MMV model, a joint DoA estimation problem is solved via SDP. Using the estimated DoA and interpolation preprocessing, an augmented support set is generated to reconstruct the measurement matrix, leading to the formulation of a sparse convex problem to accurately estimate the direction and amplitude of source signals. To speed up the computing time, an iterative algorithm based on FISTA is proposed with an adaptive regularization parameter selector by minimizing the proposed modified GCV. Simulation results provide an indication of the performance of the proposed techniques relative to CRLB and MUSIC.

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