CLOSED-FORM SOLUTION TO DIRECTLY DESIGN FACE WAVEFORMS FOR BEAMPATTERNS USING PLANAR ARRAY

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ABSTRACT

In multiple-input multiple-output radar systems, it is usually desirable to steer transmitted power in the region-of-interest. To do this, conventional methods optimize the waveform covariance matrix, $\mathbf{R}$, for the desired beampattern, which is then used to generate actual transmitted waveforms. In this paper, we provide a low complexity closed-form solution to design covariance matrix for the given planar beampattern using the planar array, which is then used to derive a novel closed-form algorithm to directly design the finite-alphabet constant-envelope waveforms. The proposed algorithm exploits the two-dimensional fast-Fourier-transform. The performance of our proposed algorithm is compared with the existing methods that are based on semi-definite quadratic programming with the advantage of a considerably reduced complexity.

***Index Terms*—** Multiple-input multiple-output radars, beampattern design, closed-form solution, waveform design, two-dimensional fast-Fourier-transform.

1. INTRODUCTION

Colocated multiple-input multiple-output (MIMO) radar has a number of advantages over the classical phased-array radar such as improvement in parameter identifiability and enhanced flexibility to design transmit beampatterns [1–6]. In fact, the latter subject has lately attracted extensive attention where we aim to focus the transmitted power in a certain region-of-interest (ROI) [7–12]. It is known that the transmit beampattern of a colocated antenna array depends on the cross-correlation between the transmitted waveforms from different antennas. Therefore, to design variety of transmit beampatterns, early solutions have relied on the following two-step process [7–13]. In the first step, the user designs the waveforms covariance matrix such that the theoretical transmitted power matches the desired beampattern as closely as possible. The second step involves the design of the actual waveforms that can realize the designed covariance matrix. Efficient algorithms are proposed in [8, 11, 14] to synthesise the waveform covariance matrix for the given beampattern. All of them are iterative approaches optimizing some constrained problems. These algorithms are computationally very expensive for real-time applications. A closed form solution, to find the waveform covariance matrix, which is based on fast-Fourier-transform (FFT) has been lately proposed in [15]. We have noticed that the solutions proposed in the previous work deal only with linear array and the ROI is defined only by the azimuth angle θ. In the planar array radar systems, the transmitting antennas form a plan and an additional dimension called the elevation angle ϕ is taken into account in order to provide a larger radar aperture. In this paper, we present a closed-form solution to design the waveform covariance matrix, for the desired 3D beampatterns, using a planar array radar. To reduce the computational complexity, the 3D beampattern design problem is mapped onto the two dimensional fast-Fourier-transform (2D-FFT). The algorithm in [15] can be considered as a special case of our proposed algorithm. Next, by exploiting the derivations of the covariance matrix in the proposed algorithm, a novel method to directly design the finite-alphabet constant-envelop (FACE) waveforms is also proposed. The direct design of waveforms does not require the synthesis of any covariance matrix and the performance is same compared with the method using covariance matrix. Therefore, the proposed direct design of the waveforms yields significant reduction in computational complexity and can achieve the best possible performance among the existing direct waveform design algorithms.

Notations: Small letters, bold small letters, and bold capital letters respectively designate scalars, vectors, and matrices. If $\mathbf{A}$ is a matrix, then $\mathbf{A}^H$ and $\mathbf{A}^T$ respectively denote the Hermitian transpose and the transpose of $\mathbf{A}$. $v(i)$ denotes the i^{th} element of vector $\mathbf{v}$. $A(i,j)$ denotes the entry in the i^{th} row and j^{th} column of matrix $\mathbf{A}$. The Kronecker product is denoted by $\otimes$. Modulo M operation on an integer i is denoted by $\langle i \rangle_M$ and $\lfloor i \rfloor_M$ denotes the quotient of i over M. Finally, the statistical expectation is denoted by $\mathrm{E}\{\cdot\}$.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a MIMO radar system with a rectangular planar-array, composed of $M \times N$ omnidirectional antennas, placed at the origin of a unit radius sphere. As shown in Fig. 1, the inter-element-spacing (IES) between any two adjacent antennas in the x and y-axis directions is d_x and d_y, respectively. If a spatial location around this planar-array has an azimuth angle θ and an elevation angle ϕ, the corresponding Cartesian coordinates of this location can be written as

$$x = \sin(\phi)\cos(\theta) \text{ and } y = \sin(\phi)\sin(\theta). \quad (1)$$

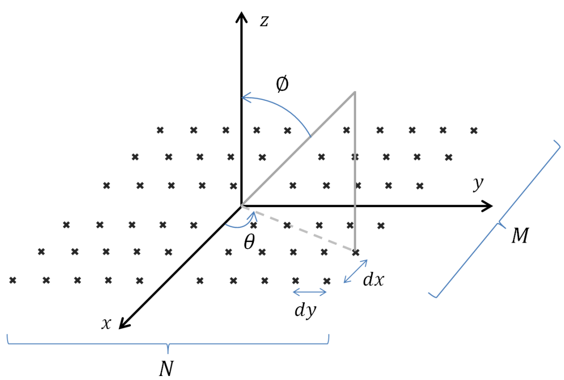

Fig. 1: Linear planar array of $M \times N$ transmit antennas.

Now we define the baseband transmitted signal vector containing the transmitted symbols from all antennas at time index n as

$$\mathbf{x}(n) = [x_{0,0}(n), \dots, x_{0,N-1}(n), \dots, x_{M-1,N-1}(n)]^T,$$

where $x_{p,q}(n)$ denotes the transmitted symbol from the antenna at the $(p,q)^{\text{th}}$ location at time index n. Assuming that the distance between any two adjacent antennas on the x-axis and y-axis direction is $\lambda/2$, the signal received by a target at a location defined by θ and ϕ can be written as

$$r(n;\theta,\phi) = \sum_{p=0}^{M-1}\sum_{q=0}^{N-1} x_{p,q}(n)\, e^{j2\pi q \frac{\sin(\phi)\cos(\theta)}{2}} e^{j2\pi p \frac{\sin(\phi)\sin(\theta)}{2}}.$$

By exploiting the relationship between the spherical and Cartesian coordinates, given in (1), one can write the received signal in terms of Cartesian coordinates as

$$r(n; f_x, f_y) = \sum_{p=0}^{M-1}\sum_{q=0}^{N-1} x_{p,q}(n) e^{j2\pi(q f_x + p f_y)}, \quad (2)$$

where

$$f_x = \frac{\sin(\phi)\cos(\theta)}{2} \text{ and } f_y = \frac{\sin(\phi)\sin(\theta)}{2} \quad (3)$$

are the normalised Cartesian coordinates of the spatial location. The received signal in (2) can be written in vector form as

$$r(n; f_x, f_y) = \mathbf{a}_s^H(f_x, f_y)\, \mathbf{x}(n), \quad (4)$$

where

$$\mathbf{a}_s(f_x, f_y) = \begin{bmatrix} 1 \\ e^{j2\pi f_y} \\ \vdots \\ e^{j2\pi(M-1)f_y} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ e^{j2\pi f_x} \\ \vdots \\ e^{j2\pi(N-1)f_x} \end{bmatrix}. \quad (5)$$

Using (2), the received power at the location (f_x, f_y) can be easily written as

$$\begin{aligned} B(f_x, f_y) &= \mathrm{E}\{\mathbf{a}_s^H(f_x, f_y)\, \mathbf{x}(n)\, \mathbf{x}(n)^H\, \mathbf{a}_s(f_x, f_y)\} \\ &= \mathbf{a}_s^H(f_x, f_y)\, \mathbf{R}\, \mathbf{a}_s(f_x, f_y), \end{aligned} \quad (6)$$

where $\mathbf{R} = \mathrm{E}\{\mathbf{x}(n)\mathbf{x}(n)^H\}$ is the $MN \times MN$ covariance matrix of the transmitted waveforms. In the conventional transmit beampattern design problem, a covariance matrix, $\mathbf{R}$, is synthesized to match the transmitted power $B(f_x, f_y)$ to the desired beampattern which involves the minimization of the following cost function

$$\begin{aligned} J(\mathbf{R}) = \sum_{l=1}^{L}\sum_{k=1}^{K} \Big| &\mathbf{a}_s^H(f_x(l), f_y(k))\mathbf{R}\mathbf{a}_s(f_x(l), f_y(k)) \\ &-\alpha P_d(f_x(l), f_y(k))\Big|_2^2, \end{aligned} \quad (7)$$

where $P_d(f_x(l), f_y(k))$ is the desired beampattern defined over the two dimensional grid $\left(\{f_x(l)\}_{l=1}^{L}, \{f_y(k)\}_{k=1}^{K}\right)$ and α is a scaling factor. For practical reasons the covariance matrix $\mathbf{R}$ should be positive semi-definite with equal diagonal elements. Therefore, we define the following minimization problem

$$\begin{cases} \min\ J(\mathbf{R}) \\ \text{subject to} \\ \mathrm{C}_1:\ \mathbf{R} \succeq 0 \\ \mathrm{C}_2:\ R(n,n) = c,\ n = 1, 2, \dots, MN. \end{cases} \quad (8)$$

The constrained problem in (8) can be optimally solved using an iterative SQP method [8]. However, for large number of antennas the computational complexity of SQP method becomes prohibitively high. Therefore, such solutions are not feasible for planar arrays of high sizes. In order to reduce the computational cost by exploiting 2D-FFT algorithm, a closed-form solution to find the matrix $\mathbf{R}$ is proposed in the following section.

3. PROPOSED COVARIANCE MATRIX DESIGN

Given an $M \times N$ time domain matrix $\mathbf{H}_t$, the $M \times N$ frequency domain matrix $\mathbf{H}_f$ can be easily generated. The relationship between the time domain coefficients $H_t(m,n)$ and the frequency domain coefficients $H_f(k_1,k_2)$ is given by the following 2D discrete-Fourier-transform (2D-DFT) formula

$$H_f(k_1,k_2) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} H_t(m,n)\, e^{-j2\pi k_1 m/M} e^{-j2\pi k_2 n/N}. \quad (9)$$

Similarly, for given frequency domain coefficients, the time domain coefficients are obtained with the 2D inverse discrete-Fourier-transform (2D-IDFT) as follows

$$H_t(m,n) = \frac{1}{MN}\sum_{k_1=0}^{M-1}\sum_{k_2=0}^{N-1} H_f(k_1,k_2)\, e^{j2\pi k_1 m/M} e^{j2\pi k_2 n/N}. \quad (10)$$

Using (9) we obtain the following lemma

Lemma 1 *Let $\mathbf{H}_f$ be an $M \times N$ matrix with real positive frequency domain coefficients and define the vectors $\mathbf{e}_M(k_1)$ and $\mathbf{e}_N(k_2)$ as*

$$\begin{aligned} \mathbf{e}_M(k1) &= \begin{bmatrix} 1 & e^{j2\pi k_1/M} & \ldots & e^{j2\pi k_1(M-1)/M} \end{bmatrix}^T, \\ \mathbf{e}_N(k_2) &= \begin{bmatrix} 1 & e^{j2\pi k_2/N} & \ldots & e^{j2\pi k_2(N-1)/N} \end{bmatrix}^T, \end{aligned} \tag{11}$$

where $k_1 = 0, 1, \ldots, M-1$ and $k_2 = 0, 1, \ldots, N-1$. If we construct a matrix $\mathbf{R}_{hh}$ as

$$\mathbf{R}_{hh} = \frac{1}{(MN)^2} \sum_{k_1=0}^{M-1} \sum_{k_2=0}^{N-1} H_f(k_1, k_2)\, \mathbf{e}(k_1, k_2)\, \mathbf{e}^H(k_1, k_2), \tag{12}$$

where $\mathbf{e}(k_1, k_2) = \mathbf{e}_N(k_2) \otimes \mathbf{e}_M(k_1)$, then $\mathbf{R}_{hh}$ will be positive semi-definite and all of its diagonal elements will be equal. Moreover, the individual elements of $\mathbf{H}_f$ are related to the entries of $\mathbf{R}_{hh}$ using the following quadratic form

$$H_f(l_1, l_2) = \mathbf{e}^H(l_1, l_2)\mathbf{R}_{hh}\mathbf{e}(l_1, l_2). \tag{13}$$

Proof of lemma 1 is provided in [16]. Finding $\mathbf{R}_{hh}$ using (12) can be computationally very expensive since it requires the outer product of MN vectors and addition of MN matrices. To reduce the computational complexity, using (12), the individual elements of $\mathbf{R}_{hh}$ can be written as

$$\begin{aligned} R_{hh}(i_1, i_2) = \frac{1}{(MN)^2} & \sum_{k_1=0}^{M-1} \sum_{k_2=0}^{N-1} H_f(k_1, k_2) \\ & \times e^{j\frac{2\pi k_1 \langle i_1 - i_2 \rangle_M}{M}} e^{j\frac{2\pi k_2 (\lfloor i_1 \rfloor_M - \lfloor i_2 \rfloor_M)}{N}}, \end{aligned} \tag{14}$$

where $i_1, i_2 = 0, 1, \ldots, MN-1$. Comparing (14) with (10), we can write

$$R_{hh}(i_1, i_2) = \frac{1}{MN} H_t(\langle i_1 - i_2 \rangle_M, \lfloor i_1 \rfloor_M - \lfloor i_2 \rfloor_M) \tag{15}$$

Since the matrix $\mathbf{R}_{hh}$ is positive semi-definite and all of its diagonal elements are equal, it satisfies both the C_1 and C_2 constraints of the optimization problem in (8) for designing the desired beampattern. Therefore, if $\mathbf{R}_{hh}$ is considered to be the waveform covariance matrix, by comparing (6) with (13), it can be easily seen that the problem of transmit beampattern design can be mapped to the result obtained in the *Lemma* 1. This transformation only requires the mapping of the steering vector $\mathbf{a}_s(f_x, f_y)$ to $\mathbf{e}(k_1, k_2)$. This can be done by mapping the values of f_x and f_y to k_1 and k_2 using the following expressions

$$\begin{cases} f_x \mapsto -0.5 + \frac{k_1}{M-1}, & k_1 = 0 \ldots M-1 \\ f_y \mapsto -0.5 + \frac{k_2}{N-1}, & k_2 = 0 \ldots N-1. \end{cases} \tag{16}$$

The three dimensional space can then be defined by a two dimensional grid $\left(\{(f_x)(l)\}_{l=1}^{M}, \{(f_y)(k)\}_{k=1}^{N}\right)$ represented by an $M \times N$ matrix $\mathbf{H}_f$. Thus, the entry $H_f(m, n)$ corresponds to $f_x = -0.5 + \frac{m}{M-1}$ and $f_y = -0.5 + \frac{n}{N-1}$. In order to define the ROI of the desired beampattern, we just have to assign 1 to the entries of $\mathbf{H}_f$ which are inside the ROI and 0 everywhere else.

4. COMPUTATIONAL COMPLEXITY

The only computational complexity of the proposed method comes from the IDFT computation step. The NM IDFT coefficients are computed using one of the famous FFT algorithms which have a complexity equal to $O(MN\ \log(MN))$ operations. However, the SQP method has a complexity of the order $O(\log(\frac{1}{\eta})\ (MN)^{3.5})$ for a given accuracy η [17]. As shown in Fig. 2, the gap of computational complexity between the FFT-based and SQP-based algorithms increases with the number of antennas which makes our method more suitable for real time radar applications.

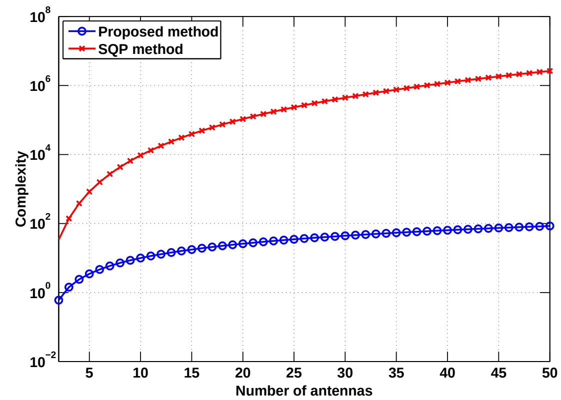

Fig. 2: Computational complexity comparison between the FFT-based algorithm and the SQP method.

5. DIRECT DESIGN OF WAVEFORMS FOR THE DESIRED BEAMPATTERN

In this section, a closed-form expression to directly design the waveforms for the desired beampattern is proposed. We start from (12), which can also be written as

$$\begin{aligned} R(i_1, i_2) = & \sum_{k_1=0}^{M-1} \sum_{k_2=0}^{N-1} \left(\frac{\sqrt{H_f(k_1, k_2)}}{MN} e^{j\frac{2\pi k_1 \langle i_1 \rangle_M}{M}} e^{j\frac{2\pi k_2 \lfloor i_1 \rfloor_M}{N}} \right) \\ & \times \left(\frac{\sqrt{H_f(k_1, k_2)}}{MN} e^{j\frac{2\pi k_1 \langle i_2 \rangle_M}{M}} e^{j\frac{2\pi k_2 \lfloor i_2 \rfloor_M}{N}} \right)^*. \end{aligned} \tag{17}$$

Choosing $k = k_1 + Mk_2 = \langle k \rangle_M + M\lfloor k \rfloor_M$, both terms in the above equation can be considered as the kth elements of the waveforms $\mathbf{s}_{i_1}$ and $\mathbf{s}_{i_2}$ that can be written as

$$\begin{aligned} s_{i_1}(k) &= \frac{\sqrt{H_f(\langle k \rangle_M, \lfloor k \rfloor_M)}}{MN} e^{j\frac{2\pi \langle k \rangle_M \langle i_1 \rangle_M}{M}} e^{j\frac{2\pi \lfloor k \rfloor_M \lfloor i_1 \rfloor_M}{N}}, \\ s_{i_2}(k) &= \frac{\sqrt{H_f(\langle k \rangle_M, \lfloor k \rfloor_M)}}{MN} e^{j\frac{2\pi \langle k \rangle_M \langle i_2 \rangle_M}{M}} e^{j\frac{2\pi \lfloor k \rfloor_M \lfloor i_2 \rfloor_M}{N}}, \end{aligned}$$

where the time index $k = 0, 1, \ldots, MN-1$. Thus, the cross-correlation between the waveforms $\{s_{i_1}(k)\}$ and $\{s_{i_2}(k)\}$ is written as

$$R(i_1, i_2) = \sum_{k=0}^{MN-1} s_{i_1}(k)\, s_{i_2}(k)^*. \tag{18}$$

The corresponding waveform vector can be written as

$$\mathbf{s}_i = \begin{bmatrix} \frac{\sqrt{H_f(0,0)}}{MN} e^{j\frac{2\pi(0)\lfloor i \rfloor_M}{N}} e^{j\frac{2\pi(0)\langle i \rangle_M}{M}} \\ \vdots \\ \frac{\sqrt{H_f(0,N-1)}}{MN} e^{j\frac{2\pi(N-1)\lfloor i \rfloor_M}{N}} e^{j\frac{2\pi(0)\langle i \rangle_M}{M}} \\ \vdots \\ \vdots \\ \frac{\sqrt{H_f(M-1,0)}}{MN} e^{j\frac{2\pi(0)\lfloor i \rfloor_M}{N}} e^{j\frac{2\pi(M-1)\langle i \rangle_M}{M}} \\ \vdots \\ \frac{\sqrt{H_f(M-1,N-1)}}{MN} e^{j\frac{2\pi(N-1)\lfloor i \rfloor_M}{N}} e^{j\frac{2\pi(M-1)\langle i \rangle_M}{M}} \end{bmatrix} \quad (19)$$

Therefore, for any transmitting element of the rectangular array at location (m, n) where $m = 0 \ldots M-1$ and $n = 0 \ldots N-1$, we assign the waveform $\mathbf{s}_i$ defined in (19) with $i = m + nM$. It should be noted here that depending on the desired beampattern some elements of the waveform $\mathbf{s}_i$ may be equal to zero. If N_a is the number of non-zero elements in the matrix $\mathbf{H}_f$ only $N_a < MN$ snapshots will be required to achieve the desired beampattern.

6. NUMERICAL SIMULATIONS

In this section, the performance of the proposed FFT-based algorithm is investigated. For simulation, a rectangular planar array composed of $M \times N$ antennas is considered. The spacing between any two adjacent antennas on the $x-$ and $y-$axis of the planar-array is kept $\lambda/2$. The MSE between the desired and designed beampatterns is defined as

$$\begin{aligned} \text{MSE} \;\; = \;\; & \sum_{l=1}^{L}\sum_{k=1}^{K} |\mathbf{a}_s^H(f_x(l), f_y(k))\mathbf{R}\mathbf{a}_s(f_x(l), f_y(k)) \\ & -\alpha P_d(f_x(l), f_y(k))|^2/KL. \end{aligned}$$

In the following simulation, the ROI is defined as $-0.1 \leq f_x \leq 0.1$ and $-0.1 \leq f_y \leq 0.1$ and we use $N = M = 10$. To design this beampattern, we use our proposed closed-form 2D-FFT based algorithm. The corresponding designed beampattern, using the covariance matrix $\mathbf{R}$ obtained by our algorithm, is shown in Fig. 3. Note that the beampattern is normalized by dividing α. The algorithm to directly design the waveforms corresponding to this beampattern is proposed in Sec. 5. In order to compare the performance of our algorithm with the iterative SQP method, we compute the corresponding MSE for different planar array dimensions and for the ROI defined by $-0.1 \leq f_x \leq 0.1$ and $-0.1 \leq f_y \leq 0.1$. The MSE of both methods with respect to the total number of antennas MN is shown in Fig. 4. We note that for low number of antennas the performance of the FFT-based method is affected. This is due to the fact that the ROI (represented by the matrix $\mathbf{H}_f$) is constructed in the two dimensional grid

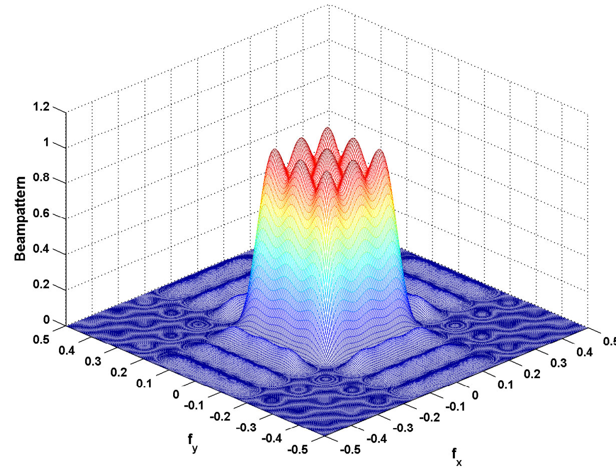

Fig. 3: The designed beampattern using the proposed FFT-based algorithm. Here, the ROI is $-0.1 \leq f_x \leq 0.1$ and $-0.1 \leq f_y \leq 0.1$ and $M = N = 10$.

$\left(\{(f_{k1})_l\}_{l=1}^{M}, \{(f_{k2})\}_{k=1}^{N}\right)$ whose resolution is related to the number of antennas. However, as the dimensions of the rectangular array increase the proposed method achieves lower MSE level approaching the SQP-based method with the advantage of being much less complex.

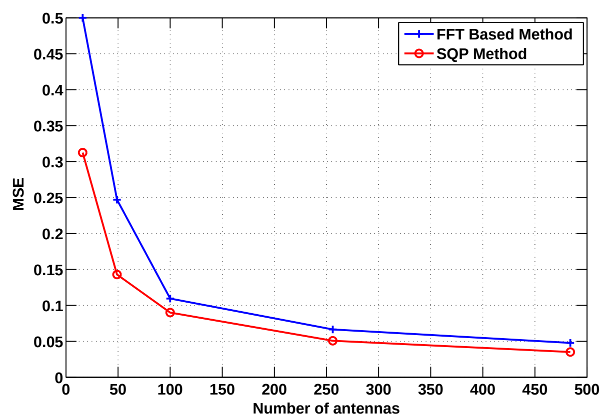

Fig. 4: MSE comparison between the FFT-based algorithm and the SQP method for different planar array dimensions.

7. CONCLUSION

In this paper we have presented a closed-form method of covariance matrix design for the planar MIMO transmit beamforming problem that exploits the IDFT coefficients. The positive semi-definition and uniform element power constraints are verified by the designed matrix. Next, a method of direct waveform design exploiting the expression of the covariance matrix that we found is proposed. The numerical simulations presented confirm that the proposed method is computationally efficient and performs closely to the SQP-based method as the number of antennas increases.

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