APERIODIC WAVEFORMS WITH MISMATCHED FILTERING FOR TARGET DETECTION IN HEAVY CLUTTER. PART II - MIMO RADAR ARCHITECTURE

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ABSTRACT

In a companion paper we have investigated the applicability of aperiodic noise-like waveforms with a "thumb-tack" ambiguity function processed using mismatched compression filters for strong clutter rejection. In this paper we investigate the role of the same principle in mode-selective MIMO OTHR applications. It has been recently demonstrated that periodic waveforms currently used in HF-OTHR for efficient ground clutter mitigation have very limited scope for MIMO radar applications. This is due to the fact that the sidelobe-free area in the appropriate MIMO ambiguity function is K times smaller than for a single periodic waveform, where K is the number of MIMO waveforms.

Index Terms— HF radar, OTHR, waveforms, mis-match filtering, aperiodic, MIMO

1. INTRODUCTION

In previous studies on the principles of MIMO HF-OTHR mode-selective operations [1, 2, 3] we considered matched processing of K orthogonal waveforms capable of strong ground clutter mitigation. Since the required sub-clutter visibility (SCV) even for "normal" clutter (i.e. not "spread" in Doppler frequency) is much higher than the time-bandwidth product (compression gain) of any available waveform, periodic waveforms such as LFMCW have been considered [1, 2, 3]. Each of such waveforms has the maximal (practically) "sidelobe-free" area around the main peak of the auto-ambiguity function [4] equal to four, that allows for unambiguous SIMO target detection anywhere within the "unambiguous" search area

$$A_1 \in \begin{cases} \tau_{\max} - \tau_{\min} = \Delta\tau \\ f_{\max} - f_{\min} = \Delta f \end{cases} \quad (1)$$

when

$$\Delta\tau\Delta f \leq 1 \quad (2)$$

and all clutter scatterers and targets are confined within this area.

In [1, 2, 3, 5], we demonstrated that for MIMO operations with K "orthogonal" waveforms, similar conditions for "sidelobe-free" area for all K auto-ambiguity functions χ_{jj}, $j = 1, \ldots, K$ and all cross-ambiguity functions χ_{kj}, $k, j = 1, \ldots, K$ and $k \neq j$ may hold only within K-times smaller "range-Doppler" area.

$$\Delta\tau\Delta f \leq 1/K \quad (3)$$

Even for SIMO operations, traditional limitations equation (2), already introduce problems for fast target detection within the required range search area, and the quite dramatic reduction of the sidelobe-free area represented by equation (3) significantly limits the scope of practical MIMO applications. In fact, most practically considered scenarios are limited to slow-moving (surface) targets [1, 3, 6].

Note, that the "range-Doppler" area actually occupied by "normal" clutter, propagated via the same ionospheric layers as "wanted" target returns, is not that great. According to [7], from resolution cells on the ground located at ranges greater than 8000 nautical miles (nm), one gets little, if any, backscattered energy. Since "normal" clutter that propagates via an unperturbed ionospheric layer has Doppler spectrum with bandwidth rarely exceeding 2Hz, the actual "range-Doppler" area occupied by (multi-hop) clutter returns does not exceed [8].

$$\tau^c_{\max}\Delta f_c \leq 0.2 \quad (4)$$

However, if the search area for a target is limited by, say, 500nm, and the maximal target Doppler frequency can reach no more than 31Hz, then

$$\tau_t\Delta f_t = 1 \quad (5)$$

which means that dimensions of "sidelobe-free" "range-Doppler" area within this approach is dominated by a large search area requirement, rather than by the actual dimension of the range-Doppler space occupied by clutter.

In our companion paper [8], we suggested an approach for SIMO operations, that can provide practically unlimited search area for actual clutter (A) confined in "range-Doppler" space. Specifically, we demonstrated that noise-like waveforms with the "thumb-tack"-like ambiguity function and compression gains equal to those currently used in air-mode OTH radar operational waveforms (BW=8–25kHz, T_{CIT}=2–4s), are capable of providing the required high-level SCV support (>70dB), when an appropriately designed mismatch filter is used.

Ultimately, for each range and Doppler frequency resolution cell (τ_0, f_0) within the search area, specific optimal mismatched filters have to be applied. Moreover, mismatch between filter and radar waveform is naturally associated with some signal-to-white-noise ratio (SWNR) loss. Yet, for the clutter zone that does not exceed 0.2, equation (4), these losses are less than 2dB in most cases, which is comparable to the SWNR loss for currently used tapers that control range and Doppler frequency sidelobes in traditional LFMCW waveforms [9].

In [8] we introduced our experimental results that demonstrated this methodology could be practically implemented for SIMO radar operations. In this paper we investigate the role mismatched filtering may play for MIMO operations with much more severe restriction on the "sidelobe-free" area that is given in equation (3).

2. MISMATCHED FILTERS FOR GROUND CLUTTER MITIGATION IN MIMO RADAR APPLICATIONS

Let us consider a transmit (Tx) 2D antenna architecture, where each of the K-MIMO waveforms is transmitted by individual antenna elements (or sub-arrays). We assume these elements are identical and are described by the beampattern $G_T(\phi, \theta|\phi_0, \theta_0)$, where ϕ is the elevation angle (El) calculated off the zenith, θ is the azimuth (Az) calculated off the boresight; (ϕ_0, θ_0) is the beamsteering direction. Let these identical Tx modules be arranged into a 2D array with coordinates (x_{T_k}, y_{T_k}) of its k-th element $(k = 1, \ldots, K)$ on the plane.

The manifold of this Tx array is calculated as

$$E_K(\phi, \theta) = \exp\left[i\frac{2\pi}{\lambda}(u\ x_{T_k} + v\ y_{T_k})\right] \quad (6)$$

where $u = \sin\phi\sin\theta$ and $v = \sin\phi\cos\theta$

Similarly, we may introduce a 2D L-element receive (Rx) antenna array with the beampattern of its elements $G_R(\phi, \theta|\phi_0, \theta_0)$, and their positions on a plane (x_{R_l}, y_{R_l}) $(l = 1, \ldots, L)$. Correspondingly, the manifold of this 2D Rx array may be calculated as

$$E_L(\phi, \theta) = \exp\left[i\frac{2\pi}{\lambda}(u\ x_{R_l} + v\ y_{R_l})\right]\ \ l = 1, \ldots, L \quad (7)$$

If each of K Tx antenna modules transmits its "own" MIMO waveform $u_k(t)$, $k = 1, \ldots, K$, then the scalar signal backscattered by an omni-directional point scatterer with random scattering coefficient ε_c, located in the direction of departure (DoD) from the Tx array (ϕ, θ), at the slant (group) range τ, may be presented as

$$s(t, \tau, f) = \varepsilon_c G_T(\phi, \theta|\phi_0, \theta_0)\sum_{k=1}^{K} u_k(t - \tau)E_k(\phi, \theta) \quad (8)$$

Note, that propagation losses, power, etc, is incorporated into the random coefficient ε_c. As in [1], we consider a traditional Tx array, so that all waveforms are delayed and shifted in Doppler frequency by the same value (τ, f).

Also note, that in fact (ϕ, θ) is, strictly speaking, the angles of arrival of the plane wave upon the considered scatterer. Due to tilts and gradients in the ionosphere, these angles are not necessarily equal to the actual angles of departure from the transmitter, and this distinction may be important in scenarios with perturbed propagation conditions. Since the beampattern $G_T(\phi, \theta|\phi_0, \theta_0)$ is usually reasonably broad, this distinction may be ignored in what follows.

With equation (8) describing the scalar signal backscattered by a point scatterer in all directions, the signal arriving at the L-element Rx antenna ports (element, or, sub-arrays) is therefore described as a scaled version of the following L-element vector:

$$X_L(t, \tau, f) = s(t, \tau, f)G_R(\phi', \theta'|\phi_0, \theta_0)E_L(\phi', \theta') \quad (9)$$

with

$$s(t, \tau, f) = s(t, \tau)\exp[i2\pi f t] \quad (10)$$

where f is the Doppler frequency specified by the radial velocity of the point scatterer with respect to Tx and Rx arrays. In general, $\phi' \neq \phi$, $\theta' \neq \theta$, yet, with propagation along the great circle, we usually have $\theta' = \theta$ but difference in elevation angles of departure and arrivals in multi-mode propagation environment may be significant [1]. Also note, that τ in equation (9) now accounts for the entire group range.

If we consider an M-element FIR mismatched filter, then in the most general case for target detection at the coordinates $(\tau_0, f_0, \phi_0, \theta_0)$, we have to consider a KxLxM-variate MIMO space-time filter with the point scatterer equation (4) output scalar signal, presented as follows:

$$x_{c_{\text{out}}}(\tau_0, f_0, \phi_0, \theta_0) = \sum_{k=1}^{K}\int_{\tau_0}^{\tau_0+T} W_{Lk}^H(t, \tau_0|\tau_0, f_0, \phi_0, \theta_0)X_L(t, \tau, f)\mathrm{d}t \quad (11)$$

where

$$W_{Lk} = w_k(t - \tau_0|\tau_0, f_0, \phi_0, \theta_0)\exp[i2\pi f_0 t] \times W_{LK}(\tau_0, f_0, \phi_0, \theta_0) \quad (12)$$

in equation (12) $w_K(t - \tau_0|\tau_0, f_0, \phi_0, \theta_0)$ is the impulse response of the M-variate mismatched filter, tailored to target detection at the coordinates $(\tau_0, f_0, \phi_0, \theta_0)$. Of course we may consider some filter for all target angle coordinates

(ϕ_0, θ_0). Similarly W_{LK} is the L-variate Rx antenna beamforming vector, also tailored to this particular resolution cell $(\tau_0, f_0, \phi_0, \theta_0)$. In this general case, with respect to equations (9) and (12), we get

$$\begin{aligned} x_{c_{\text{out}}}(\tau_0, f_0, \phi_0, \theta_0) &= \\ & \sum_{k=1}^{K} \int_{\tau_0}^{\tau_0+T} W_{Lk}^H(t, \tau_0|\tau_0, f_0, \phi_0, \theta_0) \\ & \times X_L(t, \tau, f) \mathrm{d}t \\ &= \varepsilon_c \sum_{k=1}^{K} \sum_{j=1}^{K} W_{LK}^H(\tau_0, f_0, \phi_0, \theta_0) E_L(\phi', \theta') \\ \times & \int_{\tau_0}^{\tau_0+T} w_K^*(t-\tau_0) u_j(t-\tau) \exp[i2\pi(f-f_0)t] \mathrm{d}t \\ & \times E_j(\phi, \theta) G_T(\phi, \theta|\phi_0, \theta_0) G_R(\phi', \theta'|\phi_0, \theta_0) \end{aligned} \quad (13)$$

Let us now introduce the KxK-variate cross-ambiguity matrix function $\chi_{WU}(\Delta\tau, \Delta f|\phi_0, \theta_0)$

$$\chi_{WU}(\Delta\tau, \Delta f|\phi_0, \theta_0) = \left\{ \chi_{w_k u_j}(\Delta\tau, \Delta f|\phi_0, \theta_0) \right\} \quad k, j = 1, \ldots, K \quad (14)$$

where

$$\begin{aligned} \chi_{w_k u_j}(\Delta\tau, \Delta f|\phi_0, \theta_0) &= \int_{\Delta\tau=\tau-\tau_0, \Delta f=f-f_0}^{\tau_0+T} w_k^*(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0) \\ & \times u_j(t-\tau) \exp[i2\pi(f-f_0)] \mathrm{d}t \end{aligned} \quad (15)$$

Therefore, if we introduce a target coordinate-dependent KL-variate vector $W_{KL}(\tau_0, f_0, \phi_0, \theta_0)$

$$W_{KL}(\tau_0, f_0, \phi_0, \theta_0) = [W_{L1}^H, \ldots, W_{LK}^H] \in \mathcal{C}^{1 \times KL} \quad (16)$$

we may re-write equation (13) as

$$\begin{aligned} x_{c_{\text{out}}}(\tau_0, f_0, \phi_0, \theta_0) &= \varepsilon_c G_T(\phi, \theta|\phi_0, \theta_0) G_R(\phi', \theta'|\phi_0, \theta_0) \\ & \times W_{KL}^H(\tau_0, f_0, \phi_0, \theta_0) \\ & \times [E_L(\phi', \theta') \otimes \chi_{WU}(\Delta\tau, \Delta f|\phi_0, \theta_0) E_K(\phi, \theta)] \end{aligned} \quad (17)$$

In equation (17), $E_K(\phi, \theta)$ is the Tx array manifold specified by the clutter scatterers DoD (ϕ, θ). $E_L(\phi', \theta')$ is the Rx array manifold specified by its DoA (ϕ', θ'). Therefore, for the power of this clutter point scatterer at the output of the MIMO receiver matched to the target at the coordinates $(\tau_0, f_0, \phi_0, \theta_0)$ we get

$$\begin{aligned} \sigma_{c_{\text{out}}}^2(\tau_0, f_0, \phi_0, \theta_0) &= \sigma_c^2 |G_R(\phi', \theta'|\phi_0, \theta_0)|^2 \\ & \times |G_T(\phi, \theta|\phi_0, \theta_0)|^2 W_{KL}^H(\tau_0, f_0, \phi_0, \theta_0) \\ & \times E_L(\phi', \theta') E_L^H(\phi', \theta') \\ & \otimes \chi_{WU}(\Delta\tau, \Delta f|\phi_0, \theta_0) E_K(\phi, \theta) \\ & \times \left[E_K^H(\phi, \theta) \otimes \chi_{WU}^H(\Delta\tau, \Delta f|\phi_0, \theta_0) \right] \\ & \times W_{KL}(\tau_0, f_0, \phi_0, \theta_0) \end{aligned} \quad (18)$$

where $\sigma_c^2 = \mathrm{E}[|\varepsilon_c|^2]$.

For the target at $\phi_0, \theta_0, \Delta\tau = 0, \Delta f = 0$ we get

$$\begin{aligned} \sigma_{t_{\text{out}}}^2 &= \sigma_t^2 |G_R(\phi_0, \theta_0)|^2 |G_T(\phi_0, \theta_0)|^2 \\ & \times W_{KL}^H(\tau_0, f_0, \phi_0, \theta_0) \\ & \times \left[E_L(\phi_0, \theta_0) E_L^H(\phi_0, \theta_0) \otimes \chi_{W,U}(0, 0, \phi_0, \theta_0) \right] \\ & \times \left[E_K(\phi_0, \theta_0) E_K^H(\phi_0, \theta_0) \otimes \chi_{W,U}^H(0, 0, \phi_0, \theta_0) \right] \\ & \times W_{KL}(\tau_0, f_0, \phi_0, \theta_0) \end{aligned} \quad (19)$$

Now note that integration in equation (18) over clutter point scatterer coordinates $[\phi, \theta, \phi', \theta', \tau, f]$ within the space-time-Doppler frequency cube occupied by clutter will provide the total clutter power at the output of the MIMO receiver designed to detect a target with coordinates $(\tau_0, f_0, \phi_0, \theta_0)$.

Following the same approach as in [1] for full mitigation of clutter scatterers within the clutter area A_c at the output of this MIMO receiver, we assume that the following condition on the cross-ambiguity matrix function applies:

$$|\chi_{WU}(\Delta\tau, \Delta f, \phi_0, \theta_0)|^2 = 0 \quad (20)$$

$$(\Delta\tau, \Delta f) \in A_c$$

The later means that the mismatched filters

$$w_K(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0)$$

have to be designed such that

$$\left| \int_{\tau_0}^{\tau_0+T} w_K^*(t-\tau_0) u_j(t-\tau) \exp[i2\pi(f-f_0)t] \mathrm{d}t \right|^2 = 0 \quad (21)$$

where $w_K^*(t-\tau_0)$ is short-form of $w_K(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0)$ and

for all $(\tau-\tau_0, f-f_0) \in A_c$

For SIMO applications $(k = j)$ we demonstrated in [8] that the maximal zone $(\tau-\tau_0, f-f_0) \in A_c$ where noise-like waveform sidelobes could be successfully rejected with SCV≥70dB with SWNR losses ≤2dB, is equal to S_A= 0.2. Obviously, condition equation (21) requires that this maximal zone has to be reduced to

$$S_A \leq 0.2/K \quad (22)$$

for MIMO applications with K MIMO waveforms. Therefore, similarly to the matched filter MIMO technique with total "sidelobe-free" zone reduced by K times, equation (3), we observe the same cleared area reduction in the considered approach.

Note, that as in [1, 2, 3, 5], conditions in equation (20) and (21) provide clutter mitigation in range/Doppler resolution cell (τ_0, f_0) completely irrespective of the transmit antenna manifold and DoD (ϕ, θ), since every waveform $u_j(t)$ response gets rejected by the filter matched with the particular waveform $u_k(t)$ in equation (21).

For the considered here and in [1] problem of mode selection, this requirement may be treated as excessive. Indeed, recall that the main idea of the mode-selective MIMO technique, is to provide backscattered signal selection, based upon the elevation of departure for mixed mode returns propagated back to the receive antenna on the same "wanted" propagation mode. For example, "wanted" clutter (and target) returns propagated via E–E mode, should be separated from "unwanted" clutter returns, propagated via F–E mode.

For each "clutter" resolution cell with delay τ we may introduce the synthetic clutter waveform

$$u_{\Sigma}(t-\tau|\phi,\theta) = \sum_{j=1}^{K} u_j(t-\tau) E_j(\phi,\theta) G_T(\phi,\theta|\phi_0,\theta_0) \quad (23)$$

Obviously, for each τ and the related (ϕ,θ), this waveform is different. Note, that with respect to equation (23), the expression equation (13) may be re-written as follows:

$$\begin{aligned} x_{c_{\text{out}}}(\tau_0, f_0, \phi_0, \theta_0) \ &= \ \sum_{k=1}^{K} W_{LK}^{H}(\tau_0, f_0, \phi_0, \theta_0) \\ &\times E_L(\phi', \theta') G_R(\phi', \theta'|\phi_0, \theta_0) \\ &\times \int_{\tau_0}^{\tau_0+T} w_K^*(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0) \\ &\times u_{\Sigma}(t-\tau|\phi,\theta) \exp[i2\pi(f-f_0)t] \mathrm{d}t \qquad (24) \end{aligned}$$

The main difference with respect to the matched filter approach in [1, 2, 3, 5] is that the mismatched filter $w_K(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0)$ may be designed for sidelobe mitigation created by the different (for each sidelobe) waveforms $u_{\Sigma}(t-\tau|\phi,\theta)$ in equation (24). Indeed, the discussed above MIMO mode selection requires the mismatched filter in equation (24) to reject all sidelobes in $A_c = (\tau, \phi)$ created by different sidelobes in A_c synthetic waveforms $u_{\Sigma}(t-\tau)$. For example, if the clutter area A_c is represented by the finite set of points (τ_c, f_c) for $c = 1, \ldots, Q$ selected over some fine grid, the mismatched filter optimisation problem may be formulated as finding

$$\min \left| \int_{\tau_0}^{\tau_0+T} w_K(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0) \mathrm{d}t \right|^2 \qquad (25)$$

subject to

$$\begin{aligned} &\int_{\tau_0}^{\tau_0+T} w_K^*(t-\tau_0|\tau_0, f_0, \phi_0, \theta_0) \\ &\times u_{\Sigma}(t-\tau_c|\phi_c,\theta_c) \exp[i2\pi(f-f_c)t] \mathrm{d}t = 0 \qquad (26) \end{aligned}$$

Practically, equations (25) and (26) are solved in vector/matrix form after waveform and filter response sampling [8]. From equation (26) follows that introduction of a DoD-dependent synthetic waveform $u_{\Sigma}(t-\tau)$, has allow for retaining the same maximal "sidelobe free" zone delivered by mismatched filtering equation (4) as in the SIMO application case. Note, that since for each delay this synthetic waveform is different, this property is achieved only due to invariance of the mismatch filter rejection capability with respect to the noise-like waveform.

3. CONCLUSIONS

In this paper, we investigated the potential role of noise-like MIMO waveforms with "thumb-tack"-like ambiguity functions complemented by mismatched compression filtering for HF-OTHR mode-selection applications [1, 2, 3]. We demonstrated that DoD-invariant clutter mitigation by mismatched filters is efficient only if the range-Doppler area actually occupied by "normal" clutter is K-times smaller than the maximal rejection zone in single-waveform (SIMO) applications. Since this maximal zone for SIMO mismatched filtering is quite limited [8]

$$(\tau_{\max} - \tau_{\min})(f_{\max} - f_{\min}) \leq 0.2 \qquad (27)$$

its K-times reduction in MIMO applications with K noise-like waveforms significantly reduces the scope of practical scenario addressed by this approach.

We demonstrated that for the given beamsteer directions of the MIMO beamformer, and with the proper geometry selection of the 2D MIMO Tx and Rx antenna arrays as considered in [1], an alternative approach with DoD-dependent "normal" clutter suppression is enabled by the properties of mismatched filters.

Specifically, for a given resolution cell occupied by "normal" clutter in a direction (ϕ,θ) associated in HF propagation geometry with its (group) range τ, we can introduce a (range dependent) synthetic waveform. This synthetic waveform is the coherent sum of K individual MIMO waveforms weighted by the Tx antenna manifold vector, specified by DoD (ϕ,θ). A mismatched filter optimised for sidelobes mitigation with different sidelobes created by different waveforms, is as effective as the filter optimised for the case when all rejected sidelobes are created by the same waveform. For this reason, the maximal dimensions of the clutter zone may remain the same as per the SIMO application.

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