MOVING SOUND SOURCE PARAMETER ESTIMATION USING A SINGLE MICROPHONE AND SIGNAL EXTREMA SAMPLES

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ABSTRACT

Estimating the parameters of moving sound sources using only the source signal is of interest in low-power, and contact-less source monitoring applications, such as, industrial robotics and bio-acoustics. The received signal embeds the motion attributes of the source via Doppler effect. In this paper, we analyze the Doppler effect on mixture of time-varying sinusoids. Focusing, on the instantaneous frequency (IF) of the received signal, we show that the IF profile composed of IF and its first two derivatives can be used to obtain source motion parameters. This requires a smooth estimate of IF profile. However, the numerical implementation of traditional approaches, such as analytic signal and energy separation approach, gives oscillatory behavior hence a non-smooth IF estimate. We devise an algorithm using non-uniformly spaced signal extrema samples of the received signal for smooth IF profile estimation. Using the smooth IF profiles for a source moving on a linear trajectory with constant velocity, an accurate estimate of moving source parameters is obtained. We see promise of this approach for an arbitrary trajectory motion parameter estimation.

Index Terms— Moving Sources, Doppler effect, IF estimation, IF profile, AM-FM signal.

1. INTRODUCTION

Most of the existing techniques for analyzing moving sources use the concept of transmitting a signal of known characteristics and observing the variations in the reflected signal. These techniques include RADAR [1] and laser Doppler velocimetry (LDV) [2]. In the case of active moving sources, however, since the source itself is an emitter (of light or sound), the obvious question is: Can the motion of the source be analyzed using the source signal captured by a passive receiver? This avoids the need to ping the source with a known signal. In this paper, we address the question for a moving sound source when its acoustic signal is captured by a single microphone. Once the source gets into motion, the captured signal embeds the attributes characterizing motion of the source. Denoting the captured signal by $x_{\mathcal{D}}(t)$, we have: $x_{\mathcal{D}}(t) = \mathcal{D}[x(t)]$ where, the operator \mathcal{D} models the effect of motion of the source on the transmitted signal x(t). The source signal x(t) can in general be a periodic, modulated, or transient signal. The class of signals x(t) belongs to can make a difference in terms of how easily the motion attributes of the source can be deciphered from $x_{\mathcal{D}}(t)$.

The effect of source motion can introduce a time-varying amplitude scaling and time-delay in the captured signal, $x_{\mathcal{D}}(t)$. For a source emitting a stationary tone of frequency f_T and moving with a radial velocity $v_r(t)$ towards an omnidirectional receiver, $x_{\mathcal{D}}(t)$ is given

by [3]:

$$x_{\mathcal{D}}(t) = \mathcal{D}[a\sin 2\pi f_T t] = a(t)\sin\phi_R(t) \tag{1}$$
$$f_R(t) = \frac{1}{2\pi} \frac{\mathrm{d}\phi_R(t)}{\mathrm{d}t} = f_T \left(1 - \frac{v_r(t)}{v_s}\right)^{-1} \approx f_T \left(1 + \frac{v_r(t)}{v_s}\right) \tag{2}$$

where v_s is the velocity of sound (taken as 340 m/s in air), a(t) is the amplitude modulation modeling the slow increase in amplitude as the source approaches the receiver, and slow decrease as the source passes away, $f_R(t)$ is the instantaneous frequency of the recieved signal. The approximation in (2) holds for practical acoustic moving sources of interest with $v_r < v_s/10$. For source approaching the receiver, $v_r > 0$, and for source moving away from the receiver, $v_r < 0$. In this paper, we first present an analysis of $\mathcal{D}[.]$ operated on nonstationary signals. Focus is on nonstationary signals which allow compact modeling with a mixture of time-varying amplitude and frequency sinusoids (see Section 2). This can enable analysis of $x_{\mathcal{D}}(t)$ specifically for x(t) which favours a multi-component amplitude modulated-frequency modulated (AM-FM) decomposition, such as chirp harmonics, speech, and music [4].

Next (in section 3), we address the problem of moving source parameter estimation. The source is considered moving with a constant velocity on a linear path. The parameters of interest are $\mathcal{P}=\{\text{range}(d), \text{speed}(v), \text{ and transmitted frequency}(f_T)\}$. We show that in this simplified source motion set-up, profiling the IF with its first-and second- order derivative estimates gives a fairly straight-forward approach to retrieve \mathcal{P} . Analysis highlights the need to estimate a smooth estimate of IF in the received signal. Although, approaches such as analytic signal formulation [5] and energy separation approach (ESA) [6] have been used widely for instantaneous frequency (IF) [7] estimation, these approaches and their numerical estimates do not suffice for smooth IF estimate for the case in hand. Further, Doppler frequency shift can result in wideband signals due to fast and large IF deviations, and such conditions are not suited for existing IF estimation approaches [8–11].

In section 4, we propose a new approach to estimate IF using nonuniform samples of $x_{\mathcal{D}}(t)$. We show that non-uniform samples, here signal extrema samples, encode both AM and IF information. An algorithm resorting to weighted local polynomial regression (LPR) using the non-uniform samples is developed to get smooth AM and IF profile estimate. Estimation of \mathcal{P} obtained using the proposed IF profile estimate is found close to the ground truth. The traditional IF estimates do not perform well when compared to the proposed approach (see section 5).

The simplified set-up of source moving in a linear path with a constant velocity is a constrained case of moving source parametrization when the source is moving in an arbitrary trajectory. Such scenarios often arise in monitoring maneuvers in bio-acoustics (insect and bird flights), and industrial robotics. We hypothesize that estimating arbitrary trajectory can be treated as a generalization of the linear case, and conclude in Section 6.

2. ANALYSIS

In this section, we will consider the operation of $\mathcal{D}[.]$ on x(t) for different class of signals, so as to generalize to a mixture of time-varying sinusoids.

2.1. Signals with Amplitude Modulation (AM)

Consider, $x(t) = a(t) \sin 2\pi f_T t$ where a(t) > 0 is a lowpass AM [12]. Operating $\mathcal{D}[.]$ on x(t) we get:

$$\begin{aligned} x_{\mathcal{D}}(t) &= \mathcal{D}[x(t)] = \mathcal{D}[a(t)\sin 2\pi f_T t] \\ &= \mathcal{D}\left[\left(a_0 + \sum_{k=1}^K a_k \cos\left(2\pi f_k t + \phi_k\right)\right) \sin 2\pi f_T t\right] \\ &= \mathcal{D}\left[a_0 \sin 2\pi f_T t + \sum_{k=1}^K a_k \cos\left(2\pi f_k t + \phi_k\right) \sin 2\pi f_T t\right] \\ &= \mathcal{D}\left[a_0 \sin 2\pi f_T t\right] + \frac{1}{2} \sum_{k=1}^K a_k \mathcal{D}\left[\sin\left(2\pi \left(f_T + f_k\right) t + \phi_k\right) + \frac{1}{2} \sum_{k=1}^K a_k \mathcal{D}\left[\sin\left(2\pi \left(f_T - f_k\right) t - \phi_k\right)\right] \end{aligned}$$

Using (1) and (2) and tracing back upwards,

$$x_{\mathcal{D}}(t) = \left(a_0(t) + \sum_{k=1}^{K} a_k(t) \cos \phi_{k,R}(t)\right) \sin \phi_R(t)$$
$$x_{\mathcal{D}}(t) = a_{\mathcal{D}}(t) \sin \phi_R(t).$$

We see the effect of $\mathcal{D}[.]$ on both the AM and transmitter frequency separately. The effect on transmitted frequency remains same as in (1) for a tone.

2.2. Signals with Frequency Modulation (FM)

Consider, $x(t) = a \sin \phi(t)$, where $\phi(t) = 2\pi f_c t + 2\pi \kappa_f \int_0^t f_m(\tau) d\tau$, $f_m(\tau)$ is the FM, and f_c is the carrier frequency [13]. $x_{\mathcal{D}}(t) = \mathcal{D}[x(t)] = a(t) \sin \phi (t - \alpha(t))$, where $\alpha(t) = d(t)/v_s$ models the time-varying delay introduced due to source motion. Analyzing received frequency, we get:

$$f_{R}(t) = \frac{1}{2\pi} \frac{\mathrm{d}}{\mathrm{d}t} \phi(t - \alpha(t))$$

$$= f_{c} \left(1 - \alpha'(t)\right) + \kappa_{f} f_{m}(t - \alpha(t))(1 - \alpha'(t))$$

$$= f_{c} \left(1 + \frac{v_{r}(t)}{v_{s}}\right) + \kappa_{f} f_{m} \left(t - \frac{d(t)}{v_{s}}\right) \left(1 + \frac{v_{r}(t)}{v_{s}}\right)$$

$$= f_{c,R}(t) + \kappa_{f} f_{m,\mathcal{D}}(t) \qquad (4)$$

$$x_{\mathcal{D}}(t) = a(t) \sin 2\pi \left(\int_{0}^{t} f_{R}(\tau) d\tau\right)$$

$$x_{\mathcal{D}}(t) = a(t) \sin \phi_{\mathcal{D}}(t) \qquad (5)$$

where $f_{m,\mathcal{D}}(t) = \mathcal{D}[f_m(t)]$. We see that the effect of $\mathcal{D}[.]$ can be decomposed on the received frequency $f_R(t)$ as acting separately on f_c and on FM $f_m(t)$.

2.3. Additive mixture of AM-FM signals

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Consider, $x(t) = \sum_{k=1}^{K} a_k(t) \sin \phi_k(t)$ as a mixture of K monocomponent AM-FM signals non-overlapping in frequency content. Operating $\mathcal{D}[.]$ on x(t) we get:

$$x_{\mathcal{D}}(t) = \mathcal{D}[x(t)] = \mathcal{D}\left[\sum_{k=1}^{K} a_k(t) \sin \phi_k(t)\right]$$

sing (3) and (5), $x_{\mathcal{D}}(t) = \sum_{k=1}^{K} a_{\mathcal{D},k}(t) \sin \phi_{\mathcal{D},k}(t)$. (6)

We see that each mono-component in the multi-component mixture independently encodes the motion parameters. Hence, an approach to motion parameter estimation is to first do a multi-component decomposition using approaches such as empirical mode decomposition [14], and then analyze each obtained mono-component separately. Although both AM and FM encode motion parameters, we will focus on using FM in the received signal. For a stationary sound source, the AM has been shown to be more affected than FM due to reverberation [15, 16]. Hence, the motion parameter estimates when obtained using AM may be less robust in a general acoustic environment.

In the next section we analyze a tone signal to understand how the motion parameters are encoded in FM, and how to decipher them.

3. SOURCE PARAMETER ESTIMATION

Consider, a hypothetical scenario of a sound source moving on a (3]inear path with a constant velocity. The goal is to estimate the moving source parameters using only a single fixed microphone. The microphone is omnidirectional, and the source moves at a velocity v, emitting a tone at f_T . The parameters of interest are \mathcal{P} ={range (d), v, and f_T }. The setup is illustrated in Fig. 1. We assume the



Fig. 1. [In color] Schematic of moving source and receiver. source trajectory makes a perpendicular distance of d_0 with the microphone, and the instant at which $d(t) = d_0$ will be referred to as the cross-over point. The time-instant of cross-over will be denoted by t_c . Fig. 1 shows the gradual decrease in IF as the source approaches the microphone on the linear path (blue straight path). The rate of variation in IF will depend on d_0 and v; a smaller d_0

with higher v gives a higher rate of IF variation than a larger d_0 with smaller v. This is shown for three sets of $\{d_0, v\}$ with $f_T = 100$ Hz in Fig. 1. We analyze the Doppler shift, or the variation in received IF (denoting by $f_R(t)$), below.

$$f_R(t) = f_T\left(1 + \frac{v_r(t)}{v_s}\right) = f_T\left(1 + \frac{v\cos\theta(t)}{v_s}\right)$$
(7)

$$\cos\theta(t) = v \frac{(t_c - t)}{d(t)}, \ d(t) = \sqrt{d_0^2 + v^2(t_c - t)^2} \tag{8}$$

$$\Rightarrow f_R(t) = f_T \left(1 + \frac{v(t_c - t)}{v_s \sqrt{(d_0/v)^2 + (t_c - t)^2}} \right).$$
(9)

3.1. Estimation of f_T

Analyzing the variation in slope of $f_R(t)$, it is clear that $f_R(t)$ has an inflection point at the cross-over instant t_c . This is illustrated in Fig. 2. Thus, the extrema instant in derivative of $f_R(t)$ or the zerocrossing instant in second derivative of $f_R(t)$ gives t_c . At $t = t_c$,



Fig. 2. [In color] Received signal IF profile composed of IF and its first two derivatives. The plots have been rescaled to illustrate the variation at the cross-over instant t_c .

 $\theta(t_c) = \pi/2$, that is the velocity vector of the source is perpendicular to the line joining the source and the receiver. Hence, the radial velocity v_r becomes zero at this time instant. Putting $t = t_c$ in (9) we get $f_R(t_c) = f_T$.

3.2. Estimation of v and d

Analyzing first two derivatives of $f_R(t)$ we get:

$$f_R'(t) = \frac{-f_T v d_0^2}{v_s \left[(d_0/v)^2 + (t_c - t)^2 \right]^{3/2}}$$
(10)

$$f_R''(t) = \frac{-3f_T v d_0^2(t_c - t)}{v_s \left[(d_0/v)^2 + (t_c - t)^2 \right]^{5/2}}$$
(11)

$$\frac{f_R'(t)}{f_R''(t)} = \frac{\left[(d_0/v)^2 + (t_c - t)^2 \right]}{3(t_c - t)} \tag{12}$$

We obtain the ratio d_0/v by substituting any value of t around t_c in (12). Substituting d_0/v in (9) we obtain v. From the ratio d_0/v we get d_0 , and d(t) is obtained by $d(t) = \sqrt{d_0^2 + v^2(t_c - t)^2}$. Thus, we see that using IF profile of the received signal, \mathcal{P} can be obtained in a straight forward manner. The FM in the received signal $x_{\mathcal{D}}(t)$ however is a wideband signal (due to Doppler shifts induced by motion of the source) and may have large frequency deviations depending on v, d_0 , and f_T . Also, the IF profile estimation requires smooth IF estimate. In the next section, we propose a new algorithm to get a smooth estimate of IF overcoming performance limitations of traditional methods under these cases.

4. IMPROVED AM AND IF ESTIMATION

Consider a mono-component AM-FM signal $x(t) = a(t) \sin \phi(t)$ and let S_e denote the captured data-set over a finite duration using signal extrema sampling.

$$\{x(t_i), t_i\} \in \mathcal{S}_e \Rightarrow x'(t_i) = \frac{\mathrm{d}x}{\mathrm{d}t}\Big|_{t_i} = 0$$

For a ZC instant t_i , $x(t_i) = a(t_i) \sin \phi(t_i) = 0$. As $a(t) > 0 \forall t$, hence $\phi(t_i) = i\pi$ (unwrapped phase). Thus, only instantaneous phase (IP) information is contained in zero-crossing (ZC) instants, and AM information is absent. Using this, IF estimation has been proposed in [17] for phase signals. Between two successive ZC instants, we have an extrema sample which can aid in tracking AM variations as well as IP variations. At any extrema instant t_i we have,

$$x(t_i) = a(t_i)\sin\phi(t_i) \tag{13}$$

$$x'(t_i) = a'(t_i)\sin\phi(t_i) + a(t_i)\phi'(t_i)\cos\phi(t_i) = 0$$
(14)

For lowpass AM, $\sup_{t} \left| \frac{a'(t)}{a(t)\phi'(t)} \right| << 1$ holds [18]. In other words, a(t) variation in a pseudo period of x(t) i.e $2\pi/\phi'(t)$ is small. Under this assumption, at extrema instants we have: $\phi(t_i) = (2i+1)\pi/2$, and $a(t_i) = \frac{x(t_i)}{\operatorname{sign}[x(t_i)]}, \forall t_i \in S_e$.

4.1. Algorithm

Using the sampled data-set S_e , the estimation of envelope and IP can be based on a global polynomial regression [17, 19]. In practice however, variations in envelope and IP can be arbitrary, this makes polynomial order choice a model estimation problem. Assuming no information about the nature of variations in IP, we use a local polynomial regression (LPR) [20]. Here LPR, for IP estimation at any instant, is carried out by making a least squares (LS) fit to the k-nearest neighbor samples from S_e with a d degree polynomial. Smooth estimates of IF and its first two derivatives are obtained by performing derivative operations on the obtained local polynomial fit for IP. Further to characterize a wideband FM, we associate a diagonal weight matrix with the LS formulation in LPR. The diagonal entries are dependent inversely on the distance of each of the k nearest neighbor samples from the required time instant. We find this essential to track the wideband FM with sudden frequency deviations more precisely, and such scenarios can occur due to Doppler effects depending on \mathcal{P} . A comparison of LPR with a weighted LPR is shown in Fig. 3. As can be seen, in LPR higher k gives more smoothness but at the expense of loss of good tracking of IF. However, this is not the case with weighted LPR approach which increases smoothness maintaining good tracking with increase in k. The AM is a slow varying signal relative to the underlying phase signal. The extrema instants, hence, serve as densely sampled set for AM. The AM estimate is obtained by using LPR on signal amplitude values in S_e .

5. SIMULATION

As introduced in section 3, we here present the estimation of \mathcal{P} from the IF of the received signal.

5.1. Source Signal: Tone signal

Fig. 4 shows the received IF and its first two derivatives obtained using the ES-based approach described in Algorithm 1 (choosing $d_e =$

Algorithm 1 ES (Extrema Samples)-based approach

step 0: Set the LPR parameters k and d to: $\{k_e, d_e\}$ and $\{k_f, d_f\}$ for AM and IF estimates respectively. Get S_e data-set. Pool extrema instants to T_e .

step 1: For $t = nT_s$, choose k_e and k_f samples from \mathcal{T}_e as nearest neighbor in euclidean distance. Make \mathbf{x}_e and $\phi_{\mathbf{z}\mathbf{e}}$ denote the absolute amplitude and IP column vectors.

step 2: AM estimation: Make Vandermonde matrix \mathbf{V} ($k_e \times (d_e + 1)$) with the k_e samples. $\mathbf{a} = \mathbf{V}^{\dagger} \mathbf{x}_{\mathbf{e}}$, and $\hat{a}[nT_s] = \sum_{k=0}^{d_e} a_k (nT_s)^k$.

step 3: IF profile: Make the Vandermonde matrix \mathbf{V} $(k_f \times (d_f + 1))$ with the k_f samples. $\mathbf{b} = (\mathbf{W}\mathbf{V})^{\dagger}\mathbf{W}\phi_{\mathbf{ze}}$, where $\mathbf{W} = \mathbf{diag}(|nT_s - t_i|^{-1})$. $\hat{f}_R[nT_s] = \sum_{k=1}^{d_f} b_k k(nT_s)^{k-1}$, $\hat{f}_R'[nT_s] = \sum_{k=2}^{d_f} b_k k(k-1)(nT_s)^{k-2}$, and $\hat{f}_R''[nT_s] = \sum_{k=3}^{d_f} b_k k(k-1)(k-2)(nT_s)^{k-3}$,

return:
$$\hat{a}(t)|_{t=nT_s}, f_R(t)|_{t=nT_s}, f'_R(t)|_{t=nT_s}, \text{and } f''_R(t)|_{t=nT_s}.$$



Fig. 3. [In color] Sigmoidal IF is synthesized using sigmoidal parameter $\alpha = 10000$, and signal AM is $a(t) = (2 + .5 \sin 65\pi t)$; in simulation $T_s = 1/32$ ms, and p = 3. (a) IF estimation using LPR, (b) IF estimation using weighted LPR.

 $d_f = 4$, and $k_e = k_f = 14$). The IF profile estimated using ESbased approach is less noisy for first two derivatives when compared to numerically implemented analytic signal based, and discrete-ESA based approaches. This makes ES-based IF profile estimate more suitable for estimation of \mathcal{P} . We obtained $\mathcal{P}_{est}=\{d_0 = 2.03 \text{ m}, v = 20.15 \text{ m/s}, f_T = 100.00 \text{ Hz}\}$ for $\mathcal{P}_{true}=\{d_0 = 2 \text{ m}, v = 20 \text{ m/s}, f_T = 100 \text{ Hz}\}$.

5.2. Source Signal: AM-FM signal

Fig. 5 shows the received IF for an AM-FM transmitted signal. The transmitted signal has a gaussian AM centered at t_c and FM given by $f_T = f_c + f_m(t)$, where $f_c = 100$ Hz, and $f_m(t) = 5 \sin 15\pi t$. Based on (4), the received IF can be decomposed as:

$$f_R(t) = f_{c,R}(t) + f_{m,\mathcal{D}}(t) \tag{15}$$

In (15), for fast-varying FM in source signal, $f_{c,R}(t)$ can be analyzed as a slow variation relative to $f_{m,\mathcal{D}}(t)$. In such cases, $f_R(t)$ can be decomposed using EMD [21] to extract $f_{c,R}(t)$. This is illustrated in Fig. 5 (middle). The extracted IF for the carrier frequency f_c was found to closely match $f_{c,R}(t)$, and hence can be used for estimation of \mathcal{P} . The IF estimate of $f_{m,\mathcal{D}}(t)$ was found to be close to $f_m(t)$ and can be used to analyze acoustic features of the source signal (shown in Fig. 5 bottom).



Fig. 4. [In color] The ES-based estimates are compared against analytic signal (AS) based and discrete ESA (DESA-I). In the bottom plot AS and DESA-I gave highly noisy estimates hence are omitted from the plot display.



Fig. 5. [In color] Received IF estimate and its components obtained after an EMD. The transmitted signal is an AM-FM signal with FM being $f_T = f_c + f_m(t)$ where $f_c = 100$ Hz, and $f_m(t) = 5 \sin 15\pi t$.

6. DISCUSSION

The paper analyzed the Doppler effect on AM-FM signals. Focusing on IF of the received signal, an approach to obtain the moving source parameters was presented using IF and its first two derivatives. A new algorithm based on extrema samples was used to obtain a smooth estimate of received IF, and estimate \mathcal{P} via IF profiling. The approach was demonstrated for source signal being a tone. A performance analysis of extrema sample for noisy signal, and its impact in AM-FM estimation with comparison to traditional methods is done in [22]. In future, we intend to extend the approach to estimation of \mathcal{P} for source moving in arbitrary trajectory and emitting any acoustic signal. An arbitrary trajectory can be approximated as a connected segments of small linear trajectories. The full analysis of such a case will be a generalization of the linear trajectory formulation used in this paper.

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