MULTI-MODULUS ALGORITHMS USING HYPERBOLIC AND GIVENS ROTATIONS FOR BLIND DECONVOLUTION OF MIMO SYSTEMS

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ABSTRACT

The issue of blind Multiple-Input and Multiple-Output (MIMO) deconvolution of communication system is addressed. Two new iterative Blind Source Separation (BSS) algorithms are presented, based on the minimization of Multi-Modulus (MM) criterion. A pre-whitening filter is utilized to transform the problem into finding a unitary beamformer matrix. Then, applying iterative Givens and Hyperbolic rotations results in Givens Multi-modulus Algorithm (G-MMA) and Hyperbolic G-MMA (HG-MMA), respectively. Proposed algorithms are compared with several BSS algorithms in terms of Signal to Interference and Noise Ratio (SINR) and Symbol Error Rate (SER) and it was shown to outperform them.

***Index Terms*—** blind source separation, constant modulus algorithm, multi-modulus algorithm, Givens and Hyperbolic rotations

1. INTRODUCTION

Blind Source Separation (BSS) is an important tool as it allows us to get rid of training sequences which might not be available or too complex to implement and reduces the effective data rate. BSS uses *a priori* information regarding the statistics of transmitted source signals. In the context of MIMO systems, BSS algorithms aim in finding a filtering matrix using only the received signals in order to estimate source signals and unknown channel matrix. Various BSS cost functions can be found in the literature [1, 2]. Among them, Constant Modulus (CM) criterion for phase modulated signals such as PSK and MM criterion for higher QAM signals have attracted an important interest.

The CM criterion restricts the squared modulus of the output to be constant. However, the MM criterion improves the CM one by utilizing dispersion of real and imaginary parts separately, so it is more suitable for higher QAM constellation. In the context of blind equalization, several algorithms are presented in [3] and [4] to minimize the CM and MM criteria, respectively. For MIMO systems, out of numerous implementations of CM criterion, the algebraic solution named as Analytical Constant Modulus Algorithm (ACMA) [5] provides an exact solution in the noise free case. It is capable of separating all sources in a batch mode using only few samples by solving a genaralized eigenvalue problem. To overcome the drawback of numerical complexity of ACMA, two algorithms G-CMA and HG-CMA are presented in [6], which outperform ACMA. Similarly for MIMO systems, a Multi-Modulus Algorithm (MMA) is presented in [7], which outperforms the Multi-User Kurtosis (MUK) algorithm [8].

In this paper, we propose two algorithms to minimize MM criterion using unitary Givens and non-unitary Hyperbolic rotations [9, 10]. The received signals are passed through a pre-whitening filter so that the problem can be reduced to finding a unitary beamformer matrix. Then, the complex received filtered signals are converted into real one before applying the rotations iteratively to find the desired beamformer matrix.

To the best of our knowledge, the MM criterion has never been minimized using Givens and Hyperbolic rotations in the context of BSS. Previously, Stochastic Gradient Algorithm (SGA) and analytical techniques are used to find the beamformer matrix for MIMO systems, such as MIMO MMA [7] and Analytical MMA (AMMA) [11]. As SGA is slow in convergence so our proposed algorithms are faster than MIMO MMA. Moreover, the SINR and SER curves show that proposed algorithms perform much better than MUK, ACMA, G-CMA, HG-CMA and AMMA.

The remaining part of the paper is organized as follows: In Section 2 problem formulation and assumptions are presented. Section 3 and 4 introduces the G-MMA and HG-MMA algorithms, respectively. Simulation results are presented in Section 5 and Section 6 concludes the paper.

2. PROBLEM FORMULATION

Consider below given MIMO system with M sources and N receivers

$$\mathbf{y}(i) = \mathbf{x}(i) + \mathbf{n}(i) = \mathbf{A}\mathbf{s}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{s}(i) = [s_1(i), s_2(i), \ldots, s_M(i)]^T$ is $M \times 1$ source vector, $\mathbf{n}(i) = [n_1(i), n_2(i), \ldots, n_N(i)]^\mathsf{T}$ is the $N \times 1$ additive noise vector, $\mathbf{A}$ is the $N \times M$ MIMO channel matrix and $\mathbf{y}(i) = [y_1(i), y_2(i), \ldots, y_N(i)]^\mathsf{T}$ is the $N \times 1$ received vector.

It is assumed that $N \geq M$ so the channel matrix $\mathbf{A}$ is full column rank, source signals are zero mean, discrete valued, independent and identically distributed (i.i.d) random processes and the noise is additive white independent from the source signals.

The source signals are recovered blindly (i.e., relying only on received data) using $M \times N$ beamformer matrix $\mathbf{W}$. The receiver output can be written as

$$\mathbf{z}(i) = \mathbf{W}\mathbf{y}(i) = \mathbf{W}\mathbf{A}\mathbf{s}(i) + \bar{\mathbf{n}}(i) = \mathbf{G}\mathbf{s}(i) + \bar{\mathbf{n}}(i) \tag{2}$$

where $\mathbf{z}(i) = [z_1(i), z_2(i), \ldots, z_M(i)]^\mathsf{T}$ is the $M \times 1$ vector of the estimated source signals, $\mathbf{G} = \mathbf{W}\mathbf{A}$ is the $M \times M$ global system

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matrix and $\bar{\mathbf{n}}(i) = \mathbf{W}\mathbf{n}(i)$ is the filtered noise at the receiver output. Usually in BSS, the output is recovered up to a possible permutation and scaling factor [6] i.e., $\mathbf{W}\mathbf{x}(i) = \mathbf{P}\mathbf{\Lambda}\mathbf{s}(i)$, where $\mathbf{P}$ is a permutation matrix and $\mathbf{\Lambda}$ is a non-singular diagonal matrix. One proposes to estimate matrix $\mathbf{W}$ by minimizing the MM criterion defined by [12]

$$\mathcal{J}(\mathbf{W}) = \sum_{j=1}^{K}\sum_{i=1}^{M}\left((z_{ij,R})^2 - R_R\right)^2 + \left((z_{ij,I})^2 - R_I\right)^2 \tag{3}$$

where $R_R = E[\mathbf{s}_R^4(i)]/E[\mathbf{s}_R^2(i)]$ and $R_I = E[\mathbf{s}_I^4(i)]/E[\mathbf{s}_I^2(i)]$ are dispersion constants of real and imaginary parts respectively, z_{ij} is the (i,j)th element of $\mathbf{Z} = \mathbf{W}\mathbf{Y}$, with $\mathbf{Y} = [\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(K)]$ and K is the sample size. Also x_R and x_I denotes the real and imaginary parts of signal x.

To simplify the problem, a prewhitening operation [3, 6] is applied on received signals which reduces the dimension of $\mathbf{Y}$ from $N \times K$ to $M \times K$. Using underscore to denote prewhitened variables and assuming the noise free case, the received signal after prewhitening can be given as $\underline{\mathbf{Y}} = \mathbf{B}\mathbf{Y} = \mathbf{B}\mathbf{A}\mathbf{S} = \mathbf{V}\mathbf{S}$, where $\mathbf{B}$ is an $M \times N$ prefilter, $\mathbf{S}$ is the $M \times K$ source signals matrix and $\mathbf{V} = \mathbf{B}\mathbf{A}$ is a $M \times M$ unitary[1] matrix. Now, the problem of BSS is changed into finding a matrix $\mathbf{V}$ and thus the beamformer matrix can be expressed as $\mathbf{W} = \mathbf{V}^{-1}\mathbf{B}$ resulting in output

$$\mathbf{Z} = \mathbf{W}\mathbf{Y} = \mathbf{V}^{-1}\mathbf{B}\mathbf{Y} = \mathbf{V}^{-1}\underline{\mathbf{Y}} = \mathbf{V}^{-1}\mathbf{V}\mathbf{S} = \mathbf{S} \tag{4}$$

After prewhitening, to minimize the MM cost function given in (3), complex received prewhitened signal matrix $\underline{\mathbf{Y}}$ is converted into real matrices containing real and imaginary parts. If matrix $\mathbf{X}$ after transformation to real matrix is shown as $\acute{\mathbf{X}}$ then the received prewhitened transformed matrix is given as $\underline{\acute{\mathbf{Y}}} = \acute{\mathbf{V}}\acute{\mathbf{S}}$ where

$$\underline{\acute{\mathbf{Y}}} = \begin{bmatrix}\underline{\mathbf{Y}}_R \\ \underline{\mathbf{Y}}_I\end{bmatrix} 2M \times K \;\text{ and }\; \acute{\mathbf{V}} = \begin{bmatrix}\mathbf{V}_R & -\mathbf{V}_I \\ \mathbf{V}_I & \mathbf{V}_R\end{bmatrix} 2M \times 2M \tag{5}$$

3. GIVENS MMA (G-MMA)

To miminize the MM criterion in (3), matrix $\acute{\mathbf{V}}$ needs to be rewritten using Givens rotations. Similar to Jacobi-like algorithms [13, 14] and using Lemma 1 given in [15], matrix $\acute{\mathbf{V}}$ can be decomposed into a product of elementary Givens rotations $\boldsymbol{\mathcal{G}}_{p,q}(\theta)$

$$\acute{\mathbf{V}} = \prod_{N_{Sweeps}}\prod_{1\le p,q\le M} \boldsymbol{\mathcal{G}}_{p,q}(\theta)\boldsymbol{\mathcal{G}}_{p+M,q+M}(\theta)\boldsymbol{\mathcal{G}}_{p,q+M}(\acute{\theta})\boldsymbol{\mathcal{G}}_{q,p+M}(\acute{\theta}) \tag{6}$$

where N_{Sweeps} denotes the number of iterations and $\boldsymbol{\mathcal{G}}_{p,q}$ is an identity matrix except for the four elements $\mathcal{G}_{pp}, \mathcal{G}_{qq}, \mathcal{G}_{pq}$ and $\mathcal{G}_{qp}$ given by

$$\begin{bmatrix}\mathcal{G}_{pp} & \mathcal{G}_{pq} \\ \mathcal{G}_{qp} & \mathcal{G}_{qq}\end{bmatrix} = \begin{bmatrix}\cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta)\end{bmatrix} \tag{7}$$

The specific rotation matrix selection in (6) is used to preserve the structure of $\acute{\mathbf{V}}$ shown in (5). Also, rotations $\boldsymbol{\mathcal{G}}_{p,q}$ and $\boldsymbol{\mathcal{G}}_{p+M,q+M}$ are applied using same parameter (θ) while $\boldsymbol{\mathcal{G}}_{p,q+M}$ and $\boldsymbol{\mathcal{G}}_{q,p+M}$ are applied using another same parameter $(\acute{\theta})$.

[1] This is the ideal case when $K \gg 1$ and noise is negligible. In this case, $\mathbf{V}$ is searched as a unitary matrix using Givens rotations (G-MMA). Otherwise, if the sample size is small or moderate, one allows deviation from unitary condition by using both Givens and Hyperbolic rotations (HG-MMA).

In order to minimize the MM cost function in (3), we only need to find the rotation angles (θ) and $(\acute{\theta})$ in a successive way. So, consider only one unitary transformation $\tilde{\mathbf{Y}} = \boldsymbol{\mathcal{G}}_{p,q}\underline{\acute{\mathbf{Y}}}$, which according to (7) only changes the rows p and q of $\underline{\acute{\mathbf{Y}}}$ such as

$$\begin{aligned}\tilde{y}_{pj} &= \cos(\theta)\underline{\acute{y}}_{pj} + \sin(\theta)\underline{\acute{y}}_{qj} \\ \tilde{y}_{qj} &= -\sin(\theta)\underline{\acute{y}}_{pj} + \cos(\theta)\underline{\acute{y}}_{qj}\end{aligned} \tag{8}$$

where $\underline{\acute{y}}_{ij}$ denotes the (i,j)th entry of $\underline{\acute{\mathbf{Y}}}$. So using (8), only two rows $i = p$ and $i = q$ of the real part of (3) are modified and omitting the terms in $\underline{\acute{\mathbf{Y}}}$ independent from (θ), (3) can be rewritten as

$$\mathcal{J}(\boldsymbol{\mathcal{G}}_{pq}) = \sum_{j=1}^{K}\left[\left(\tilde{y}_{pj}^2 - R_R\right)^2 + \left(\tilde{y}_{qj}^2 - R_R\right)^2\right] \tag{9}$$

Using double angle identities and (8), we get

$$\begin{aligned}\tilde{y}_{pj}^2 &= \mathbf{t}_j^{\mathsf{T}}\mathbf{v} + \frac{1}{2}\left(\underline{\acute{y}}_{pj}^2 + \underline{\acute{y}}_{qj}^2\right) \\ \tilde{y}_{qj}^2 &= -\mathbf{t}_j^{\mathsf{T}}\mathbf{v} + \frac{1}{2}\left(\underline{\acute{y}}_{pj}^2 + \underline{\acute{y}}_{qj}^2\right)\end{aligned} \tag{10}$$

where

$$\begin{aligned}\mathbf{v} &= \begin{bmatrix}\cos(2\theta) & \sin(2\theta)\end{bmatrix}^{\mathsf{T}} \\ \mathbf{t}_j &= \begin{bmatrix}\frac{1}{2}(\underline{\acute{y}}_{pj}^2 - \underline{\acute{y}}_{qj}^2) & \underline{\acute{y}}_{pj}\underline{\acute{y}}_{qj}\end{bmatrix}^{\mathsf{T}}\end{aligned} \tag{11}$$

Using (10) in (9), we obtain

$$\mathcal{J}(\boldsymbol{\mathcal{G}}_{pq}) = 2\mathbf{v}^{\mathsf{T}}\sum_{j=1}^{K}\left[\mathbf{t}_j\mathbf{t}_j^{\mathsf{T}}\right]\mathbf{v} + 2\left(\frac{\underline{\acute{y}}_{pj}^2 + \underline{\acute{y}}_{qj}^2}{2} - R_R\right)^2 \tag{12}$$

Similarly, by applying rotation $\boldsymbol{\mathcal{G}}_{p+M,q+M}$ with the same angle parameter θ, one finally obtains (constant terms are again omitted)

$$\mathcal{J}(\boldsymbol{\mathcal{G}}_{pq}\,\boldsymbol{\mathcal{G}}_{p+M,q+M}) = 2\mathbf{v}^{\mathsf{T}}\sum_{j=1}^{K}\left[\mathbf{t}_j\mathbf{t}_j^{\mathsf{T}} + \acute{\mathbf{t}}_j\acute{\mathbf{t}}_j^{\mathsf{T}}\right]\mathbf{v} \tag{13}$$

where $\acute{\mathbf{t}}_j = [\frac{1}{2}(\underline{\acute{y}}_{p+M,j}^2 - \underline{\acute{y}}_{q+M,j}^2), \underline{\acute{y}}_{p+M,j}\underline{\acute{y}}_{q+M,j}]^{\mathsf{T}}$. By setting the derivative of (13) w.r.t (θ) to zero, we get

$$\begin{aligned}\theta = \frac{1}{4}\arctan\Bigg[&\frac{4\sum_{j=1}^{K}(\underline{\acute{y}}_{pj}^2 - \underline{\acute{y}}_{qj}^2)\underline{\acute{y}}_{pj}\underline{\acute{y}}_{qj} + \ldots}{\sum_{j=1}^{K}\left\{(\underline{\acute{y}}_{pj}^2 - \underline{\acute{y}}_{qj}^2)^2 - 4\,\underline{\acute{y}}_{pj}^2\underline{\acute{y}}_{qj}^2\right\} + \ldots} \\ &\frac{4\sum_{j=1}^{K}(\underline{\acute{y}}_{p+M,j}^2 - \underline{\acute{y}}_{q+M,j}^2)\underline{\acute{y}}_{p+M,j}\underline{\acute{y}}_{q+M,j}}{\sum_{j=1}^{K}\left\{(\underline{\acute{y}}_{p+M,j}^2 - \underline{\acute{y}}_{q+M,j}^2)^2 - 4\,\underline{\acute{y}}_{p+M,j}^2\underline{\acute{y}}_{q+M,j}^2\right\}}\Bigg]\end{aligned} \tag{14}$$

The remaining Givens rotations can be found similarly and applied successively on matrix $\underline{\acute{\mathbf{Y}}}$ to compute $\acute{\mathbf{V}}$ according to (6) where matrix $\acute{\mathbf{V}}$ is initialized as $\acute{\mathbf{V}} = \mathbf{I}$ as shown in Table 1.

4. HYPERBOLIC G-MMA (HG-MMA)

G-MMA does not perform well for small number of samples K, so non-unitary Hyperbolic rotations are applied alternatively along with Givens rotations to overcome this limitation, resulting in algorithm named HG-MMA. Matrix $\acute{\mathbf{V}}$ can be decomposed into a product of

Initialization: $\acute{\mathbf{V}} = \mathbf{I}_{2M}$
1. Prewhitening: $\underline{\mathbf{Y}} = \mathbf{B}\mathbf{Y}$
2. Construct real matrix $\acute{\underline{\mathbf{Y}}}$ using (5)
3. Givens Rotations:
for $i = 1 : N_{Sweeps}$ **do**
  for $p = 1 : M-1$ **do**
    for $q = p+1 : M$ **do**
      a. find θ for (p,q) & $(p+M, q+M)$ using (14)
      b. Compute $\boldsymbol{\mathcal{G}}_{p,q}\&\boldsymbol{\mathcal{G}}_{p+M,q+M}$ using (7) for same (θ)
      c. $\acute{\underline{\mathbf{Y}}} = \boldsymbol{\mathcal{G}}_{p,q}\boldsymbol{\mathcal{G}}_{p+M,q+M}\acute{\underline{\mathbf{Y}}}$
      d. $\acute{\mathbf{V}} = \boldsymbol{\mathcal{G}}_{p,q}\boldsymbol{\mathcal{G}}_{p+M,q+M}\acute{\mathbf{V}}$
      repeat steps (a **to** d) for $(p, q+M)$ & $(q, p+M)$ using same $(\acute{\theta})$
    end for
  end for
end for
4. Construct complex matrix $\mathbf{V}$ using (5)
5. Compute separation matrix: $\mathbf{W} = \mathbf{V}^{\mathsf{H}}\mathbf{B}$
6. Estimated Sources: $\hat{\mathbf{S}} = \mathbf{W}\mathbf{Y}$

Table 1. Givens MMA (G-MMA) Algorithm

elementary Hyperbolic rotations, Givens rotations and normalization transformation as follows

$$\begin{aligned} \acute{\mathbf{V}} = \prod_{N_{Sweeps}} \prod_{1\le p,q\le M} & \boldsymbol{\Gamma}_{p,q}\boldsymbol{\Gamma}_{p+M,q+M}\boldsymbol{\Gamma}_{p,q+M}\boldsymbol{\Gamma}_{q,p+M} \\ \boldsymbol{\Gamma}_{p,q} &= \boldsymbol{\mathcal{D}}_{p,q}\boldsymbol{\mathcal{G}}_{p,q}\boldsymbol{\mathcal{H}}_{p,q} \end{aligned} \tag{15}$$

where $\boldsymbol{\mathcal{D}}_{p,q}, \boldsymbol{\mathcal{G}}_{p,q}$ and $\boldsymbol{\mathcal{H}}_{p,q}$ refer to normalization, Givens and Hyperbolic transformations, respectively. $\boldsymbol{\mathcal{G}}_{p,q}$ is defined in (7) and similarly $\boldsymbol{\mathcal{H}}_{p,q}$ is an identity matrix except for the four elements $\mathcal{H}_{pp}, \mathcal{H}_{qq}, \mathcal{H}_{pq}$ and $\mathcal{H}_{qp}$ given by

$$\begin{bmatrix} \mathcal{H}_{pp} & \mathcal{H}_{pq} \\ \mathcal{H}_{qp} & \mathcal{H}_{qq} \end{bmatrix} = \begin{bmatrix} \cosh(\gamma) & \sinh(\gamma) \\ \sinh(\gamma) & \cosh(\gamma) \end{bmatrix} \tag{16}$$

where γ is a hyperbolic transformation parameter. Similar to Givens rotations, the Hyperbolic rotations $\boldsymbol{\mathcal{H}}_{p,q}$ and $\boldsymbol{\mathcal{H}}_{p+M,q+M}$ are applied using same parameter (γ) while $\boldsymbol{\mathcal{H}}_{p,q+M}$ and $\boldsymbol{\mathcal{H}}_{q,p+M}$ are applied using another same but opposite parameter $(\acute{\gamma})$ and $(-\acute{\gamma})$, respectively[2].

We will consider dispersion parameters R_R and R_I be equal to 1 and use $\boldsymbol{\mathcal{D}}_{p,q}$ for normalization where $\boldsymbol{\mathcal{D}}_{p,q}$ is a diagonal matrix with diagonal elements equal to one except for the two elements $\mathcal{D}_{pp} = \lambda_p$ and $\mathcal{D}_{qq} = \lambda_q$. Below, we give a brief of finding hyperbolic and normalization transformation parameters to minimize the MM criterion (3).

Similar to Givens rotations we will consider only one non-unitary transformation $\tilde{\mathbf{Y}} = \boldsymbol{\mathcal{H}}_{p,q}\acute{\underline{\mathbf{Y}}}$, which according to (16) only changes the rows p and q of $\acute{\underline{\mathbf{Y}}}$ such as

$$\begin{aligned} \tilde{y}_{pj} &= \cosh(\gamma)\acute{\underline{y}}_{pj} + \sinh(\gamma)\acute{\underline{y}}_{qj} \\ \tilde{y}_{qj} &= \sinh(\gamma)\acute{\underline{y}}_{pj} + \cosh(\gamma)\acute{\underline{y}}_{qj} \end{aligned} \tag{17}$$

[2] Indeed it is shown in [15] that these conditions are necessary for preserving the structure of matrix $\acute{\mathbf{V}}$ in (5) along the iterations.

Using hyperbolic double angle identities and (17), we get

$$\begin{aligned} \tilde{y}_{pj}^2 &= \mathbf{r}_j^{\mathsf{T}}\mathbf{u} + \frac{1}{2}\left(\acute{\underline{y}}_{pj}^2 - \acute{\underline{y}}_{qj}^2\right) \\ \tilde{y}_{qj}^2 &= \mathbf{r}_j^{\mathsf{T}}\mathbf{u} - \frac{1}{2}\left(\acute{\underline{y}}_{pj}^2 - \acute{\underline{y}}_{qj}^2\right) \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mathbf{u} &= \begin{bmatrix} \cosh(2\gamma) & \sinh(2\gamma) \end{bmatrix}^{\mathsf{T}} \\ \mathbf{r}_j &= \begin{bmatrix} \frac{1}{2}(\acute{\underline{y}}_{pj}^2 + \acute{\underline{y}}_{qj}^2) & \acute{\underline{y}}_{pj}\acute{\underline{y}}_{qj} \end{bmatrix}^{\mathsf{T}} \end{aligned} \tag{19}$$

Using (18) and omitting the terms of $\acute{\underline{\mathbf{Y}}}$ independent from (γ), (3) can be rewritten as

$$\begin{aligned} \mathcal{J}(\boldsymbol{\mathcal{H}}_{p,q}) &= \mathbf{u}^{\mathsf{T}}\left[\sum_{j=1}^{K}\mathbf{r}_j\mathbf{r}_j^{\mathsf{T}}\right]\mathbf{u} - 2\mathbf{u}^{\mathsf{T}}\left[\sum_{j=1}^{K}\mathbf{r}_j\right] \\ &= \mathbf{u}^{\mathsf{T}}\mathbf{R}\mathbf{u} - 2\mathbf{r}^{\mathsf{T}}\mathbf{u} \end{aligned} \tag{20}$$

The optimization problem in (20) can be solved using the Lagrange multiplier method and thus can be given as

$$\min_{\mathbf{u}} \ \mathcal{F}(\mathbf{u}) \quad \text{s.t.} \ \mathbf{u}^{\mathsf{T}}\mathbf{J}_2\mathbf{u} = 1 \tag{21}$$

where $\mathbf{J}_2 = \operatorname{diag}([1, -1])$, such a constraint is equivalent to $\cosh^2(2\gamma) - \sinh^2(2\gamma) = 1$. The solution of (21) is given as

$$\mathbf{u} = (\mathbf{R} + \lambda\mathbf{J}_2)^{-1}\mathbf{r} \tag{22}$$

where λ is the solution of

$$\mathbf{r}^{\mathsf{T}}(\mathbf{R} + \lambda\mathbf{J}_2)^{-1}\mathbf{J}_2(\mathbf{R} + \lambda\mathbf{J}_2)^{-1}\mathbf{r} = 1 \tag{23}$$

which is a 4-th order polynomial equation and desired solution λ is it's real valued root that corresponds to the minimum value of (21). Using desired λ in (22) will give us the solution $\mathbf{u} = [u_1, u_2]^{\mathsf{T}}$ and the Hyperbolic transformation matrix can be computed as

$$\mathcal{H}_{pp} = \mathcal{H}_{qq} = \sqrt{\frac{1+u_1}{2}} \ \ and \ \ \mathcal{H}_{pq} = \mathcal{H}_{qp} = \frac{u_2}{\mathcal{H}_{pp}} \tag{24}$$

After applying Hyperbolic and Givens rotations, normalization transformation is applied in order to compensate for the dispersion parameters R_R and R_I. Normalization is done for only two rows p and q modified by Givens and Hyperbolic rotations. Considering the transformation $\tilde{\mathbf{Y}} = \boldsymbol{\mathcal{D}}_{p,q}\acute{\underline{\mathbf{Y}}}$, only two parameters (λ_p, λ_q) needs to be computed for the minimization of MM criterion so the cost function can be expressed as

$$\mathcal{J}(\boldsymbol{\mathcal{D}}_{p,q}) = \sum_{j=1}^{K}\lambda_p^4\,\tilde{y}_{R_{pj}}^4 - 2\lambda_p^2\,\tilde{y}_{R_{pj}}^2 + \lambda_q^4\,\tilde{y}_{R_{qj}}^4 - 2\lambda_q^2\,\tilde{y}_{R_{qj}}^2 \tag{25}$$

Taking the derivative of (25) with respect to these two parameters and equating the result to zero gives optimal parameters as

$$\begin{aligned} \lambda_p &= \sqrt{\sum_{j=1}^{K}\tilde{y}_{R_{pj}}^2 \Big/ \sum_{j=1}^{K}\tilde{y}_{R_{pj}}^4} \\ \lambda_q &= \sqrt{\sum_{j=1}^{K}\tilde{y}_{R_{qj}}^2 \Big/ \sum_{j=1}^{K}\tilde{y}_{R_{qj}}^4} \end{aligned} \tag{26}$$

HG-MMA is presented in Table 2. To make adaptive version of HG-MMA, we need to consider a sliding window of size T, $\underline{\mathbf{Y}}^{(t-1)} = [\underline{\mathbf{y}}(t-T), \ldots, \underline{\mathbf{y}}(t-2), \underline{\mathbf{y}}(t-1)]$ which is updated adaptively at each arrival of the new symbol $\underline{\mathbf{y}}(t)$. At each time instant, only one sweep of rotations are applied on the sliding window and matrix $\mathbf{W}$ is updated accordingly.

Initialization: $\acute{\mathbf{W}} = \mathbf{I}_{2M}$
Signal subspace projection if $N > M$
1. Create real matrix $\underline{\acute{\mathbf{Y}}}$ using (5)
2. Hyperbolic, Givens and Normalization Rotations:
for $i = 1 : N_{Sweeps}$ **do**
  for $p = 1 : M - 1$ **do**
    for $q = p + 1 : M$ **do**
      a. find $\mathbf{R}_1$ & $\mathbf{r}_1$ for (p, q) using (20)
      b. find $\mathbf{R}_2$ & $\mathbf{r}_2$ for $(p + M, q + M)$ using (20)
      c. find (γ) i.e. $\mathbf{u}$ using $\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2$ and $\mathbf{r} = \mathbf{r}_1 + \mathbf{r}_2$
      d. Compute $\boldsymbol{\mathcal{H}}_{p,q}$ & $\boldsymbol{\mathcal{H}}_{p+M,q+M}$ using (24) for (γ)
      e. $\underline{\acute{\mathbf{Y}}} = \boldsymbol{\mathcal{H}}_{p,q}\boldsymbol{\mathcal{H}}_{p+M,q+M}\underline{\acute{\mathbf{Y}}}$
      f. $\acute{\mathbf{V}} = \boldsymbol{\mathcal{H}}_{p,q}\boldsymbol{\mathcal{H}}_{p+M,q+M}\acute{\mathbf{V}}$
      g. Apply Givens rotation using (a **to** d) of Table 1
      h. Compute $\boldsymbol{\mathcal{D}}_{p,q}$ & $\boldsymbol{\mathcal{D}}_{p+M,q+M}$ using (26)
      i. $\underline{\acute{\mathbf{Y}}} = \boldsymbol{\mathcal{D}}_{p,q}\boldsymbol{\mathcal{D}}_{p+M,q+M}\underline{\acute{\mathbf{Y}}}$
      j. $\acute{\mathbf{V}} = \boldsymbol{\mathcal{D}}_{p,q}\boldsymbol{\mathcal{D}}_{p+M,q+M}\acute{\mathbf{V}}$
      repeat steps (a **to** j) for $(p, q + M)$ & $(q, p + M)$ using $(\acute{\theta}, \acute{\gamma})$ & $(\acute{\theta}, -\acute{\gamma})^a$, respectively
    end for
  end for
end for
4. Construct complex matrix $\mathbf{W}$ similar to $\mathbf{V}$ using (4) and (5)
5. Estimated Sources: $\hat{\mathbf{S}} = \mathbf{W}\mathbf{Y}$

[a] $(\acute{\gamma})$ is obtained using $\mathbf{R} = \mathbf{R}_1 + \acute{\mathbf{R}}_2$ and $\mathbf{r} = \mathbf{r}_1 + \acute{\mathbf{r}}_2$, where $\acute{\mathbf{R}}_2$ and $\acute{\mathbf{r}}_2$ are obtained using (20) with $\mathbf{r}_j = [\frac{1}{2}(\underline{\acute{y}}_{pj}^2 + \underline{\acute{y}}_{qj}^2), -\underline{\acute{y}}_{pj}\underline{\acute{y}}_{qj}]^{\mathsf{T}}$

Table 2. Hyperbolic Givens MMA (HG-MMA) Algorithm

5. SIMULATION RESULTS

For comparison, we have considered a MIMO system having 2 transmitters and 4 receivers ($M = 2, N = 4$). 50 symbols ($K = 50$) are transmitted by each source from 4-QAM constellation while 200 symbols ($K = 200$) are drawn from 16-QAM. Each symbol is passed through a channel matrix $\mathbf{A}$, generated randomly at each iteration. Sources, noise and channel has the same properties as described in Section 2. The results are averaged over 1000 Monte Carlo runs.

In Figure 1, we have considered $M = 5, N = 7$ and 16-QAM constellation. It can be seen that proposed algorithms provide better performance as compared to batch algorithms HG-CMA, G-CMA and ACMA. Moreover, performance improves with the increase in number of symbols. Also, we noticed that the performance of G-MMA is much lower than HG-MMA for $K = 50$ because of the ineffective pre-whitening operation.

Figure 2 compares the SINR vs. SNR of HG-MMA, G-MMA with AMMA, ACMA and MUK for both cases. The results for 4-QAM is shown in Figure 2 (a) while (b) shows 16-QAM case. The highest SINR is obtained with the proposed HG-MMA algorithm, followed by G-MMA and AMMA, then by ACMA and the lowest SINR is obtained with the MUK. For both cases, the proposed algorithms outperform the three algorithms.

Figure 3 depicts the SER of proposed algorithms vs. SNR. We noticed that the proposed HG-MMA and G-MMA gives the best performance both for 4-QAM and 16-QAM cases. It is noticed that for higher QAM constellation, the performance of AMMA, ACMA and MUK highly deteriorate but HG-MMA and G-MMA still shows good results.

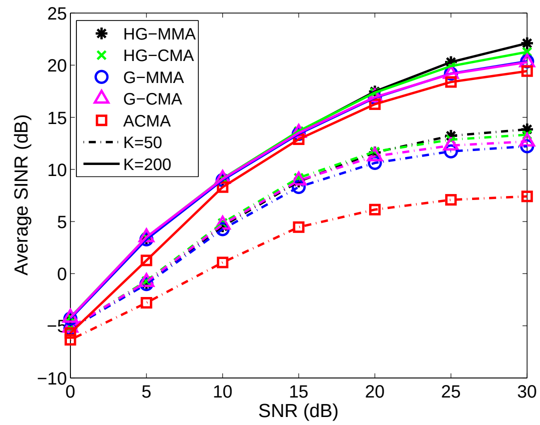
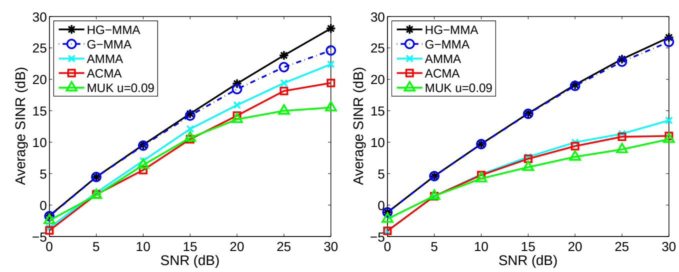
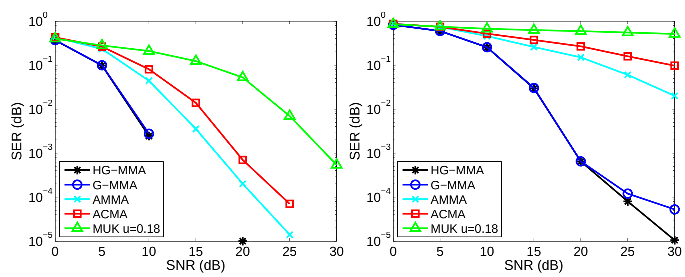


Fig. 1. SINR of HG-MMA, HG-CMA, G-MMA, G-CMA and AMMA vs. SNR for $M = 5, N = 7$ for different number of samples K, 16-QAM case.



(a) 4-QAM with $K = 50$ (b) 16-QAM with $K = 200$

Fig. 2. SINR of HG-MMA, G-MMA, AMMA, ACMA and MUK vs. SNR for $M = 2, N = 4$.



(a) 4-QAM with $K = 50$ (b) 16-QAM with $K = 200$

Fig. 3. SER of HG-MMA, G-MMA, AMMA, ACMA and MUK vs. SNR for $M = 2, N = 4$.

6. CONCLUSION

In this paper, two new BSS algorithms named G-MMA and HG-MMA are presented using unitary Givens and non-unitary Hyperbolic rotations. Both are based on the minimization of the Multimodulus criterion. G-MMA is suitable for a large number of samples but in case of small number of samples, HG-MMA should be used. These algorithms perform better than AMMA and ACMA in terms of the SINR and SER.

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