MISSING INTENSITY RESTORATION VIA ADAPTIVE SELECTION OF PERCEPTUALLY OPTIMIZED SUBSPACES

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ABSTRACT

A missing intensity restoration method via adaptive selection of perceptually optimized subspaces is presented in this paper. In order to realize adaptive and perceptually optimized restoration, the proposed method generates several subspaces of known textures optimized in terms of the structural similarity (SSIM) index. Furthermore, the SSIM-based missing intensity restoration is performed by a projection onto convex sets (POCS) algorithm whose constraints are the obtained subspace and known intensities within the target image. In this approach, a non-convex maximization problem for calculating the projection onto the subspace is reformulated as a quasi-convex problem, and the restoration of the missing intensities becomes feasible. Furthermore, the selection of the optimal subspace is realized by monitoring the SSIM index converged in the POCS algorithm, and the adaptive restoration becomes feasible. Experimental results show that our method outperforms existing methods.

***Index Terms*—** Missing intensity restoration, perceptually optimized algorithm, adaptive subspace selection, POCS algorithm.

1. INTRODUCTION

Since missing intensity restoration can afford a number of fundamental applications such as image inpainting, error concealment and old film restoration, many researchers have proposed methods related to this study [1]–[21]. A pioneering work based on texture synthesis was proposed by Efros et al. [1]. In recent years, a fragment-based restoration method and an exemplar-based method have been developed by Drori et al. [5] and Criminisi et al. [6, 7], respectively, and their methods became benchmarking methods in this field.

In addition to the above approach, several restoration methods which approximate each local patch within a target image by low-dimensional subspaces have been proposed. Since missing intensity restoration is one of ill-posed inverse problems, the derivation of its solution becomes feasible based on the approximation using such low-dimensional subspaces. Then their restoration performance tends to depend on multivariate analysis algorithms used for obtaining these subspaces. Amano et al. proposed an effective PCA-based method that reconstructs missing textures [11], and by introducing the kernel methods into PCA [22], its improvement can be also realized [12, 13, 14]. Furthermore, image restoration based on sparse representation [23, 24] has intensively been studied [15]–[18]. Mairal et al. proposed a representative work based on the sparse-representation [15], and Xu et al. also proposed an improved exemplar-based method using the sparse representation [18]. In addition, several works related to the sparse representation have intensively been proposed, e.g., the methods based on rank minimization [20] and neighboring embedding [21].

In most existing methods, generation of low-dimensional subspaces and projection onto them are performed based on minimization of mean square error (MSE). Although the MSE is the simplest metric used as a quality measure, it is well known that the MSE cannot reflect perceptual qualities [25, 26]. Recently, there have been proposed many image quality assessment algorithms [25]–[28]. Among them, the structural similarity (SSIM) index [28] is a representative measure, and it is reported that the SSIM index outperforms the MSE and its variants in several image processing applications [29, 30]. Therefore, by introducing the SSIM index, the use of perceptually optimized subspaces for the restoration can be expected. Furthermore, we have to note another important point. Since each target image is composed of several different textures, the subspace should be generated for each kind of texture, and the restoration should be also performed with adaptively selecting the optimal subspaces.

This paper presents a missing intensity restoration method via adaptive selection of perceptually optimized subspaces. Our method generates a subspace optimized in terms of the SSIM index for each kind of texture within a target image in order to restore missing areas based on the optimal subspaces. Specifically, the missing intensity restoration is performed by using a projection onto convex sets (POCS) algorithm [31] whose constraints are one selected subspace and known intensities within the target image. In this approach, a non-convex maximization problem for calculating the projection onto the subspace is reformulated as a quasi-convex problem to obtain the perceptually optimized solution. Furthermore, we monitor the SSIM index converged in the POCS algorithm to select the subspace optimal for restoring the target missing areas. Consequently, the adaptive and perceptually optimized missing intensity restoration becomes feasible.

2. STRUCTURAL SIMILARITY INDEX

The SSIM index is a similarity measure between two vectors [28]. Given two vectors $\mathbf{x}_1$ and $\mathbf{x}_2 \in \mathcal{R}^n$, the SSIM index is defined as

$$\mathrm{SSIM}(\mathbf{x}_1, \mathbf{x}_2) = \frac{\left(2\mu_{\mathbf{x}_1}\mu_{\mathbf{x}_2} + C_1\right)\left(2\sigma_{\mathbf{x}_1,\mathbf{x}_2} + C_2\right)}{\left(\mu_{\mathbf{x}_1}^2 + \mu_{\mathbf{x}_2}^2 + C_1\right)\left(\sigma_{\mathbf{x}_1}^2 + \sigma_{\mathbf{x}_2}^2 + C_2\right)},$$

where $\mu_{\mathbf{x}_i}$ and $\sigma_{\mathbf{x}_i}^2$ $(i = 1, 2)$ are the mean and the variance of $\mathbf{x}_i$, respectively, and $\sigma_{\mathbf{x}_1,\mathbf{x}_2}$ is the cross covariance between $\mathbf{x}_1$ and $\mathbf{x}_2$. Furthermore, the constants C_1 and C_2 are necessary for avoiding instability when the denominators are very close to zero. The SSIM index is defined by separately calculating three similarities in terms of luminance, variance and structure, which are derived on the basis of the human visual system (HVS) not accounted for by the MSE.

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Therefore, perceptually optimized restoration is expected by using this similarity measure.

3. ADAPTIVE AND PERCEPTUALLY OPTIMIZED MISSING INTENSITY RESTORATION

The missing intensity restoration method based on adaptive selection of perceptually optimized subspaces is presented in this section. In our method, a patch f ($w \times h (= N)$ pixels) including missing areas Ω is clipped from a target image according to the patch priority determined based on [7]. Then the proposed method tries to restore the missing intensities within Ω from the other known areas $\bar{\Omega}$ of the target patch f. For the following explanation, we define two vectors, whose elements are respectively intensities within f and $\bar{\Omega}$, as $\mathbf{x} \in \mathcal{R}^N$ and $\mathbf{y} \in \mathcal{R}^{N_{\bar{\Omega}}}$, where $N_{\bar{\Omega}}$ is the number of pixels within the known areas $\bar{\Omega}$.

The proposed method generates a subspace optimized in terms of the SSIM index for each kind of texture within the target image and restores the target patch f by adaptively selecting the optimal subspace. Therefore, we first perform clustering of known patches within the target image by using the SSIM-based subspaces (See **3.1**). Next, we perform the SSIM-based missing intensity restoration including the adaptive selection of the optimal subspace, i.e., the optimal cluster (See **3.2**).

3.1. Clustering Algorithm Using SSIM-Based Subspaces

In order to perform the clustering of textures within the target image, we clip known patches f_i ($i = 1, 2, \cdots, L$) whose size is the same as that of f in the same interval. For each patch f_i, we define a vector $\mathbf{x}_i \in \mathcal{R}^N$, which corresponds to $\mathbf{x}$. Then we perform their clustering into K clusters that maximizes the following criterion:

$$C = \sum_{k=1}^{K} \sum_{j=1}^{L^k} \mathrm{SSIM}(\mathbf{x}_j^k, \hat{\mathbf{x}}_j^k), \tag{1}$$

where $\mathbf{x}_j^k$ ($j = 1, 2, \cdots, L^k$) is $\mathbf{x}_i$ belonging to cluster k, $L = \sum_{k=1}^{K} L^k$, and $\hat{\mathbf{x}}_j^k = \hat{\mathbf{U}}^k \hat{\mathbf{a}}_j^k$. Furthermore, $\hat{\mathbf{U}}^k \in \mathcal{R}^{N \times D^k}$ is a D^k ($< N$) dimensional orthonormal basis matrix, and $\hat{\mathbf{a}}_j^k \in \mathcal{R}^{D^k}$ is a coefficient vector for representing $\mathbf{x}_j^k$. The proposed method iteratively performs assignment of each known patch maximizing Eq. (1) and update of the basis matrix $\hat{\mathbf{U}}^k$. In the assignment procedures, we calculate the optimal coefficient vector $\hat{\mathbf{a}}_j^k$ maximizing $\mathrm{SSIM}(\mathbf{x}_j^k, \hat{\mathbf{x}}_j^k)$ by the steepest ascent algorithm. Different from the algorithm shown in the following subsection, we adopt this simple optimization algorithm for reducing the complexity of our method. Furthermore, we show the details of the procedures for calculating the basis matrix $\hat{\mathbf{U}}^k$ from $\mathbf{x}_j^k$ ($j = 1, 2, \cdots, L^k$) below.

The proposed method tries to calculate the basis matrix $\hat{\mathbf{U}}^k$ including D^k orthonormal bases which span the subspace optimized in terms of the SSIM index. Since it is difficult to obtain all bases optimized with the SSIM index, simultaneously, our method adopts the simplest algorithm selecting the optimal bases one by one, which is similar to some matching pursuit algorithms [32, 33]. The details of dth ($d = 1, 2, \cdots, D^k$) optimal basis calculation are shown below. In dth iteration, i.e., dth optimal basis calculation, we first define the following vector approximating $\mathbf{x}_j^k$ ($j = 1, 2, \cdots, L^k$):

$$\mathbf{x}_{j,(d)}^k = \begin{bmatrix} \hat{\mathbf{U}}_{(d-1)}^k & \mathbf{u}_{(d)}^k \end{bmatrix} \begin{bmatrix} \mathbf{a}_{j,(d-1)}^k \\ a_{j,(d)}^k \end{bmatrix},$$

where $\hat{\mathbf{U}}_{(d-1)}^k = [\hat{\mathbf{u}}_{(1)}^k, \hat{\mathbf{u}}_{(2)}^k, \cdots, \hat{\mathbf{u}}_{(d-1)}^k] \in \mathcal{R}^{N \times (d-1)}$ is a fixed matrix containing $d-1$ bases previously calculated in the $d-1$ iterations. We estimate the optimal orthonormal basis $\hat{\mathbf{u}}_{(d)}^k$ of $\mathbf{u}_{(d)}^k$ that provides the best representation performance for all vectors $\mathbf{x}_j^k$ ($j = 1, 2, \cdots, L^k$) based on the SSIM index. Specifically, it can be calculated by solving the following problem:

$$\begin{aligned} \left\{\hat{\mathbf{u}}_{(d)}^k, \hat{\mathbf{a}}_{(d)}^k\right\} &= \arg \max_{\mathbf{u}_{(d)}^k, \mathbf{a}_{(d)}^k} \sum_{j=1}^{L^k} \mathrm{SSIM}(\mathbf{x}_j^k, \mathbf{x}_{j,(d)}^k) \\ &\text{subject to} \quad \|\mathbf{u}_{(d)}^k\|^2 = 1 \\ &\qquad \mathbf{u}_{(d)}^{k}{}' \hat{\mathbf{u}}_{(\tilde{d})}^k = 0 \quad (\tilde{d} = 1, 2, \cdots, d-1), \end{aligned} \tag{2}$$

where $\mathbf{a}_{(d)}^k$ is a set of $\mathbf{a}_{1,(d)}^k, \mathbf{a}_{2,(d)}^k, \cdots, \mathbf{a}_{L^k,(d)}^k$, and $\mathbf{a}_{j,(d)}^k = [\mathbf{a}_{j,(d-1)}^{k}{}', a_{j,(d)}^k]'$ ($j = 1, 2, \cdots, L^k$). It should be noted that vector/matrix transpose is denoted by the superscript $'$. The proposed method calculates the optimal basis $\hat{\mathbf{u}}_{(d)}^k$ and the optimal coefficient vectors $\hat{\mathbf{a}}_{j,(d)}^k$ ($j = 1, 2, \cdots, L^k$) by applying the constrained cyclic coordinate ascent algorithm to Eq. (2). Although the cyclic coordinate ascent algorithm does not necessarily provide the global optimal solution in Eq. (2), but we adopt this algorithm in the same reason as that in the assignment procedures. By iterating the above procedures D^k times, we can calculate the optimal D^k orthonormal bases $\hat{\mathbf{u}}_{(d)}^k$ ($d = 1, 2, \cdots, D^k$) to obtain $\hat{\mathbf{U}}^k$ based on the SSIM index.

3.2. Missing Intensity Restoration Algorithm

The proposed method restores the missing areas Ω within the target patch f based on the POCS algorithm [31] whose constraints are shown below.

[Constraint 1]
The intensities within $\bar{\Omega}$ are fixed since they are known within the target patch f. Thus, $\mathbf{y} = \mathbf{E}\mathbf{x}$ is satisfied, where $\mathbf{E} \in \mathcal{R}^{N_{\bar{\Omega}} \times N}$ is a binary matrix extracting only the known intensities in $\bar{\Omega}$.

[Constraint 2]
The vector $\mathbf{x}$ is in the cluster k's subspace spanned by the D^k orthonormal bases $\hat{\mathbf{u}}_{(d)}^k$ ($d = 1, 2, \cdots, D^k$) in $\hat{\mathbf{U}}^k$.

By projecting the target vector $\mathbf{x}$ onto these two closed convex sets iteratively, the proposed method estimates the restoration result $\hat{\mathbf{x}}^k \in \mathcal{R}^N$ by cluster k. In each iteration, it is necessary to calculate the projection onto the subspace spanned by the D^k orthonormal bases $\hat{\mathbf{u}}_{(d)}^k$ ($d = 1, 2, \cdots, D^k$) in $\hat{\mathbf{U}}^k$ based on the SSIM index. Specifically, in tth iteration, we have to obtain the following linear combination:

$$\hat{\mathbf{x}}_{(t)}^k = \hat{\mathbf{U}}^k \hat{\mathbf{a}}_{(t)}^k, \tag{3}$$

and $\hat{\mathbf{x}}_{(t)}^k \in \mathcal{R}^N$ is the approximation result of $\mathbf{x}_{(t)}^k$ satisfying Constraint 1, where $\hat{\mathbf{a}}_{(t)}^k \in \mathcal{R}^{D^k}$ satisfies

$$\hat{\mathbf{a}}_{(t)}^k = \arg \max_{\mathbf{a}_{(t)}^k} \mathrm{SSIM}\left(\mathbf{x}_{(t)}^k, \hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k\right). \tag{4}$$

In the above equation,

$$\begin{aligned} \mathrm{SSIM}\left(\mathbf{x}_{(t)}^k, \hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k\right) &= \left[\frac{2\mu_{\mathbf{x}_{(t)}^k} \mu_{\hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k} + C_1}{\mu_{\mathbf{x}_{(t)}^k}^2 + \mu_{\hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k}^2 + C_1}\right] \left[\frac{2\sigma_{\mathbf{x}_{(t)}^k, \hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k} + C_2}{\sigma_{\mathbf{x}_{(t)}^k}^2 + \sigma_{\hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k}^2 + C_2}\right] \\ &= \left[\frac{2\left(\frac{1}{N}\mathbf{1}'\mathbf{x}_{(t)}^k\right)\left(\mu_{\hat{\mathbf{U}}^k}{}' \mathbf{a}_{(t)}^k\right) + C_1}{\left(\frac{1}{N}\mathbf{1}'\mathbf{x}_{(t)}^k\right)^2 + \left(\mu_{\hat{\mathbf{U}}^k}{}' \mathbf{a}_{(t)}^k\right)^2 + C_1}\right] \\ &\quad \times \left[\frac{2\mathbf{x}_{(t)}^{k}{}' \mathbf{H} \hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k + NC_2}{\mathbf{x}_{(t)}^{k}{}' \mathbf{H} \mathbf{x}_{(t)}^k + \mathbf{a}_{(t)}^{k}{}' \hat{\mathbf{U}}^{k}{}' \mathbf{H} \hat{\mathbf{U}}^k \mathbf{a}_{(t)}^k + NC_2}\right], \end{aligned} \tag{5}$$

Table 1. Performance comparison (SSIM index) between the existing methods and our method.

Test image	Reference [7]	Reference [8]	Reference [18]	Reference [11]	Reference [14]	Reference [21]	Our method
Image 1	0.7031	0.6979	0.6608	0.6695	0.7670	0.6935	**0.7792**
Image 2	0.7383	0.7459	0.7419	0.6668	0.7130	0.7324	**0.7508**
Image 3	0.6648	0.6475	**0.6891**	0.5678	0.6274	0.6308	0.6772
Image 4	0.6722	0.6659	0.6948	0.5887	0.6544	0.6929	**0.7432**
Image 5	0.6998	0.6915	0.7019	0.6200	0.7320	0.6674	**0.7463**
Image 6	0.7456	0.7264	0.7497	0.6081	0.6883	0.7587	**0.7849**
Average	0.7040	0.6959	0.7064	0.6202	0.6970	0.6959	**0.7469**
Median	0.7015	0.6947	0.6984	0.6141	0.7007	0.6932	**0.7486**

and $\boldsymbol{\mu}_{\hat{\mathbf{U}}^k} = \frac{1}{N}\hat{\mathbf{U}}^{k\prime}\mathbf{1}$, where $\mathbf{1} = [1, 1, \cdots, 1]'$ is an $N \times 1$ vector. Furthermore, $\mathbf{H} = \mathbf{I} - \frac{1}{N}\mathbf{1}\mathbf{1}'$ is an $N \times N$ centering matrix, where $\mathbf{I}$ is the identity matrix.

From the definition of Eq. (5), $\mathrm{SSIM}\left(\mathbf{x}^k_{(t)}, \hat{\mathbf{U}}^k\mathbf{a}^k_{(t)}\right)$ is a non-convex function of $\mathbf{a}^k_{(t)}$. Thus, based on the calculation scheme in [29], this function is converted into a quasi-convex problem. Specifically, since the first term is a function only of $\boldsymbol{\mu}_{\hat{\mathbf{U}}^k}{}'\mathbf{a}^k_{(t)} \left(= \rho^k_{(t)}\right)$, Eq. (4) can be rewritten as

$$\max_{\mathbf{a}^k_{(t)}} \left(\frac{2\mathbf{x}^k_{(t)}{}'\mathbf{H}\hat{\mathbf{U}}^k\mathbf{a}^k_{(t)} + NC_2}{\mathbf{x}^k_{(t)}{}'\mathbf{H}\mathbf{x}^k_{(t)} + \mathbf{a}^k_{(t)}{}'\hat{\mathbf{U}}^{k\prime}\mathbf{H}\hat{\mathbf{U}}^k\mathbf{a}^k_{(t)} + NC_2} \right) \quad \text{subject to } \boldsymbol{\mu}_{\hat{\mathbf{U}}^k}{}'\mathbf{a}^k_{(t)} = \rho^k_{(t)}. \tag{6}$$

Therefore, the overall problem is reformulated to find the highest SSIM index in Eq. (4) by searching over range of $\rho^k_{(t)}$. Furthermore, Eq. (6) is converted into a quasi-convex optimization problem as

$$\min : \tau \quad \text{subject to} \left[\begin{array}{c} \max : \left(\frac{2\mathbf{x}^k_{(t)}{}'\mathbf{H}\hat{\mathbf{U}}^k\mathbf{a}^k_{(t)} + NC_2}{\mathbf{x}^k_{(t)}{}'\mathbf{H}\mathbf{x}^k_{(t)} + \mathbf{a}^k_{(t)}{}'\hat{\mathbf{U}}^{k\prime}\mathbf{H}\hat{\mathbf{U}}^k\mathbf{a}^k_{(t)} + NC_2} \right) \leq \tau \\ \text{subject to} \quad \boldsymbol{\mu}_{\hat{\mathbf{U}}^k}{}'\mathbf{a}^k_{(t)} = \rho^k_{(t)} \end{array} \right].$$

Then the above problem is further rewritten as

$$\begin{array}{l} \min : \tau \\ \text{subject to} \\ \left[\begin{array}{c} \min : \left[\tau \left(\mathbf{x}^k_{(t)}{}'\mathbf{H}\mathbf{x}^k_{(t)} + \mathbf{a}^k_{(t)}{}'\mathbf{K}^k_1\mathbf{a}^k_{(t)} + NC_2 \right) - \left(\mathbf{x}^k_{(t)}{}'\mathbf{K}^k_2\mathbf{a}^k_{(t)} + NC_2 \right) \right] \geq 0 \\ \text{subject to} \quad \boldsymbol{\mu}_{\hat{\mathbf{U}}^k}{}'\mathbf{a}^k_{(t)} = \rho^k_{(t)} \end{array} \right], \end{array}$$

where $\mathbf{K}^k_1 = \hat{\mathbf{U}}^{k\prime}\mathbf{H}\hat{\mathbf{U}}^k$ and $\mathbf{K}^k_2 = 2\mathbf{H}\hat{\mathbf{U}}^k$. Then the proposed method adopts the Lagrange multiplier approach shown as follows:

$$\begin{aligned} \mathcal{L} &= \tau \left(\mathbf{x}^k_{(t)}{}'\mathbf{H}\mathbf{x}^k_{(t)} + \mathbf{a}^k_{(t)}{}'\mathbf{K}^k_1\mathbf{a}^k_{(t)} + NC_2 \right) - \left(\mathbf{x}^k_{(t)}{}'\mathbf{K}^k_2\mathbf{a}^k_{(t)} + NC_2 \right) \\ &\quad + \lambda \left(\boldsymbol{\mu}_{\hat{\mathbf{U}}^k}{}'\mathbf{a}^k_{(t)} - \rho^k_{(t)} \right). \end{aligned}$$

By calculating the solution of the above problem, the optimal coefficient vector $\hat{\mathbf{a}}^k_{(t)}$ can be estimated, and $\hat{\mathbf{x}}^k_{(t)}$ in Eq. (3) is obtained. It should be noted that τ is obtained by using the standard bisection procedures. In this way, we can calculate the restoration result $\hat{\mathbf{x}}^k$ by cluster k based on the POCS algorithm.

The subspace spanned by the bases in $\hat{\mathbf{U}}^k$ used in Constraint 2 enables the optimal SSIM-based approximation of cluster k's elements. Therefore, if we can classify $\mathbf{x}$ of the target patch f into the optimal cluster k^{opt}, the proposed method can perform accurate restoration by using its optimal subspace. Unfortunately, since $\mathbf{x}$ contains missing areas Ω, we cannot classify it by the algorithm shown in the previous subsection. Therefore, in order to achieve the classification of $\mathbf{x}$, the proposed method monitors the SSIM index in Eq. (5) converged after performing the POCS algorithm. Since this converged SSIM index is the maximum similarity from a vector satisfying Constraint 2, we use it as the criterion for the classification of $\mathbf{x}$. Then the adaptive selection of the optimal cluster k^{opt} for the target patch including the missing areas becomes feasible. The proposed method regards the result $\hat{\mathbf{x}}^{k^{\mathrm{opt}}}$ obtained by the selected cluster k^{opt} as the final output. Consequently, by performing the non-conventional approach, which adaptively selects the optimal subspace, we can perform the adaptive and accurate restoration.

4. EXPERIMENTAL RESULTS

In order to verify the performance of our method, experimental results are shown in this section. We prepared six text images shown in Fig. 1 and added missing areas to these images. For these test images, we restored the missing areas by using the proposed method and the existing methods [7, 8, 18, 11, 14, 21]. We used the representative exemplar-based method [7] and its improved versions [8, 18]. Furthermore, the subspace-based methods based on PCA [11], kernel PCA [14] and neighboring embedding [21] were also adopted. The exemplar-based methods [7, 8, 18] select the optimal patches based on the MSE, and the subspace-based methods [11, 14, 21] also perform the MSE-based subspace generation and restoration. Therefore, we used these existing methods for the comparisons of our method. In this experiment, the patch size was fixed to 15, and the number of training patches became smaller. Since this comparison scheme was adopted in several papers, we also adopted such difficult conditions in order to make the difference in the performance of the proposed method and the existing methods clearer.

The results obtained by the existing methods and our method are shown in the third and forth columns of Fig. 1, respectively. We only show the result of one existing method for each test image due to the limitation of spaces[1]. As shown in Fig. 1, the proposed method realizes successful missing intensity restoration.

Furthermore, in order to quantitatively evaluate the performance of the proposed method, we show the SSIM index calculated from the restoration results in Table 1. The results shown in this table were calculated from only the restored areas. From the obtained results, it can be seen that the proposed method also achieves the improvement in terms of the SSIM index. As shown in the previous section, the proposed method realizes the subspace generation and the adaptive restoration based on the optimal subspace according to the better quality measure, i.e., the SSIM index. Therefore, the perceptually optimized restoration becomes feasible by the proposed method.

[1] All of the restoration results obtained from the six test images in Fig. 1 by our method and the existing methods [7, 8, 18, 11, 14, 21] can be confirmed in the following Web site.
http://www-lmd.ist.hokudai.ac.jp/wp/wp-content/uploads/ICASSP2015-Ogawa.pdf

Fig. 1. Restoration results obtained by the existing methods and our method. Six test images are used, and they respectively correspond to Images 1–6. Note that the existing methods used for restoring Images 1–6 are respectively Ref [7], Ref [8], Ref [18], Ref [11], Ref [14] and Ref [21]. The sizes of Images 1–6 are 640×480 pixels, 640×480 pixels, 480×360 pixels, 480×360 pixels, 640×480 pixels and 480×360 pixels, respectively. The percentages of missing areas are 5.5%, 5.4%, 11.3%, 10.7%, 6.2% and 8.9% in Images 1–6, respectively.

5. CONCLUSIONS

This paper has presented a missing intensity restoration method via adaptive selection of perceptually optimized subspaces. The proposed method performs the generation of subspaces optimized in terms of the SSIM index. Then, by using the POCS algorithm, whose constraints are the obtained subspace and the known intensities, the SSIM-based restoration becomes feasible. In this approach, the adaptive selection of the optimal subspace for the restoration is realized by monitoring the SSIM index converged in the POCS algorithm. Consequently, the improvement of our method over the existing methods can be confirmed.

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