AN IMPROVED CROSS-CORRELATION APPROACH TO PARAMETER ESTIMATION BASED ON FRACTIONAL FOURIER TRANSFORM FOR ISAR MOTION COMPENSATION

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ABSTRACT

Motion compensation (MOCOMP) is a key procedure in inverse synthetic aperture radar (ISAR) imaging because the accuracy of estimated parameter has a strong influence on the imaging quality. Generally, the backscattered signal of a moving target is sampled in fast time dimension, which can be approximated as the combination of multiple Chirp signals with a proper Chirp rate. Compared with the Fourier transform, the fractional Fourier transform (FrFT) performs better compression property due to its unique energy focus ability to Chirp signals. An improved Cross-correlation method based on FrFT for parameter estimation is presented in this paper. It employs the correlation between range profiles compressed by FrFT to enhance the quality of parameter estimation for ISAR applications. The method takes good balance between accuracy and complexity, and is robust to noise. Simulation results show that the proposed method outperforms the conventional Cross-correlation Method in terms of ISAR translational MOCOMP.

Index Terms— fractional Fourier transform, ISAR, cross-correlation, motion compensation.

1. INTRODUCTION

Inverse synthetic aperture radar (ISAR) [1], [2] is a powerful signal processing technique for imaging targets in range-Doppler domains [3]. In order to get a clear and focused image, it is necessary to estimate the motion parameters of observed targets, and then conduct the motion compensation procedure with the so-obtained parameters before imaging process [4], [5]. The cross-correlation method (CCM), the minimum entropy method (MEM), and the joint time-frequency method [6], [7] are some of the popular MOCOMP algorithms in the literature. The CCM is one of the most applied range tracking algorithms because of its low computational complexity. The MEM performs more accurate estimation than CCM, but it requires a prior knowledge of estimated parameters. The JTF based MOCOMP method is effective because of making use of both time and frequency information, nevertheless, computational complexity is increased at the same time.

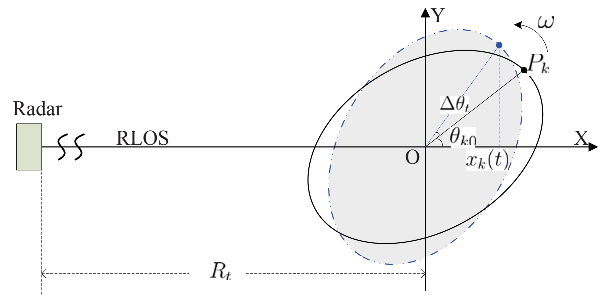

Fig. 1. 2-D ISAR model

In an ISAR imaging system, radar echo signals can be approximated as a linear combination of multi-component Chirp signals because the phase term is modulated by a unique chirp rate induced by the moving characteristics of target. The fractional Fourier transform (FrFT) [8] is a generalization of the classical Fourier transform, and can be viewed as the Chirp-basis expansion directly from its definition [9]. The FrFT performs a more centralized compression capability because of its unique energy aggregation characteristics to Chirp signals [10], [11]. In this paper an improved cross-correlation parameter estimation method based on FrFT is proposed. We make good use of the high correlation of range profiles processed by FrFT compression, and take into consideration the balance between complexity and precision in searching for the matched-order of FrFT. The theoretical analysis and simulation results reveal that the proposed scheme can effectively improve the parameter estimation accuracy compared to the CCM, thus enhancing the translational MOCOMP quality in ISAR imaging. In particular, it does not acquire prior knowledge and the consequent complexity is much less than that by JTF method.

2. ISAR ECHO MODEL

In general, the radar line of sight (RLOS), i.e. X axis, is assumed to be along the radial direction, while Y axis is along the cross-range direction. Considering that the observed target consists of a number of strong scattering points, the baseband radar echo signal can be given by

$$s(t) = \sum_k A_k \exp[-j\frac{4\pi f_t}{c} R_k(t)] \quad (1)$$

The instantaneous range from the scattering point P_k of far-field target to radar is defined as $R_k(t), 0 \le t \le T_a$, where T_a is the observing duration. Depicting the initial position of P_k as $P_k(x_{k0}, y_{k0}) = P_k(\rho_{k0}\angle\theta_{k0})$, where x_{k0}, y_{k0}, ρ_{k0}, θ_{k0} are the Cartesian coordinates and the Polar coordinates, respectively. For far-field targets, $R_k(t)$ can be written as

$$\begin{aligned} R_k(t) &= R_t + x_k(t) \\ &= R_t + \rho_{k0}\cos[\theta_{k0} + \Delta\theta_t] \\ &= R_t + x_{k0}\cos\Delta\theta_t - y_{k0}\sin\Delta\theta_t \end{aligned} \quad (2)$$

where R_t is the instantaneous range from the phase center of the target to radar, $x_k(t)$ is the instantaneous projection of P_k along RLOS, and $\Delta\theta_t$ is rotation angle during the coherent processing interval (CPI), as shown in Fig. 1. In consideration of the non-uniform motion of the target, both the distance R_t and rotation angle $\Delta\theta_t$ can be expanded in the form of Taylor series as

$$\begin{cases} R_t = R_0 + vt + \frac{1}{2}v't^2 + \frac{1}{3}v''t^3 + \cdots \\ \theta_t = \omega t + \frac{1}{2}\omega' t^2 + \frac{1}{3}\omega'' t^3 + \cdots \end{cases}$$

For short CPI, as the rotation angle of the target is very small during the CPI, a pair of approximate expression $\sin\theta_t \approx \theta_t$ and $\cos\theta_t \approx 1$ can be adopted. Therefore we can rewrite (1) as

$$\begin{aligned} s(t) \approx \sum_k A_k \exp\{-j\frac{4\pi f_t}{c}[R_0 + x_{k0} \\ + (v - y_{k0}\omega)t + \frac{1}{2}(v' - \mathrm{y_{k0}}\omega')\mathrm{t}^2]\} \end{aligned} \quad (3)$$

In ISAR imaging processing, radar echo signal is sampled at time $t = (m + nM)T_r$ to form an ISAR image. Here $m = 0, \ldots, M-1$, $n = 0, \ldots, M-1$, are the fast and slow time dimensional index, respectively. We assume that the stepped frequency radar transmit a sequence of N bursts, and that each burst includes M narrow frequency band pulses with pulse repetition interval (PRI) T_r. Then the sampled data are arranged into a 2-D data array of size $N \times M$. During the fast time period, the rotation angle can be approximated as zero and carrier frequency is $f_t = f_0 + m\Delta f$. In practice, the phase terms containing T_r^2 are much smaller than other ones, then the echo signal for each burst sampled at fast time $t = mT_r$ can be expressed as

$$\begin{aligned} s(m) \approx \sum_k A_k \exp\{-j\frac{4\pi}{c} \\ \times [R_k f_0 + (\Delta f R_k + f_0 v T_r)m + \Delta f v T_r m^2]\} \end{aligned} \quad (4)$$

From(5), the echo signal sampled at the fast time dimension can be approximated as the combination of a set of LFM signals with an unique Chirp rate $\mu_1 = \Delta f v T_r$.

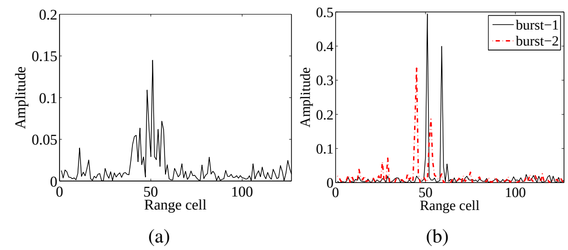

Fig. 2. Range compression. (a) Range profile of 1 burst with FFT method. (b) Range profiles of 2 adjacent bursts with FrFT method.

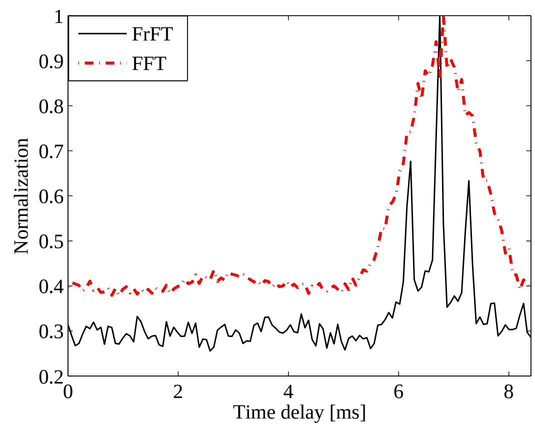

Fig. 3. Cross-correlation of two adjacent range profiles.

3. FRFT CHIRP-BASIS DECOMPOSITION CHARACTERISTICS

Consider a single component Chirp signal

$$x(t) = \exp(j2\pi f_0 t + j\pi\mu_0 t^2) \quad (5)$$

where μ_0 is the Chirp rate. Its p th-order FrFT is defined as

$$\begin{aligned} X_p(u) &= \int_{-\infty}^{+\infty} x(t)K_p(t,u)dt \\ &= A_0 \int_{-\infty}^{+\infty} \exp\{j\pi[(u^2\cot\alpha) \\ &\quad - 2(u\csc\alpha - f_0)t + (\mu_0 + \cot\alpha)t^2]\}dt \end{aligned} \quad (6)$$

where $K_p(t,u)$ is the transform kernel function, u is the fractional Fourier domain, $p = 2\alpha/\pi$ is called matched-order, and $A_0 = \sqrt{1 - j\cot\alpha}$. The corresponding inverse transformation is described as

$$x(t) = \int_{-\infty}^{+\infty} X_p(u)K_{-p}(t,u)du \quad (7)$$

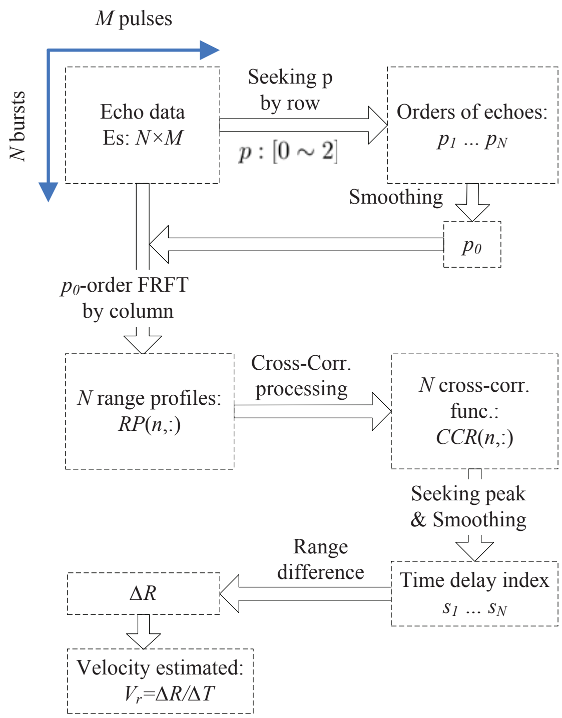

Fig. 4. Data flow chart of the proposed method.

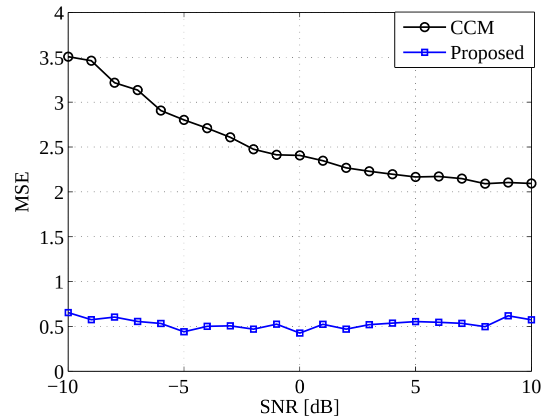

Fig. 5. MSE of estimated velocity.

If the signal argument μ_0 and the FrFT argument α satisfies the formula

$$\mu_0 = -\cot\alpha \tag{8}$$

then (7) can be rewritten as

$$\begin{aligned} X_p(u) =& A_1 \int_{-\infty}^{+\infty} \exp[-j2\pi(u\csc\alpha - f_0)t]dt \\ =& A_2(u)\delta(u - f_0 \sin\alpha) \end{aligned} \tag{9}$$

where $A_2(u) = 2\pi|\sin\alpha|\sqrt{1-j\cot\alpha}\exp(j\pi u^2\cot\alpha)$.

From (10), the FrFT expression of a single component Chirp signal is a Dirac function with the impulse position at its weighted center frequency $f_0 \sin\alpha$, i.e., the transformation has the ability of maximum energy aggregation for a Chirp signal at some unique fractional Fourier domain, which can be interpreted from the perspective of signal function space. From (8), the p th-order FrFT function $X_p(u)$ of signal $x(t)$ can be viewed as the expansion on the kernel function space. The kernel function $\{K_{-p}(t,u)\}$ is a set of orthogonal bases with Chirp forms in u domain. From this viewpoint, the FrFT might be interpreted as the decomposition in terms of Chirp signals [12], [13], which makes FrFT to be particularly suitable for processing the Chirp signals.

In order to verify the characteristic of FrFT, numerical simulations of a target consisting of two scatter points were performed. By transmitting a set of stepped frequency signals and collecting the echo data with SNR=-3dB, then performing range compression to the data with FFT and FrFT separately, we can get corresponding range profiles as shown in Fig. 2. The range spectrum in Fig. 2(a) is expanded because of the radial movement of the target, while that in Fig. 2(b) shows sharp amplitude peaks at range cells of relative scatter points. Thus FrFT method provides higher spatial resolution and more robust to noise.

Generally, because the rotation angle of the target is smaller than 0.01° during a burst, two adjacent echoes are quite similar in real-valued envelope. As shown in Fig. 2(b), the curves of the two bursts have different peak positions due to time delay. Thus, the cross-correlation coefficient of their range profiles would yield the maximum value at the position of some correlation time, which is the theoretical foundation of the conventional CCM. Fig. 3 shows the cross-correlation curves of two kinds of range profiles above-mentioned. Compared to the traditional Fourier method, the curve of the FrFT presents sharper peaks and higher resolution, which demonstrates that the range compression with FrFT instead of FFT can make better use of data correlation of different bursts.

4. PROPOSED METHOD

According to previous analysis, a parameter estimation method based on the FrFT is proposed here. The data flow chart is shown in Fig. 4, and the main steps of the proposed method are detailed as follows:

Step 1: Sample the baseband echo to get the data array of size $N \times M$.

Step 2: Search for the fractional order p_0 corresponding to the Chirp rate μ_0 in (6) in the range dimension.

Step 3: Perform the range compression on M frequency samples by p_0-FrFT row-by-row and get N range profiles.

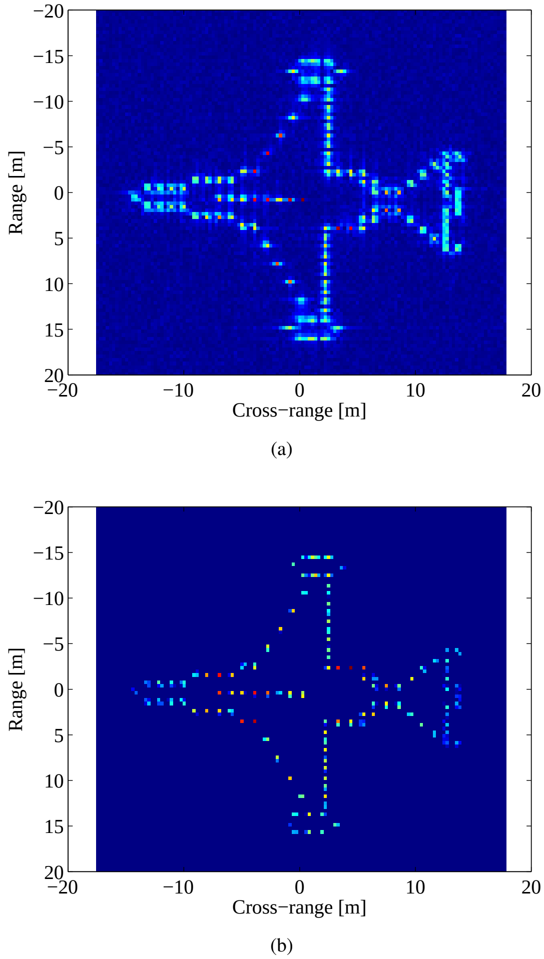

Fig. 6. ISAR images. (a) ISAR image after MOCOMP by CCM. (b) ISAR image after MOCOMP by proposed method.

Step 4: Taking the first range profile RP_1 as the reference, and calculate the cross-correlations of other $N-1$ range profiles via computing the following cross-correlation factor: $CCR_n = |IFFT(FFT(|RP_1|) \cdot FFT(|RP_n|)^*)|$.

Step 5: Estimate the target moving parameter by finding the range walk ΔR between two range profiles which corresponds to the location of the peak value for the calculated cross correlations.

It is important to note that the algorithm of seeking the FrFT-order plays a pivotal role in the performance of the proposed method. In step 2, different intervals scanning modes are employed to reduce the computational complexity. Moreover, Robust Lowess method [14] is utilized for data smoothing in order to get rid of the influence of outburst values.

5. SIMULATION RESULTS

Numerical simulations were performed based on the ISAR model and radar system described previously. Fig. 5 shows the mean square errors (MSEs) of estimated velocity by the proposed method compared with that of the traditional CCM. The initial frequency of transmitted stepper frequency signal is 10 GHz. There are 128 burst in total and each consists of 128 narrow-band pulses with PRF=20 KHz. The bandwidth of each burst is 128 MHz. The moving parameters of the target are $V_r = 65m/s$ and $\omega = 0.03rad/s$. The number of Monte-Carlo trials is 150.

It is observed that the proposed method provides a considerable improvement in terms of MSE for all the SNRs. Furthermore, because the noises in radar system have no feature of energy focusing in the fractional domain, the curve suffers small floatability when the SNR varies from -10dB to 10dB. It is worth mentioning that using the MEM method to achieve the same estimating precision requires such a small computational intervals that it takes more time to figure out the data, even through estimating range is provided.

To evaluate the performance of the proposed method in ISAR imaging, we examine the contrasting results of ISAR imaging by employing motion compensation with different parameter estimation methods. As is shown in Fig. 6, the proposed approach outperforms the traditional method in ISAR imaging quality. The ISAR image in Fig. 6(b) looks more clear and clean than the image in Fig. 6(a), as it benefits from the high estimation accuracy and effective denoising ability of the proposed method.

6. CONCLUSION

An improved cross-correlation method of parameter estimation based on the FrFT is proposed for ISAR translational MOCOMP. It takes advantage of energy focus ability of the FrFT to Chirp signal. To certain extent, the proposed method may induce some computational complexity due to the searching of the fractional order compared to the CCM, but it effectively reduces the parameter estimation error, and thus improves the precision of translational compensation. Actually, it still has obvious advantage of low complexity compared to the JTF method as well as the MEM. Therefore, the proposed method take a good balance in estimation accuracy and computational complexity. Secondly, it is a blind estimation technique, so it can be practical applied in the case of lacking the prior information of estimated parameters. Moreover, this method yields low sensitivity to the noise. That is to say, it is able to provide a good performance even at low SNR conditions.

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