ACTIVE MATCHING FOR PATCH ADAPTIVITY IN NONLOCAL MEANS IMAGE DENOISING

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ABSTRACT

Baseline nonlocal means denoising scheme may be improved by incorporating more adaptivity, like locally varying filtering window, smoothing constants and patch size or shape. In this paper, we presents a novel adaptive nonlocal means filtering scheme, the key idea of which is that before computing the similarity between two pixels, active matching is performed to determine optimally matched patch shape and size. Systematic analysis and detailed simulation results show that the proposed algorithm achieves excellent trade-off between bias and variance, and obtains superior denoising performance compared with the state of the art.

***Index Terms*—** Nonlocal means, active matching, local polynomial approximation (LPA), intersection of confidence interval (ICI)

1. INTRODUCTION

Image denoising is widely considered as one of the most fundamental low-level vision problems. Besides its evident practical significance, image denoising is also an ideal test bed for evaluating the regularization efficacy of image models.

In terms of the corresponding operation domain, image denoising schemes may be roughly categorized into two classes: spatial domain methods and transform domain methods. Most spatial domain image denoising algorithms can be essentially cast in the adaptive kernel regression framework [1], among which nonlocal means (NLM) [2] represents relatively new development.

The novelty of NLM is twofold. First, NLM explicitly introduces the nonlocal filtering paradigm to exploit the long-range self-similarity of natural images, while most of its predecessors focus on local smoothing (although in practice the distinction is not so clear-cut). Second, NLM determines the similarity between image pixels and filtering weights solely based on patch information around the corresponding pixels. Both of these are strongly inspired by the success of the nonparametric texture synthesis techniques [3] and have in turn given birth to a flurry of so called nonlocal patch-based restoration algorithms, with BM3D [4] being the most well-known.

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Despite its popularity, NLM is far from being optimal. While there have been theoretical analysis of the sub-optimality of baseline NLM scheme [5], intuitively this sub-optimality could be attributed partly to its weak adaptivity. In fact, the efforts to incorporate more adaptivity into NLM framework have constituted the bulk of post-Buades NLM-related research. Kervrann and Boulanger [6] applied adaptive smoothing theory to NLM, to make filtering window adaptive with respect to local image characteristics. Duval et al. [7] explored the possibility of smoothing constants adaptivity based on local SURE. Deledalle et al. [8] considered replacing the usual square patch with various shapes to take advantages of local image geometry. Specifically, they combined estimates associated with various shapes to achieve patch adaptivity, with local combination weights determined by SURE minimization. Salmon et al. [9] advocated patch-wise NLM + reprojection denoising paradigm to alleviate the rare patch effect of original NLM. This can also be considered as a kind of patch shape adaptivity, namely using decentered square patch rather than the usual centered counterpart. The same authors also proposed to globally combine two estimates corresponding to different patch sizes, which may be seen as a weak form of patch size adaptivity.

In this paper, we are concerned only with the issue of patch adaptivity, that is, making patch size and shape adapt to local image geometry. However, unlike previous works mentioned above, we take a novel active matching approach. Specifically, before estimating the similarity between two pixels we first perform active matching to arrive at optimally matched patch size and shape. Bearing in mind that only noisy image is given, we carefully design the matching scheme based on LPA-ICI criterion [10]. Simulation results confirm the effectiveness of the proposed approach.

The rest of the paper is organized as follows. Section 2 introduces the principle of baseline NLM and illustrates the need for patch adaptivity and active matching. Section 3 outlines the practical active matching-based NLM denoising algorithm tailored for noisy input. Experimental results are given in Section 4 to prove the superiority of the new denoising scheme. We conclude the paper in the last section.

2. MOTIVATION FOR PATCH ADAPTIVITY AND ACTIVE MATCHING IN NLM

2.1. Bias-variance Perspective on NLM

In this paper, we focus on removing additive noise from image, with the data model as

$$y(i) = x(i) + n(i), \tag{1}$$

where $x(i)$ and $y(i)$ are the ith pixel value of the original and corrupted image respectively, and $n(i)$ is the AWGN noise with variance of σ^2 at the same location. Most of modern denoising filters [1] take the following form:

$$\hat{x}(i) = \frac{\sum_j w_{ij} y(j)}{\sum_j w_{ij}}, \tag{2}$$

where w_{ij} denotes filtering weight, which depends on the proximity or similarity between pixels i and j. Specifically, in NLM the weight is set as

$$w_{ij} = K\left(D\left(P_i, P_j\right)\right), \tag{3}$$

where $K(\cdot)$ is the kernel function and $D\left(P_i, P_j\right)$ measures the similarity (or dissimilarity) between two patches of fixed size and shape surrounding pixels i and j. The mean squared error resulting from (1) and (2) can be derived as

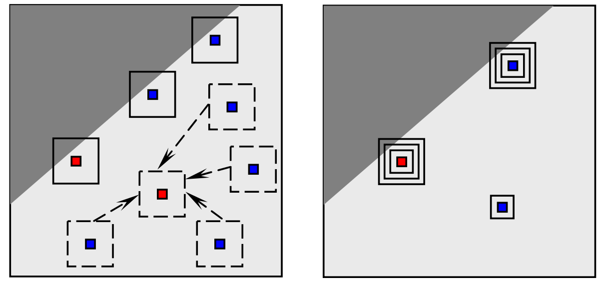
$$\begin{aligned} e(i) &= E\left[\left\{x(i) - \hat{x}(i)\right\}^2\right] \\ &= \left[x(i) - \sum_j c_{ij} x(j)\right]^2 + \sigma^2 \sum_j c_{ij}^2 \end{aligned} \tag{4}$$

where c_{ij} is the normalized weight corresponding to w_{ij}, and the first and second term of the last line correspond to bias and variance respectively. From this expression, it is clear that to guarantee good denoising performance we should *include as many similar pixels as possible into averaging operation, while at the same time exclude as many dissimilar pixels as possible from the same process*. Inclusion of dissimilar pixels will lead to large bias, while too few similar pixels will cause high variance. The next subsection will illustrate this point by a simple example.

By the way, there also exists patch-wise NLM denoising paradigm [9], corresponding to (2) replaced by

$$\hat{\boldsymbol{x}}(i) = \frac{\sum_j w_{ij} \boldsymbol{y}(j)}{\sum_j w_{ij}}, \tag{5}$$

where the boldfaced letters stand for patches surrounding the center pixels. Due to the redundancy of patch space with respect to pixel space, reprojection of the denoised patch is needed to yield the final denoised pixel value. The important observations made in the last paragraph hold for the patch-wise filters too. We will revisit this issue later.



(a) (b)

Fig. 1. Need for patch adaptivity and active matching
(a) Rare patch effect (b) Benefits from active matching

2.2. Rare Patch Effect—The Need for Patch Adaptivity

A fundamental assumption underlying NLM is patch regularity [7], that is, similar patches have similar central pixels. However, given the locally varying geometry in natural images, the fixed patch size and shape adopted in baseline NLM is clearly problematic. Fig. 1(a) shows the situation when NLM is applied to denoising an ideal edge image (what is shown is actually oracle image, while in practice only noisy image is known). The red points denote current pixels to be denoised and the blue points stand for pixels which are patch-wise similar to the red pixels. One red pixel is near edge, and its patch-wise similar pixels are confined to those which are aligned along the edge with that red pixel. The other red pixel has its patch completely contained within one of uniform regions, so any pixel with its patch totally lying within the same uniform region is patch-wise similar to this kind of red pixel. Therefore, with baseline NLM the pixels near the edge will find far less patch-wise similar pixels than the pixels inside the uniform region, while from the perspective of pixel-wise similarity two scenarios are just similar. This is referred to as rare patch effect, which yields relatively high denoising variance near the edge, appearing as noise halo.

At first glance, the rare patch effect may be alleviated by using smaller patches. However, with noisy image it is preferred to use larger patches to give more reliable similarity estimates. Therefore, better tradeoff can be struck by patch size and shape adaptivity with respect to varying local geometry. Furthermore, since patch information is exploited to estimate the similarity between two pixels, the patch adaptivity should *be built on the relationship between local neighborhoods corresponding to the two center pixels*. While the observations are quite simple, the associated criterion is not fulfilled by existent patch adaptivity approaches, where for a given pixel to be denoised the patch size and shape is fixed when estimating the similarity between this pixel and its different neighbors.

2.3. Active Matching for Patch Adaptivity

To achieve the desired patch adaptivity, we take a novel active matching approach, whose principles are described as

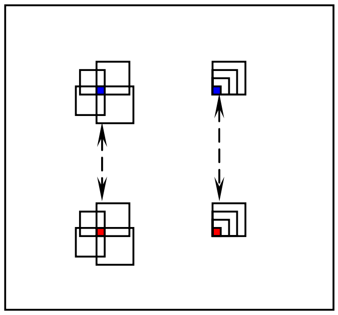


Fig. 2. Process of active matching

follows. Assume a sequence of enlarging patches $P^{(k)}, k = 1, 2, \ldots, m$. Given pixels i and j, we start from the smallest patch and gradually increase patch size until some matchedness criterion is not satisfied by corresponding patches around the two pixels currently under consideration. The process is formalized as

$$k^* = \max\left\{k \,\middle|\, \forall l \leq k, \boldsymbol{C}(P_i^{(l)}, P_j^{(l)}) == true\right\}, \quad (6)$$

where $\boldsymbol{C}$ denotes the proposition for matchedness of two patches. The result of active matching is $P^{(k^*)}$. To address patch shape adaptivity, we perform active matching independently in multiple directions, say four quadrants, and fuse the corresponding matching results, which is shown in Fig. 2, where the left half illustrates the idea of fusion of multiple directional results and the right half corresponds to the process of maximal expansion in a single direction.

The effect of active matching is well demonstrated in Fig. 1 (b), in contrast with Fig. 1 (a). Consider the same edge pixel. With edge-aligned pixels, active matching will yield a large matched patch, while with pixels far inside the uniform region the process will lead to relatively small matched patch, which is more reasonable now. The evident benefit of active matching is more accurate similarity estimates, and in turn better bias-variance tradeoff.

It is interesting to highlight the advantages of active matching approach compared with previous works. Besides richer patch adaptivity, we invest more efforts into analysis of difference between local neighborhoods, which is believed to bring out patch adaptivity of finer granularity. The effect becomes more profound for patch-wise NLM filtering paradigm.

3. PRACTICAL NLM DENOISING ALGORITHM BASED ON ACTIVE MATCHING

It remains to determine the matchedness criterion in active matching. Even more challenging is to take noise into account during the decision process. The solutions to these questions have been found in [11], where Foi et al. proposed a patch-wise shape-adaptive DCT-Wiener denoising scheme and more interestingly the local patch shapes were determined using LPA-ICI criterion. We generalize the same idea to active matching between two local neighborhoods.

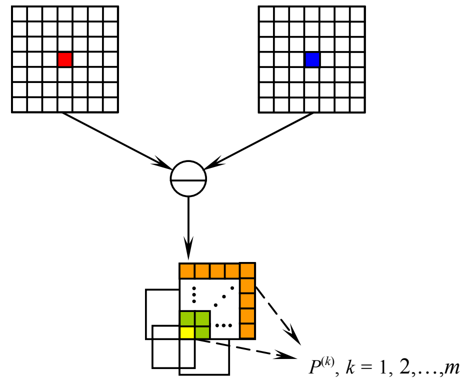


Fig. 3. Active matching for noisy input based on LPA-ICI

Given pixels i and j, we first compute the difference image between their corresponding noisy neighborhoods. If the two neighborhoods do not overlap, the results from this operation can be considered as true difference image corrupted by AWGN with variance of $\sigma_d^2 = 2\sigma^2$. We then apply LPA-ICI to this noisy difference image to obtain maximally matched patch between two pixels. Specifically, let z denote the difference image, we select 0th-order polynomial as underlying local image model. In other words, for each of enlarging patch $P^{(k)}$ we compute

$$\hat{z}_0^{(k)} = \frac{\sum_{j \in P^{(k)}} z_j}{\left|P^{(k)}\right|} \quad (7)$$

where $\left|P^{(k)}\right|$ is the number of pixels in $P^{(k)}$. The maximally matched patch is then selected based on the following criterion:

$$k^* = \max\left\{k \,\middle|\, \bigcap_{l=1}^{k} \left[\hat{z}_0^{(l)} - \Gamma\sigma_d^2, \hat{z}_0^{(l)} + \Gamma\sigma_d^2\right] \neq \emptyset\right\}, \quad (8)$$

where Γ is the constant reflecting local image smoothness and possible deviation of assumption from reality. The whole process is graphically described in Fig. 3.

It seems to be in order to briefly explain how this matching scheme works. It can be seen from the LPA-ICI criterion that the output of active matching operation can be thought of as maximally affinely matched patch between two neighborhoods, that is, the true difference image within this patch can be well approximated by constant functions. Whether two pixels are actually similar is determined finally by kernel functions.

To take full use of the benefit of reliable similarity estimates provided by active matching, we incorporate it into

Table 1. Denoising results (PSNR, in dB) of the proposed algorithm across various noise levels compared with several recent patch-adaptive NLM schemes. One box enclosed by double-line border corresponds to one combination of test image and noise level. Each box contains four figures, where the upper-left, upper-right, lower-left and lower-right part correspond to WAV, WAV-2, NLM-SAP and our method respectively. The highest performance in each scenario is boldfaced.

	Barbara		Boats		Bridge		Cameraman		Couple		Hill		House		Lena		Man		Peppers	
$\sigma = 5$	36.39	36.75	35.50	36.27	34.36	34.84	37.21	37.73	36.04	36.64	34.77	35.57	38.11	38.53	37.35	37.88	36.35	36.95	36.59	37.28
	36.94	36.89	36.35	**36.38**	34.78	**34.92**	37.78	**37.86**	36.73	**36.79**	35.45	**35.64**	38.56	**38.60**	37.96	**38.02**	37.06	**37.11**	37.46	**37.52**
$\sigma = 10$	32.73	33.03	32.57	32.91	29.28	30.15	32.71	33.55	32.63	33.03	30.73	31.54	35.19	35.36	34.67	34.97	32.67	33.19	33.47	33.90
	33.59	33.25	**33.01**	32.98	30.09	**30.23**	33.41	**33.56**	33.12	**33.12**	31.47	**31.61**	**35.37**	35.36	**35.10**	34.99	33.23	**33.77**	**34.22**	34.10
$\sigma = 20$	30.16	30.02	29.56	29.69	25.86	26.44	29.12	29.72	29.32	29.44	27.75	28.06	32.25	32.29	32.07	32.06	29.53	29.79	30.37	30.66
	30.23	30.11	29.71	**29.87**	26.23	**26.54**	29.57	**29.81**	29.43	**29.69**	27.88	**28.24**	32.38	**32.42**	32.03	**32.15**	29.75	**29.99**	30.72	**30.77**
$\sigma = 50$	25.03	24.99	25.06	25.19	22.16	22.38	24.73	24.93	24.52	24.66	23.88	24.05	26.75	26.82	27.23	27.28	25.42	25.58	24.94	25.21
	24.67	**25.08**	25.13	**25.60**	22.24	**22.71**	24.79	**25.15**	24.54	**25.16**	23.84	**24.41**	26.46	**27.29**	27.15	**27.57**	25.55	**25.98**	25.37	**25.68**

patch-wise NLM filtering structure. As noted before, patch-wise filtering tends to include more pixels into averaging operation, leading to lower residual noise variance. However, high-bias may arise due to dissimilar pixels participating in the filtering process. In fact, even though the two neighborhoods are patch-wise similar, individual pixels are not necessarily similar. The finer-granularity adaptivity of active matching is just a remedy to this potential problem.

For reprojection of denoised patch, we adopt the WAV (weighted average reprojection) method [9], where the reprojection weight of each pixel is set to be inversely proportional to its residual noise variances. It is worthwhile to note the novelty added by active matching to reprojection process. In previous works, for current pixel to be denoised, all patches involved in patch-wise weighted averaging are of the same size and shape, or when they are stacked there are vertically same number of pixels at any horizontal location. Therefore, each pixel of current denoised patch has same residual noise variance and same reprojection weights. In contrast, by active matching the patches participating in averaging are not necessarily of same size and shape. So when these patches are vertically aligned, at different horizontal locations there may be different number of pixels taking part in averaging, as such the residual noise variances and in turn reprojection weights at those locations may be different. This is believed to be a meaningful extension to traditional patch-wise denoising paradigm.

4. EXPERIMENTAL RESULTS

To demonstrate the effectiveness of the active matching-based patch adaptivity approach for NLM denoising, simulation experiments are performed on typical test images across wide range of noise intensities. The results of three recent patch adaptive NLM denoising algorithms, WAV [9], WAV-2 [9] and NLM-SAP [8] are shown for comparison.

The parameter settings of the proposed method are as follows. The minimum patch size at single quadrant is set to be 2×2, while the maximum size is set as 6×6 when noise standard deviation is less than 15 and as 8×8 otherwise. LPA-ICI parameter Γ is set to be 1.0. Due to patch adaptivity, patch similarity (dissimilarity) is measured by *mean* squared difference. Flat kernel function [9] is used to compute filtering weights, where bandwidth constant is fixed as $3\sigma^2$. In addition, we use invariably 9×9 filtering windows. The algorithms for comparison are implemented strictly as specified in the original papers or with codes shared by the authors.

Due to limit of space, only objective denoising performance indices are given, as shown by Table 1. All data are obtained by averaging results from 5 independent noise realizations. The superiority of the active matching approach is clearly seen from the table, especially for images with rich edge structures and for scenarios with severe noise.

5. CONCLUSIONS

In this paper, a novel active approach is proposed to address patch adaptivity in NLM image denoising. Before computing the similarity between two pixels, active matching is performed to obtain optimally matched patch size and shape between these pixels. The idea is successfully implemented on noisy input by building on LPA-ICI criterion from adaptive kernel regression. The modifications made lead to meaningful improvement and extension to baseline NLM and previous adaptive approaches, which have been confirmed by extensive simulation experiments.

Further performance boost can be achieved by making the process of active matching more adaptive. For example, the key parameter Γ of LPA-ICI has been set as global constant for simplicity in experiment, which is suboptimal. Further improvements can be made possible by letting Γ vary according to local image characteristics.

Potential application of active matching beyond NLM can also be envisioned. An immediate application is to incorporate active matching into BM3D framework to arrive at truly 3D shape-adaptive denoising structure. Applications outside denoising are also possible. For example, in image super-resolutions based on neighbor embedding (NE) [12], low-resolution patch retrieval is performed before synthesizing high-resolution results. Active matching is believed to be of interest for this task.

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