NONLOCAL MEANS IMAGE DENOISING BASED ON BIDIRECTIONAL PRINCIPAL COMPONENT ANALYSIS

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ABSTRACT

In this paper, a very efficient image denoising scheme, which is called nonlocal means based on bidirectional principal component analysis, is proposed. Unlike conventional principal component analysis (PCA) based methods, which stretch a 2D matrix into a 1D vector and ignores the relations between different rows or columns, we adopt the technique of bidirectional PCA (BDPCA), which preserves the spatial structure and extract features by reducing the dimensionality in both column and row directions. Moreover, we also adopt the coarse-to-fine procedure without performing nonlocal means iteratively. Simulations demonstrated that, with the proposed scheme, the denoised image can well preserve the edges and texture of the original image and the peak signal-to-noise-ratio is higher than that of other methods in almost all the cases.

Index Terms— Image denoising; nonlocal means; principal component analysis (PCA); bidirectional principal component analysis (BDPCA); 2-D signal processing

1. INTRODUCTION

Nonlocal means (NLM) image denoising has attracted much attention since the work of Buades et al. [1]. Many powerful image denoising methods based on the nonlocal principle, which exploits the nonlocal self-similarities among the patches in an image, were proposed in recent years [1-10].

The ability of NLM for denoising is due to the patch regularity assumption, e.g., similar patches have similar center pixels. However, this assumption does not always hold in inhomogeneous regions. In addition, NLM often fails to obtain proper weights, especially in complex scenes. Therefore, many powerful methods were developed for calculating better weights [2-10]. In [2], Zhong *et al.* proposed to use both structure and homogeneous patch similarity to determine the weights in NLM (NLM-SHPS). In [3, 4], Deledalle et al. adopted shape-adaptive patches and the probability patch-based filter (PPB) in the NLM algorithm. In [5] and [6], the NLM methods based on the Foveated patch distance (Fov-NLM) and calculating the minimized variance using average reprojections with two sizes of patches (2-WAV) were proposed, respectively. In [7-10], several methods applying the geometrical structures of an image were proposed. In [7], Zhang et al. proposed a PCA method with local pixel grouping (LPG-PCA). The training patches with similar local spatial structures are grouped before PCA transformation. The principal neighborhood dictionary was adopted in [8]. In [9], Luisier et al. applied the wavelet transform together with Stein's unbiased risk estimation (SURE-LET). A wavelet shrinkage method using Stein's unbiased risk estimation (Shrink-SURE) was proposed by Zhou and Cheng in [10].

In this paper, we propose an efficient denoising scheme, which is called nonlocal means based on bidirectional principal component analysis (NLM-BDPCA).

Unlike conventional PCA-based methods, which directly stretch 2D image patches into 1D image vectors and hence ignores the spatial relations between different rows (or columns), BDPCA projects 2D image patches using row and column projectors to generate feature matrices. Therefore, it can capture the better intrinsic geometrical structure embedded in the original image spatial domain [11, 12]. We incorporate BDPCA into the framework of nonlocal means for better weight calculation and perform coarse-to-fine steps to improve the denoising performance. From all simulation results, the proposed scheme achieves highly superior performance when compared with classical nonlocal means methods and several recent powerful image denoising methods from subjective and objective measures of image quality in a wide range of noise addition.

The remainder of this paper is organized as follows. In Section 2, we describe the proposed image denoising algorithm with coarse-to-fine steps, which incorporates BDPCA into nonlocal means. Section 3 shows several simulations. A conclusion is made in Section 4.

2. PROPOSED NONLOCAL MEANS BASED ON BIDIRECTIONAL PRINCIPAL COMPONENT ANALYSIS (NLM-BDPCA)

2.1. Nonlocal Means

A noisy image can always be modeled as:

$$y_i = x_i + n_i \tag{1}$$

where n_i is the zero-mean white Gaussian noise of variance σ^2, y_i and x_i are the i^{th} pixel intensities in noisy and noise-free images, respectively. Then, in the NLM algorithm, the noise-free image is estimated from

$$\hat{x}_i = \frac{1}{Z_i}\sum_{j\in N_i} w_{i,j} y_j \qquad (2)$$

where N_i is the square search window centered around pixel i, $w_{i,j}$ denotes the weight, which is computed by using the vectorized noisy patches $\mathbf{y}_i$ and $\mathbf{y}_j$, and $Z_i = \sum_{j\in N_i} w_{i,j}$ is a normalization factor. The weight is defined as

$$w_{i,j} = \exp\left(-\frac{1}{k\sigma^2}\left\|\mathbf{y}_i - \mathbf{y}_j\right\|_2^2\right) \qquad (3)$$

where k is the smoothing parameter, and $\|\ \|_2^2$ denotes the Euclidean distance to measure patch similarities. NLM is powerful in denoising, but it causes a bias of the weights due to the noisy patches used for similarity measure, especially when few repetitive structures exist in an image. To overcome this problem, a new method for weight calculation is proposed to compute patch distance in the low-dimensional feature domain using BDPCA. More details are described in the following subsection.

2.2. Bidirectional Principal Component Analysis

As a generalization of two-dimensional PCA (2D PCA), BDPCA is a straightforward projection technique, which is based on 2D image data matrices rather than 1D vectors [11, 12]. That is, an image covariance is constructed directly using the original 2D image matrices and adopts the concept of row and column eigenvectors to create the feature matrix, which effectively captures the intrinsic image structure.

In this paper, we perform BDPCA on image patches in an image and incorporate it to nonlocal means for better weight calculation. Let $\{Y_1, Y_2,\ldots, Y_N\}$, $Y_i \in R^{m\times n}$ be a set of training image patches in a noisy image (the image patches that has been processed are selected as training patches). Assume that the noisy image is divided into N blocks. Denote the row total scatter matrix and the column total scatter matrix respectively by

$$S_t^{row} = \frac{1}{Nm}\sum_{i=1}^{N}(Y_i - \bar{Y})^T (Y_i - \bar{Y}),$$

$$S_t^{col} = \frac{1}{Nn}\sum_{i=1}^{N}(Y_i - \bar{Y})(Y_i - \bar{Y})^T \qquad (4)$$

where $\bar{Y}$ is the mean matrix of all training image patches. Denote the eigenvalues for S_t^{row} by $\{\lambda_i^{row}\}$ (i = 1, 2, …, n), which are sorted in descending order, and the corresponding eigenvectors by $\{W_i^{row}\}$. Similarly, the eivgenvalues and eigenvectors for S_t^{col} are represented by $\{\lambda_i^{col}\}$ and $\{W_i^{col}\}$ (i = 1, 2, …, m), respectively. Then, the row and column projectors are constructed by choosing the row eigenvectors corresponding to the first q largest eigenvalues of S_t^{row} and the column eigenvectors corresponding to the first r largest eigenvalues of S_t^{col}, respectively:

$$W_{row} = \left[W_1^{row}, W_2^{row}, \ldots, W_q^{row}\right], \quad W_{col} = \left[W_1^{col}, W_2^{col}, \ldots, W_r^{col}\right].$$

The new feature matrix Y' with dimensionality reduction can be obtained from

$$Y' = W_{col}^T \times Y \times (W_{row}). \qquad (5)$$

Note that the image blocks centered at each pixel in an image are collected for training to construct the row and column projectors. In this study, square image patches are exploited, i.e., $m = n$. Obviously, the denoising performance depends largely on the number of row and column eigenvectors retained. The eigenvectors with small eigenvalues generally correspond to noise components. Hence, to preserve the most important geometrical structure, we first determine the number of row eigenvectors, which is retained by a threshold value, TH, as follows

$$q = \arg\max_i \left|\bar{\lambda}_i^{row} - \bar{\lambda}_{i+1}^{row}\right| > TH \qquad (6)$$

where $\bar{\lambda}_i^{row} = \lambda_i^{row} / \sum_{i=1}^{n} \lambda_i^{row}$ is a normalized eigenvalue.

The smaller the value TH is, the more details of an image are preserved but with a larger amount of noise. In contrast, the larger the value TH is, the more noise is removed but the image geometrical structure (e.g., edge and texture) may be destroyed. TH is set as 0.0014 in all simulations.

Generally, the number of column eigenvectors is equal to or larger than that of row eigenvectors but should be selected as small as possible to keep the denoising ability. Hence, we set $r = q$ if $j^* < q$ and set $r = q + 2$ if $j^* > q + 2$ where $j^* = \arg\max_j \lambda_j^{col} \geq \lambda_q^{rol}$.

After the row and column projection of image patches, new features for each pixel in an image are generated and stretched to d-dimensional vectors ($d = q{\cdot}r$). Hence, a new weight can be obtained by (7) and applied to the NLM framework as follows:

$$w'_{i,j} = \exp\left(-\frac{1}{k'\sigma^2}\left\|\mathbf{f}_i - \mathbf{f}_j\right\|_2^2\right) \qquad (7)$$

where $\mathbf{f}_i$ and $\mathbf{f}_j$ denote the d-dimensional new features at pixels i and j, and k' is the smoothing parameter. Finally, the estimated pixel value $\hat{x}'_i$ is calculated from

$$\hat{x}'_i = \frac{1}{Z'_i}\sum_{j\in N_i} w'_{i,j} y_j \quad \text{where } Z'_i = \sum_{j\in N_i} w_{i,j}. \qquad (8)$$

2.3. Coarse-to-fine NLM-BDPCA Image Denoising

The proposed coarse-to-fine NLM-BDPCA image denoising contains two steps, i.e., the coarse step and the fine step. In the coarse step, we exploit the parameters, including the search window of size S_1xS_1, the smoothing parameter k_1' and the row and column projectors (i.e., $W'_{row} \in R^{n_1 \times q_1}$, $W'_{col} \in R^{n_1 \times r_1}$), to perform image denoising by (8). In this step, the denoised image $\hat{x}'$ is obtained by coarsely smoothing the noisy image to reduce a large amount of noise using large parameter values. However, there is still visually unpleasant noise residual in the denoised image due to the strong noise corrupted total

Algorithm 1 Coarse-to-fine NLM-BDPCA image denoising

Input: Noisy image y_i.

-Step 1:

- Extract all image patches at every pixel i to calculate the row and the column total scatter matrices using (4).
- Generate the row projector and the column projector.
- Project each image patch centered at i to generate features using (5) and (6) and stretch it to a 1D vector $\mathbf{f}_i$.
- For every pixel i, do
  (a) Apply (7) to calculate $w_{i,j}^{l}$ for every j in the search window centered at i.
  (b) Obtain the pixel value estimation result $\hat{x}_i'$ using (8).

-Noise estimation: Use (9) to estimate the noise variance of the denoised image $\hat{x}' = (\hat{x}_i')$ obtained by Step 1.

-Step 2:

- Apply the same procedure as in Step 1 to each pixel $\hat{x}_i'$ but using smaller parameters, which are discussed in more details in Section 2.3, to obtain the final denoised pixel $\hat{x}_i''$.

Output: Denoised image $\hat{x}_i''$.

scatter matrices, which leads to a bias of the row and column projectors in BDPCA and hence deteriorates the denoising performance. To overcome the above problem, the same procedure discussed above is applied to finely smooth the denoised image $\hat{x}'$ for the preservation of details and reduction of visual artifacts, but smaller parameters are used. That is, the search window of size, the smoothing parameter, the row and column projectors and image patches grouping for BDPCA are all smaller than that in the first step. Hence, the final denoised image $\hat{x}''$ is obtained.

The denoised image $\hat{x}'$, which is estimated by (9), is also applied in the second step. More details in the noise estimation can be referred to [7]

$$\sigma_r^2 = c_r \cdot \sqrt{\sigma^2 - E[\tilde{x}^2]} \qquad (9)$$

where $c_r < 1$ is a constant. $\tilde{x}$ is the method noise [1], which is defined as the difference between the noisy image y and the denoised image $\hat{x}'$, i.e., $\tilde{x} = y - \hat{x}'$. c_r is set as 0.95 in all simulations. The overall procedure of the proposed scheme is summarized in Algorithm 1.

The proposed algorithm can preserve the edge and the structure information because the image patch that has higher similarity with the current patch is assigned a larger weight, as in (7).

3. SIMULATIONS

The performance of the proposed NLM-BDPCA algorithm for image denoising is evaluated on four test images: *Lena*, *House*, *Peppers*, and *Airplane* and the 68 images in the Berkeley Database [13]. The noisy images are generated by adding zero-mean white Gaussian noise with standard deviation $\sigma \in \{20, 40, 60, 80, 100\}$. Throughout this paper, the search window sizes, S_1xS_1 and S_2xS_2, are set as 21x21 and 5x5, respectively. The smoothing parameters k_1' is 36 and k_2' is 25. The parameters q_1 and r_1 are determined by (6). It should be noted that q_1 is at least 6 in our scheme. In the fine step, the image patch size is 3x3 and (q_2, r_2) = (3, 3). To illustrate the effectiveness of the proposed denoising algorithm, we compare the proposed scheme with the NLM method [2] and several other existing methods: NLM-SHPS [3], NLM-SAP [4], PPB [5], FovNLM [6], 2-WAV [7], LPG-PCA [8], SURE-LET [9], and Shrink SURE [10].

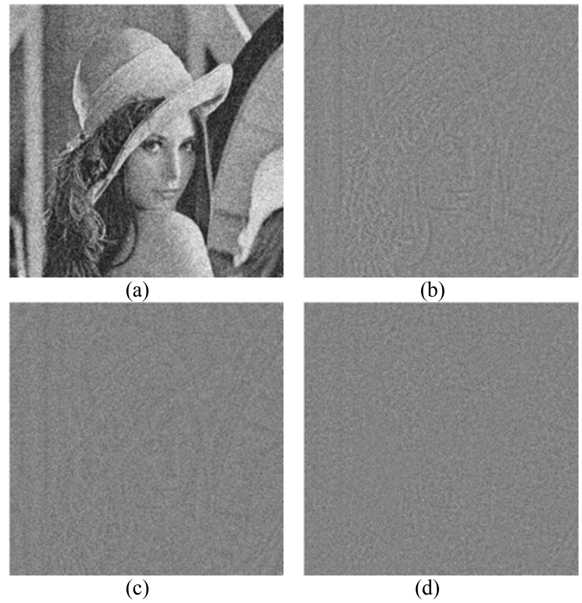

Figure 1. Comparison of the method noises (i.e., the difference between the noisy image and the denoised image) for *Lena* image using different algorithms. (a) Noisy image corrupted with noise when σ = 40, (b) NLM, (c) LPG-PCA, (d) Proposed NLM-BDPCA.

Fig. 1 shows an example for comparing the method noises using different denoising methods for *Lena* image corrupted by the noise with σ = 40. The method noise indicates whether the geometrical structures or details are preserved or eliminated in the denoised image. It looks like white Gaussian noise if most of the structures and the edges of the original image are preserved after denoising. Note that some edges, such as the texture of the hair and the border of the face, appear in the method noises in Figs. 1(b) and 1(c) but do not appear in Fig. 1(d). It shows that, compared with NLM and LPG-PCA, the proposed scheme can well preserve the edges of an image.

Fig. 2 shows the cropped and zoom-in denoising results of the proposed scheme and existing state-of-the-art denoising methods. It can be seen that the proposed scheme have better visual quality than all of the other methods.

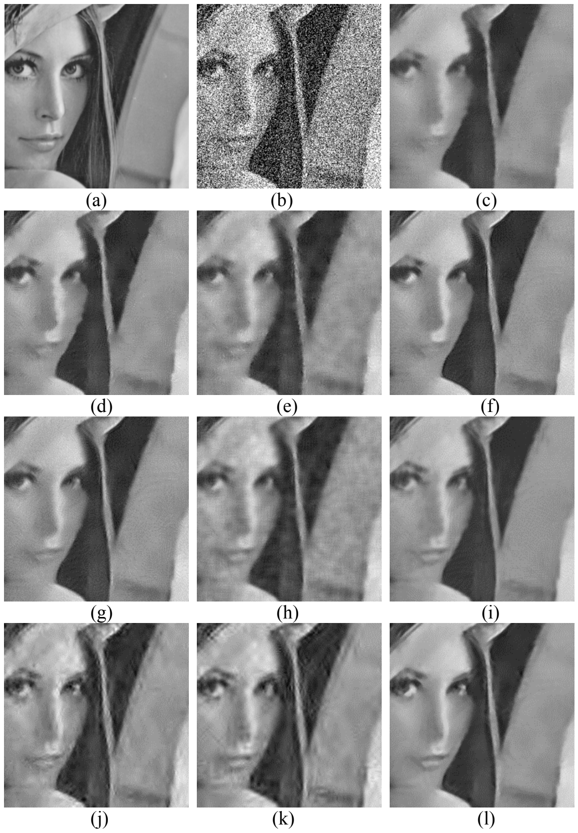

Figure 2. Denoising results for *Lena* image by different schemes. (a) Original image, (b) Noisy image corrupted with noise when σ= 60, (c) NLM, (d) NLM-SHPS, (e) NLM-SAP, (f) PPB, (g) FovNLM, (h) 2-WAV, (i) LPG-PCA, (j) SURE-LET, (k) NeighShrink SURE, (l) Proposed NLM-BDPCA.

Since the NLM-BDPCA denoising algorithm can capture local image structure for better weight calculation, the noise in images is removed without introducing too much artifact.

From Tables I and II, one can obviously see that the proposed scheme significantly outperforms the classical nonlocal means method and several powerful image denoising methods, especially when the image is seriously corrupted by noise. Although a little more computation time is required, the denoising performance is much better.

4. CONCLUSION

This paper presents a very effective image denoising scheme, which incorporates bidirectional principal component analysis (BDPCA) into the framework of nonlocal means. A coarse-to-fine algorithm is designed and its effectiveness is shown in simulation results, which demonstrated that the proposed scheme is significantly superior to the classical nonlocal means method and several recent powerful methods in a wide range of noise addition.

TABLE I
PEAK SIGNAL-TO-NOISE RATIO (PSNR) COMPARISON OF IMAGE DENOISING RESULTS. THE BEST RESULTS FOR EACH CASE ARE HIGHLIGHTED.

Images	Noise σ	NLM	NLM-SHPS	PPB	Fov-NLM	2-WAV	LPG-PCA	Shrink SURE	**Proposed NLM-BDPCA**
Lena	20	31.65	31.93	31.49	32.44	32.10	**32.62**	31.52	32.48
	40	28.34	28.79	28.67	29.09	28.55	29.27	28.41	**29.80**
	60	26.39	26.87	26.81	27.02	26.38	27.33	26.64	**28.00**
	80	25.01	25.50	25.40	25.51	24.94	25.94	25.35	**26.70**
	100	23.98	24.42	24.26	24.31	23.82	24.85	24.43	**25.67**
House	20	32.46	32.46	31.80	32.71	32.27	**33.07**	31.07	32.95
	40	28.46	29.01	28.90	29.43	28.30	29.71	27.88	**29.82**
	60	25.81	26.65	26.85	26.93	25.78	27.46	26.04	**27.87**
	80	24.12	24.97	24.98	25.09	24.27	25.79	24.76	**26.37**
	100	22.98	23.72	23.59	23.71	23.16	24.49	23.75	**25.14**
Air-plane	20	30.96	31.01	30.54	31.47	31.36	**32.15**	30.95	31.61
	40	27.38	27.94	27.58	28.17	27.43	28.48	27.95	**28.67**
	60	25.02	25.73	25.46	25.78	25.05	26.27	26.11	**26.54**
	80	23.52	24.20	23.91	24.12	23.60	24.73	24.89	**25.08**
	100	22.49	23.08	22.76	22.95	22.58	23.60	24.02	**24.00**
Peppers	20	31.64	31.71	31.31	32.05	32.10	**32.26**	30.75	32.11
	40	28.49	28.96	28.67	29.11	28.66	29.20	27.26	**29.60**
	60	26.38	26.92	26.58	26.95	26.26	27.13	25.36	**27.73**
	80	24.84	25.41	25.10	25.33	24.73	25.60	24.11	**26.30**
	100	23.70	24.25	23.95	24.07	23.61	24.41	23.16	**25.20**

TABLE II
PSNR COMPARISON OF IMAGE DENOISING RESULTS ON THE 68 IMAGES IN THE BERKELEY DATABASE [13].

Noise σ	NLM	NLM-SHPS	PPB	Fov-NLM	2-WAV	LPG-PCA	Shrink SURE	**Proposed NLM-BDPCA**
60	23.29	23.75	23.62	23.76	23.53	23.99	23.61	**24.17**
80	22.36	22.73	22.61	22.70	22.41	22.96	22.64	**23.24**
100	21.69	21.96	21.81	21.89	21.56	22.22	21.92	**22.56**

REFERENCES

[1] A. Buades, B. Coll, and J.M. Morel, “A review of image denoising algorithm, with a new one,” *Multiscale Modeling and Simulation*, vol. 4, issue 2, pp. 490–530, 2005.

[2] H. Zhong, P.P. Han, X.H. Zhang, and Y.Q. Yu, “Hybrid patch similarity for image denoising,” *Electron. Lett.*, vol. 48, issue 4, pp. 212–213, 2012.

[3] C.A. Deledalle, V. Duval, and J. Salmon, “Non-local methods with shape-adaptive patches (NLM-SAP),” *Journal of Mathematical Imaging and Vision*, vol. 43, pp. 103–120, 2012.

[4] C.A. Deledalle, L. Denis, and F. Tupin, “Iterative weighted maximum likelihood denoising with probabilistic patch-based weights,” *IEEE Trans. Image Processing*, vol. 18, issue 12, pp. 2661–2672, 2009.

[5] A. Foi and G. Boracchi, “Foveated self-similarity in nonlocal image filtering,” in *Proc. SPIE Electronic Imaging, Human Vision and Electronic Imaging*, pp. 829110-1 - 829110-12, 2012.

[6] J. Salmon and Y. Strozecki, “Patch reprojections for non-local methods,” *Signal Processing*, vol. 92, issue 2, pp. 477–489, 2012.

[7] L. Zhang, W. Dong, D. Zhang, and G. Shi, “Two-stage image denoising by principal component analysis with local pixel grouping,” *Pattern Recogn.*, vol. 43, pp. 1531–1549, 2010.

[8] T. Tasdizen, “Principal components for non-local means image denoising,” *Proc. 15th IEEE Int. Conf. Image Processing*, pp. 1728–1731, 2008.

[9] F. Luisier, T. Blu, and M. Unser, “A new SURE approach to image denoising: interscale orthonormal wavelet thresholding,” *IEEE Trans. Image Processing*, vol. 16, issue 3, pp. 593–606, 2007.

[10] D.W. Zhou and W.G. Cheng, “Image denoising with an optimal threshold and neighbouring window,” *Pattern Recognition Letters*, vol. 29, issue 11, pp. 1694–1697, 2008.

[11] J. Yang, D. Zhang, A.F. Frangi, and J.Y. Yang, “Two-dimensional PCA: a new approach to appearance-based face representation and recognition,” *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 26, issue 1, pp. 131-137, 2004.

[12] W. Zuo, D. Zhang, and K. Wang, “Bidirectional PCA with assembled matrix distance metric for image recognition,” *IEEE Trans. Syst. Man Cybern., Part B, Cybern.*, vol. 36, issue 4, pp. 863-872, 2006.

[13] D. Martin, C. Fowlkes, D. Tal, and J. Malik, “A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics,” in *IEEE Int. Conf. Computer Vision*, vol. 2, pp. 416-423, 2001.