

# COUPLED FISHER DISCRIMINATION DICTIONARY LEARNING FOR SINGLE IMAGE SUPER-RESOLUTION

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## ABSTRACT

Image Super-resolution (SR) reconstruction techniques based on sparse representation have attracted ever-increasing attentions in recent years, where the choice of over-complete dictionary is of prime important for reconstruction quality. However, most of the image SR methods based on sparse representation fail to consider the discrimination and the redundancy of the dictionaries, which lead to obvious SR reconstruction artifacts. In this paper, we propose a novel image SR framework using coupled fisher discrimination dictionary learning (CFDDL). With CFDDL, a pair of discriminative dictionaries are first learned for the same class of high-resolution (HR) image patches and corresponding low-resolution (LR) image patches, respectively. Then, we utilize the identical sparse representation for the same class of HR and LR image patches, which can not only discover the inherent relationship between the HR and LR image patches but also enhance the computational efficiency. Extensive experiments compared with several other SR methods demonstrate the superiority of the proposed method in terms of subjective evaluation as well as objective evaluation.

**Index Terms**— Super-resolution (SR), sparse coding, coupled fisher discrimination dictionary

## 1. INTRODUCTION

Image super-resolution (SR) reconstruction is currently a very active field of research [1]. It is the inverse problem of recovering a high-resolution (HR) image from one or more low-resolution (LR) observation images. Due to the physical limitation of relevant imaging devices (e.g., digital cameras, cell phone cameras or surveillance cameras), it is hard to obtain the desired HR images. Therefore, when the physical devices can not work, people resort signal processing techniques to restore the potential information hidden in the source. Hence, lots of image SR reconstruction methods [2, 3, 4, 5, 6, 7, 8, 9, 10] have been reported in recent years.

Currently, sparse representation techniques [4, 6, 7, 8] are employed to perform image SR. In the work of Yang et al. [4], by enforcing  $l_1$ -norm sparsity prior regularization, an over-complete dictionary is learned to perform image SR reconstruction, namely ScSR. LR image patches have the same

sparse representation with their corresponding HR counterparts, which can reduce the computational complexity but ignore the structure difference between the HR and LR image patches. Wang et al. [7] proposed a semi-coupled dictionary learning (SCDL) scheme by learning a linear mapping for connecting HR and LR image patches sparse representation. Their method protects the structure of HR and LR images successfully, but at the cost of high computational time. Huang et al. [8] further proposed a full coupled dictionary learning method for image SR, in which a common feature space is learned for connecting the sparse representation of HR and LR image patches. However, the main drawback of this method is low computational efficiency.

In this paper, we focus on studying the sparse representation method for image SR. Motivated by [11], we find that learning a fisher discriminative dictionary can not only preserve the diversity of the dictionary for different classes of image patches but also make the dictionary more discriminative. Therefore, we propose a novel single image SR reconstruction framework based on coupled fisher discrimination dictionary learning (CFDDL). Specifically, we first learn the coupled discriminative dictionaries for pairs of HR and LR image patches. Then, the HR image patch is reconstructed over the HR dictionary with sparse coefficients coded by the same class of LR image patch over the LR dictionary, which not only discover the inherent relationship between HR image patches and their corresponding counterparts but also enhance the computational efficiency. Fig. 1 illustrates the flowchart of the proposed method.

The rest of the paper is organized as follows. Section 2 presents a form of preparation. Section 3 presents our algorithm. Experimental results are given in Section 4. Section 5 concludes the paper.

## 2. PRELIMINARY

### 2.1. Sparse Coding

Sparse coding aims to approximate each input signal  $\mathbf{x} \in \mathbb{R}^m$  with a weighted linear combination of a few elementary signals called basis atoms, often chosen from an over-complete dictionary  $\mathbf{D} \in \mathbb{R}^{m \times k}$  ( $m < k$ ). The classical objective func-

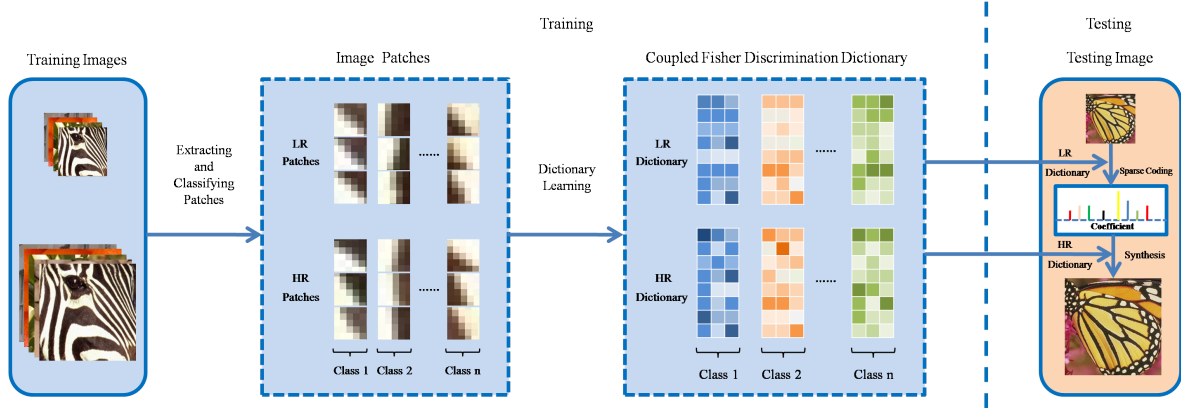


Fig. 1. Flowchart of the proposed image SR method.

tion of sparse coding can be defined as:

$$\min_{D, \{\alpha_i\}_{i=1}^N} \sum_{i=1}^N \|x_i - D\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1 \quad (1)$$

$$s.t. \quad \|d_i\|_2^2 \leq 1, i = 1, 2, \dots, k,$$

where  $d_i$  is the  $i$ -th column of  $D$ ,  $\lambda$  is the regularization parameter, the term  $\|\alpha_i\|_1$  is to enforce sparsity, and the constraint term  $\|d_i\|_2^2$  removes the scaling ambiguity.

## 2.2. Joint Sparse Coding

Unlike the sparse coding, Yang et al. [4] used joint sparse coding to learn the coupled dictionary pair  $D_y$  and  $D_x$  for HR image patches  $\{y_i\}_{i=1}^N$  and LR image patches  $\{x_i\}_{i=1}^N$ , respectively, and the sparse representation of  $x_i$  in terms of  $D_x$  should be same as that of  $y_i$  in terms of  $D_y$ . Since the coupled dictionary pair are learned, the HR image patch can be reconstructed over the HR dictionary with sparse coefficients coded by corresponding LR image patch over the LR dictionary. The above problem can be defined as:

$$\min_{D_x, D_y, \{\alpha_i^x\}_{i=1}^N} \sum_{i=1}^N \left\{ \frac{1}{2} \|x_i - D_x \alpha_i^x\|_2^2 + \lambda \|\alpha_i^x\|_1 \right. \\ \left. + \frac{1}{2} \|y_i - D_y \alpha_i^y\|_2^2 + \lambda \|\alpha_i^y\|_1 \right\} \quad (2)$$

$$s.t. \quad \|d_i^x\|_2^2 \leq 1, \|d_i^y\|_2^2 \leq 1, \alpha_i^x = \alpha_i^y, i = 1, 2, \dots, k,$$

which is equivalent to

$$\min_{\bar{D}, \{\alpha_i\}_{i=1}^N} \sum_{i=1}^N \|\bar{x}_i - \bar{D}\alpha_i\|_2^2 + \lambda \|\alpha_i\|_1 \quad (3)$$

$$s.t. \quad \|\bar{d}_i\|_2 \leq 1, i = 1, 2, \dots, k,$$

where  $\bar{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$  and  $\bar{D} = \begin{bmatrix} D_x \\ D_y \end{bmatrix}$ ,  $\bar{d}_i$  is the  $i$ -th column of  $\bar{D}$ . Since only a pair of over-complete dictionaries  $D_x$  and  $D_y$  are learned for all the various HR and LR image patches, the learned dictionaries have no capacities of discrimination and representation.

## 3. PROPOSED METHOD

In this section, we propose a novel image SR method based on CFDDL. We will detail this method in the following sub-sections.

### 3.1. Problem Formulation

Instead of learning an over-complete dictionary for all classes, we learn a coupled fisher discrimination dictionary  $D_x = [D_{x,1}, \dots, D_{x,c}]$  and  $D_y = [D_{y,1}, \dots, D_{y,c}]$ , where  $D_{x,i}$  and  $D_{y,i}$  are the  $i$ -th class of HR and LR sub-dictionary, respectively, and  $c$  is the total number of classes. Let image patch sets  $X = [X_1, \dots, X_c] \in \mathbb{R}^{d_1 \times n}$  and  $Y = [Y_1, \dots, Y_c] \in \mathbb{R}^{d_2 \times n}$  be  $n$  data pairs extracted from HR and LR images, respectively, where  $X_i$  and  $Y_i$  are the subsets of LR and HR image patches from class  $i$ , respectively.

Our CFDDL-based image SR can be approached as solving the following minimization problem:

$$\min_{D_x, D_y, A} E_{DL}(X, D_x, A) + E_{DL}(Y, D_y, A) \quad (4)$$

$$+ \lambda_1 \|A\|_1 + \lambda_2 F(A).$$

In Eq. (4),  $\lambda_1, \lambda_2$  are regularization parameters to balance the terms in the objective function, and  $E_{DL}$  denotes the discriminative fidelity term and can be defined as follows:

$$E_{DL}(X_i, D_x, A_i) = \|X_i - D_x A_i\|_F^2 + \|X_i - D_{x,i} A_i^i\|_F^2 \\ + \sum_{j=1, j \neq i}^c \|D_{x,j} A_i^j\|_F^2, \\ E_{DL}(Y_i, D_y, A_i) = \|Y_i - D_y A_i\|_F^2 + \|Y_i - D_{y,i} A_i^i\|_F^2 \\ + \sum_{j=1, j \neq i}^c \|D_{y,j} A_i^j\|_F^2, \quad (5)$$

where  $A_i = [A_i^1; \dots; A_i^j; \dots; A_i^c]$  is the representation of  $X_i$  (or  $Y_i$ ) over  $D_x$  (or  $D_y$ ),  $A_i^j$  is the sparse coefficients of  $X_i$  (or  $Y_i$ ) over the sub-dictionary  $D_{x,j}$  (or  $D_{y,j}$ ). The first term denotes that the dictionary  $D_x$  (or  $D_y$ ) should be

able to well represent  $X_i$  (or  $Y_i$ ). If we add a constraint to make the second term small, better discrimination will be achieved. Since  $A_i^i$  has some significant coefficients, i.e.,  $\|X_i - D_{x,i}A_i^i\|_F^2$  (or  $\|Y_i - D_{y,i}A_i^i\|_F^2$ ) is small, the  $A_i^j$  is nearly zero which makes the last term small.

In Eq. (4),  $F(A)$  is the discriminative coefficient term. To make dictionary  $D_x$  (or  $D_y$ ) be discriminative for the samples in  $X$  (or  $Y$ ), we can make the sparse coefficients be discriminative by using Fisher discrimination criterion [12].  $F(A)$  can be rewritten as:

$$F(A) = \text{tr}(W(A)) - \text{tr}(B(A)) + \mu\|A\|_F^2, \quad (6)$$

where  $\|A\|_F^2$  is added to make  $F(A)$  convex and stable,  $\mu$  is a parameter, and  $W(A)$  and  $B(A)$  are denoted as the within-class scatter of  $A$  and the between-class scatter of  $A$ , respectively, which can be defined as:

$$\begin{aligned} W(A) &= \sum_{i=1}^c \sum_{a_k \in A_i} (a_k - m_i)(a_k - m_i)^T, \\ B(A) &= \sum_{i=1}^c n_i(m_i - m)(m_i - m)^T, \end{aligned} \quad (7)$$

where  $n_i$  is the number of samples in  $A_i$ , and  $m_i$  and  $m$  are denoted as the mean vector of  $A_i$  and  $A$ , respectively.

Grouping the two  $E_{DL}$  together and denoting

$$\bar{X}_i = \begin{bmatrix} X_i \\ Y_i \end{bmatrix}, \bar{D} = \begin{bmatrix} D_x \\ D_y \end{bmatrix}, \quad (8)$$

we can convert Eq. (4) to the standard fisher discrimination dictionary learning [11] problem:

$$\begin{aligned} \min_{\bar{D}, A} \sum_{i=1}^c \{ \|\bar{X}_i - \bar{D}A_i\|_F^2 + \|\bar{X}_i - \bar{D}_iA_i^i\|_F^2 \\ + \sum_{j=1, j \neq i}^c \|\bar{D}_jA_i^j\|_F^2 \} + \lambda_1\|A\|_1 + \lambda_2F(A) \quad (9) \\ \text{s.t. } \|\bar{d}_k\|_2 \leq 1, \forall k. \end{aligned}$$

### 3.2. Optimization

While the objective function in Eq. (9) is not jointly convex to  $\bar{D}$  and  $A$ , it is convex when  $\bar{D}$  is fixed or  $A$  is fixed. Therefore, we separate the objective function into two sub-problems: updating  $A$  by fixing  $\bar{D}$ ; and updating  $A$  by fixing  $\bar{D}$ . Given training image data  $X$  and  $Y$ , we apply an iterative algorithm to optimize the dictionaries  $\bar{D}$  and coefficients  $A$ , respectively.

#### 3.2.1. Updating $A$

We first fix  $\bar{D}$ , and the objective function in Eq. (9) becomes a sparse coding problem to compute  $A$ . We calculate  $A_i$  class by class. When calculate  $A_i$ , all  $A_j (j \neq i)$ , are fixed. Thus we can calculate the sparse coefficients  $A_i$  as follows:

$$\begin{aligned} \min_{A_i} \|\bar{X}_i - \bar{D}A_i\|_F^2 + \|\bar{X}_i - \bar{D}_iA_i^i\|_F^2 \\ + \sum_{j=1, j \neq i}^c \|\bar{D}_jA_i^j\|_F^2 + \lambda_1\|A_i\|_1 + \lambda_2F_i(A_i). \end{aligned} \quad (10)$$

Many optimization methods can solve Eq. (10) effectively, such as proximal methods [13], Iterative Projection Method (IPM) [14], etc. In this paper, we choose the IPM [14] to solve it.

#### 3.2.2. Updating $\bar{D}$

When the sparse coefficients  $A$  are fixed, we update the dictionary  $\bar{D}_i$  class by class. On calculating  $\bar{D}_i$ , all  $\bar{D}_j (j \neq i)$ , are fixed. Thus, we convert (9) into the following problem:

$$\begin{aligned} \min_{\bar{D}_i} \|\bar{X}_i - \bar{D}_iA_i^i\|_F^2 - \sum_{j=1, j \neq i}^c \bar{D}_jA_j^j\|_F^2 \\ + \|\bar{X}_i - \bar{D}_iA_i^i\|_F^2 + \sum_{j=1, j \neq i}^c \|\bar{D}_iA_j^j\|_F^2, \end{aligned} \quad (11)$$

where  $A_i^i$  is the sparse coefficients of  $\bar{X}$  over  $\bar{D}_i$ . Eq. (11) is a quadratic programming problem and can be solved by using the algorithm in [15].

### 3.3. Synthesis

After learning the discriminative dictionary pair  $D_x$  and  $D_y$ , for a given LR image  $y$ , we can easily convert it into an HR image  $x$  by solving the following optimization:

$$\begin{aligned} \min_A \sum_{i=1}^c \{ \|\bar{x}_i - \bar{D}A_i\|_F^2 + \|\bar{x}_i - \bar{D}_iA_i^i\|_F^2 \\ + \sum_{j=1, j \neq i}^c \|\bar{D}_jA_i^j\|_F^2 \} + \lambda_1\|A\|_1 + \lambda_2F(A), \end{aligned} \quad (12)$$

where  $\bar{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$  and  $\bar{D} = \begin{bmatrix} D_x \\ D_y \end{bmatrix}$ ,  $y_i$  is a patch of  $y$  and  $x_i$  is the corresponding patch in the intermediate estimate of  $x$  to be synthesized. The solving method of Eq. (12) is as same to Eq. (10). Finally,  $x_i$  can be reconstructed as:

$$\hat{x}_i = D_x \hat{A}_i. \quad (13)$$

After all the patches are estimated, the estimation of the desired HR image  $x$  can then be obtained. In our synthesis method, we need an initial estimation of  $x$ . For example,  $x$  can be simply initialized by bicubic interpolation.

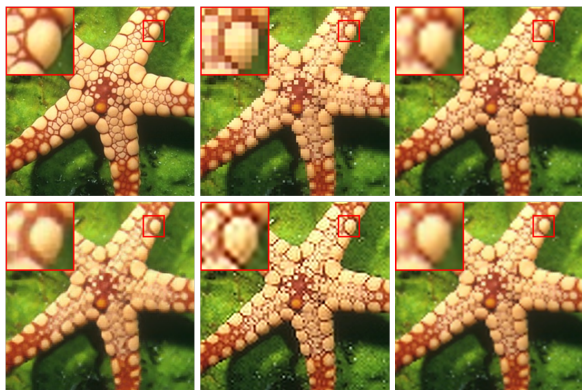
## 4. EXPERIMENTS

In this section, we verify the performance of our CDFFL-based image SR method. The training image patch pairs are collected from the Kodak PhotoCD dataset, which has no relation with the testing images. The size of image patch is  $5 \times 5$ . Pre-clustering is done and the cluster number is set to be 6. The numbers of dictionary atoms for both  $D_x$  and  $D_y$  are set to be 200 for each cluster. In the following experiments, we empirically set regularization parameter  $\lambda_1 = 0.005$ ,  $\lambda_2 = 0.05$ , and  $\mu = 0.05$ .

Considering the limited space, we only compare our method with several representative methods, such as nearest,

**Table 1.** The PSNR(dB) and FSIM results (luminance components) of reconstructed HR images (scaling factor = 3).

Image	Nearest		Bicubic		NE [16]		ScSR [4]		Proposed	
	PSNR	FSIM	PSNR	FSIM	PSNR	FSIM	PSNR	FSIM	PSNR	FSIM
Butterfly	18.69	0.7406	20.48	0.7645	19.20	0.6946	20.73	0.7688	<b>23.63</b>	<b>0.8254</b>
Lena	24.45	0.8041	25.86	0.8602	24.06	0.8318	25.92	0.8461	<b>29.11</b>	<b>0.9090</b>
Parrots	23.05	0.8492	24.73	0.8996	23.15	0.8783	24.83	0.8863	<b>27.18</b>	<b>0.9257</b>
Starfish	21.42	0.7690	22.95	0.8379	21.35	0.8063	22.93	0.8159	<b>26.25</b>	<b>0.8906</b>
Flower	22.42	0.7369	23.84	0.8078	22.12	0.7773	23.80	0.7814	<b>26.70</b>	<b>0.8652</b>
Girl	27.51	0.8067	28.33	0.8692	27.04	0.8347	28.94	0.8560	<b>31.66</b>	<b>0.9032</b>
Hat	24.90	0.8020	26.31	0.8418	24.29	0.8125	26.51	0.8298	<b>28.42</b>	<b>0.8753</b>
Parents	23.12	0.7947	24.57	0.8581	23.30	0.8335	24.52	0.8395	<b>28.05</b>	<b>0.9082</b>
Plants	25.58	0.8065	27.07	0.8533	25.53	0.8194	27.19	0.8425	<b>30.51</b>	<b>0.9047</b>
Racoon	24.08	0.7804	25.42	0.8477	23.46	0.8209	25.61	0.8166	<b>27.48</b>	<b>0.8692</b>

**Fig. 2.** Reconstructed HR images of *Butterfly* by different methods. Top row: Original, Nearest, Bicubic. Bottom row: NE [16], ScSR [4], Proposed.**Fig. 3.** Reconstructed HR images of *Starfish* by different methods. Top row: Original, Nearest, Bicubic. Bottom row: NE [16], ScSR [4], Proposed.

bicubic, NE [16], and ScSR [4]. All the codes are downloaded from the authors' personal websites. For fair comparisons, the test LR input images are downgraded from the ground-truth HR images by the same way.

To objectively assess the quality of the SR reconstruction, PSNR and FSIM [17] are adopted to evaluate the quality of SR reconstruction. For color images, we only calculate PSNR values for the luminance channel. The PSNR and FSIM results are listed in Table 1, meanwhile Fig. 2 and Fig. 3 show the comparison results of our proposed method and several other representative methods. As shown in Fig. 2 and Fig. 3, some ringing artifacts for the edges can be found in ScSR. It can also be observed that the proposed method achieves the highest PSNR values for all of the test images, and generally outperforms state-of-the-art SR methods.

## 5. CONCLUSION

In this paper, we propose a novel single image SR framework by incorporating discriminative sparse coding, namely CFD-DL. The proposed method guarantees that the sparse representation, derived from the different class LR image patches with their corresponding dictionaries, can well recover their HR counterparts with the class-specific HR patch dictionaries. Compared with the several state-of-the-art SR methods, our proposed method improves the reconstruction quality significantly, and at the same time removes the artifacts.

## 6. ACKNOWLEDGMENT

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## 7. REFERENCES

- [1] S. C. Park, M. K. Park, and M. G. Kang, "Super-resolution image reconstruction: a technical overview," *IEEE Signal Processing Magazine*, vol. 20, no. 3, pp. 20–36, 2003.
- [2] W. Freeman, T. Jones, and E. pasztor, "Example-based super-resolution," *IEEE Computer Graphics and Applications*, vol. 22, no. 2, pp. 56–65, 2002.
- [3] M. Protter, M. Elad, H. Takeda, and P. Milanfar, "Generalizing the nonlocal-means to super-resolution reconstruction," *IEEE Transactions on Image Processing*, vol. 18, no. 1, pp. 36–51, 2009.
- [4] J. Yang, J. Wright, T. Huang, and Y. Ma, "Image super-resolution via sparse representation," *IEEE Transactions on Image Processing*, vol. 19, no. 11, pp. 2861–2873, 2010.
- [5] W. Dong, L. Zhang, G. Shi, and X. Wu, "Image deblurring and super-resolution by adaptive sparse domain selection and adaptive regularization," *IEEE Transactions on Image Processing*, vol. 20, no. 7, pp. 1838–1857, 2011.
- [6] X. Lu, H. Yuan, P. Yan, Y. Yuan, and X. Li, "Geometry constrained sparse coding for single image super-resolution," in *Proceedings of IEEE International Conference on Computer Vision and Pattern Recognition*, 2012, pp. 1648–1655.
- [7] S. Wang, L. Zhang, Y. Liang, and Q. Pan, "Semi-coupled dictionary learning with applications to image super-resolution and photo-sketch synthesis," in *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition*, 2012, pp. 2216–2223.
- [8] D. Huang and Y. Wang, "Coupled dictionary and feature space learning with applications to cross-domain image synthesis and recognition," in *Proceedings of IEEE International Conference on Computer Vision*, 2013, pp. 2496–2503.
- [9] X. Chen and C. Qi, "Low-rank neighbor embedding for single image super-resolution," *IEEE Signal Processing Letters*, vol. 21, pp. 79–82, 2014.
- [10] J. Xu, C. Deng, X. Gao, D. Tao, and X. Li, "Image super-resolution using multi-layer support vector regression," in *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 2014, pp. 5799–5803.
- [11] M. Yang, L. Zhang, X. Feng, and D. Zhang, "Fisher discrimination dictionary learning for sparse representation," in *Proceedings of IEEE International Conference on Computer Vision*, 2011, pp. 543–550.
- [12] R. Duda, P. Hart, and D. Stork, *Pattern classification*, Wiley-Interscience, 2000.
- [13] N. Parikh and S. Boyd, "Proximal algorithms," *Foundations and Trends in Optimization*, vol. 1, no. 3, pp. 123–231, 2013.
- [14] L. Rosasco, A. Verri, M. Santoro, S. Mosci, and S. Villa, "Iterative projection methods for structured sparsity regularization," 2009.
- [15] M. Yang, L. Zhang, J. Yang, and D. Zhang, "Metaface learning for sparse representation based face recognition," in *Proceedings of IEEE International Conference on Image Processing*, 2010, pp. 1601–1604.
- [16] H. Chang, D. Yeung, and Y. Xiong, "Super-resolution through neighbor embedding," in *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition*, 2004, pp. 275–282.
- [17] L. Zhang, D. Zhang, and X. Mou, "Fsim: a feature similarity index for image quality assessment," *IEEE Transactions on Image Processing*, vol. 20, no. 8, pp. 2378–2386, 2011.