# JOINT DIRECTIONAL-POSITIONAL MULTIPLEXING FOR LIGHT FIELD ACQUISITION BY KRONECKER COMPRESSED SENSING

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**ABSTRACT**

In this paper, we propose a joint and unified framework to compressively capture a light field in the consideration of both directional and positional multiplexing based on Kronecker compressed sensing (KCS). First of all, both of the 2D angular and 2D spatial correlations of the light field can be fully utilized in the compressive acquisition, and the multiplexing is more flexible and balanced during the acquisition. Secondly, other types of light field acquisition can be unified into our proposed framework. In the experiment, it is shown that more balanced allocation between directional and positional multiplexing achieves better reconstruction quality of light field given the same number of total acquisitions. Furthermore, the experimental result also illustrates that the proposed method can capture a light field with full resolution and achieve better reconstruction quality than other previous methods.

***Index Terms—*** Joint multiplexing, Directional-Positional multiplexing, Kronecker compressed sensing (KCS), Compressive light field acquisition

## 1. INTRODUCTION

A light field represents a set of densely-sampled light rays traveling through a 3-D space. Since two decades ago, the acquisition of a light field has attracted great attentions in the area of computational photography, computer vision and image processing [1, 2, 3, 4], and the application of light field has been stretched to the areas of image refocus [5], high dynamic range (HDR) image [6], and navigation of free viewpoint image (FVI) [7]. Moreover, recently, the appearance and booming popularity of the commercial light field cameras, such as Lytro and Raytrix, have also stimulated and promoted the imperative requirement for the study of light field acquisition.

One of the greatest challenges of light field acquisition is how to handle the huge data problem. In the recent decade, based on the theory of compressed sensing [8, 9], compressive light field acquisition has obtained more popularity in academic research and practical implementation. In this paper, we propose a joint and unified framework for compressively capturing a light field. The framework has two advantages. Firstly, both of the 2D angular and 2D spatial correlations of a light field can be fully utilized in the compressive acquisition by a more flexible and balanced multiplexing. Secondly, other types of light field acquisition can be unified into the proposed framework. Experimental result shows that a balanced allocation between directional and positional multiplexing achieves better reconstruction quality given the same number of acquisitions.

## 2. LIGHT FIELD ACQUISITION TECHNIQUES

A light field $R(u, v, s, t)$ includes four dimensional information and is parameterized by two planes, aperture plane $(u, v)$ and sensor plane $(s, t)$, as shown in **Fig. 1**. To be simple, we adopt $R(\vec{u}, \vec{s})$ to represent the light field, where $\vec{u} = (u, v)$ and $\vec{s} = (s, t)$. In this paper, $\vec{u}$ and $\vec{s}$ denote the directional and positional dimension respectively.

There are several works focusing on compressive light field acquisition based on directional multiplexing, such as [10, 11, 12]. In [10], the author proposed a compressive coded aperture to reduce the number of acquisitions. Their acquisition method is illustrated in **Fig. 2** and formalized as

$$I(\vec{s}) = \int_{\vec{u}} f_d(\vec{u}) R(\vec{u}, \vec{s}) d\vec{u}, \tag{1}$$

where $f_d(\vec{u})$ represents a directional multiplexing pattern, and $I(\vec{s})$ is one acquisition on a conventional image sensor. The light field can be reconstructed from several measurements $I(\vec{s})$ with different aperture patterns $f_d(\vec{u})$. The authors in [11, 12] proposed to locate a coded pattern between the aperture and the sensor for obtaining better directional multiplexing in compressive light field acquisition. However, this method only adopted conventional image sensor in acquisition so that the positional multiplexing can not be conducted.

Meanwhile, the single-pixel camera [13] conducted positional multiplexing by using digital micro-mirror device (DMD) to selectively integrate pixel values into a single photo detector. The imaging process is illustrated in **Fig. 3** and can be formulated as

$$I = \int \int_{\vec{u}, \vec{s}} f_p(\vec{s}) R(\vec{u}, \vec{s}) d\vec{u} d\vec{s} \tag{2}$$

where $f_p(\vec{s})$ represents a pattern of positional multiplexing. Several measurements of $I$ should be repeated with chang-

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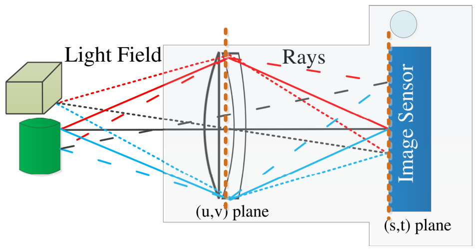



**Fig. 1**. Two-plane parameterization of 4D light field

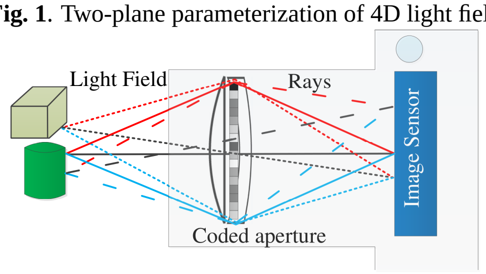



**Fig. 2**. Light field acquisition by coded aperture

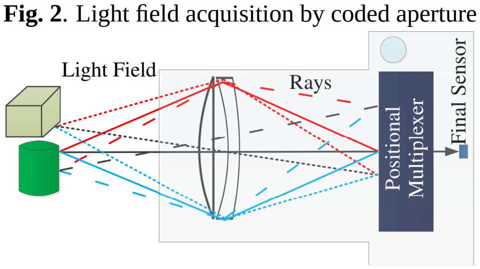



**Fig. 3**. Light field acquisition by PLM

ing the pattern $f_p(\vec{s})$ to reconstruct an image. We name this method as positional light multiplexing (PLM). However, this system only served for 2D photograph acquisition. Therefore, in order to capture a light field, all the angular information of a light field needs to be captured individually.

## 3. PROPOSED FRAMEWORK FOR COMPRESSIVE LIGHT FIELD ACQUISITION

### 3.1. Overview

Our proposal is a joint and unified framework for compressive light field acquisition based on the theory of Kronecker compressed sensing [14]. First, it is a joint framework, and it it more flexible and balanced to allocate acquisitions between directional and positional multiplexing, so that better reconstruction quality is achieved. Second, it is a unified framework, and the methods of coded aperture [10], PLM [13], multiview image acquisition [1] can be derived from the proposal. As far as we know, it is the first time to unify these acquisition models together.

In the following, we present our proposal in detail. The acquisition of a light field is split into two stages, which include directional multiplexing by coded aperture and positional multiplexing, as shown in **Fig. 4**. Slightly different

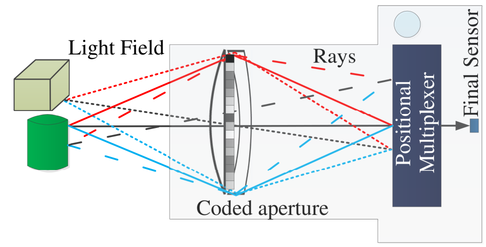



**Fig. 4**. Light field acquisition by joint directional and positional multiplexing

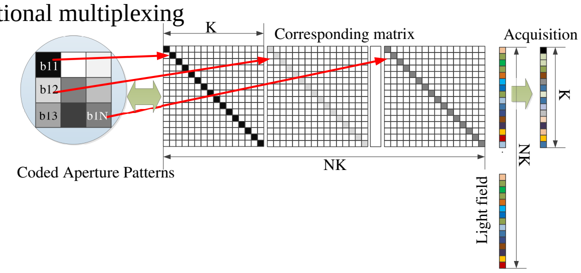



**Fig. 5**. The matrix visualization of coded aperture

from eqs. (1) and (2), a mathematical representation of our acquisition scheme is given by

$$Y = \int\int_{\vec{u},\vec{s}} f_d(\vec{u}) f_p(\vec{s}) R(\vec{u},\vec{s}) d\vec{u} d\vec{s}, \tag{3}$$

where $f_d(\vec{u})$ and $f_p(\vec{s})$ represent the directional and positional multiplexing respectively, and $Y$ is the final acquisitions in our proposed framework. Measurement of $Y$ should be repeated for several times with changing the multiplexing patters of $f_d(\vec{u})$ and $f_p(\vec{s})$ to fully reconstruct the original light field $R(\vec{u},\vec{s})$.

### 3.2. Observation model

We define a data structure of a 4D light field atom as a long concatenated vector

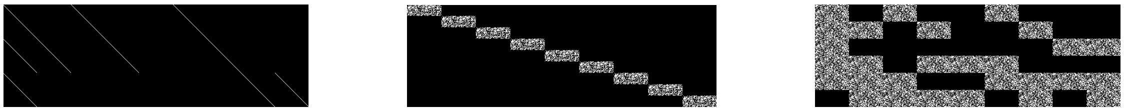
$$R_{NK\times 1} = [{X^1_{K\times 1}}^T, {X^2_{K\times 1}}^T, ..., {X^N_{K\times 1}}^T]^T$$

where each $X$ represents a vectorized patch of a discrete angular image, which is $R(\vec{u}^*, \vec{s})$ for a fixed $\vec{u}^*$, and $N$ and $K$ correspond to angular and spatial resolution of the light field atom. Therefore, directional multiplexing is represented as $Z_{K\times 1} = M_{K\times NK} R_{NK\times 1}$. The directional multiplexing matrix $M_{K\times NK}$ can be generated by Kronecker product between $B_{1\times N}$ and $I_{K\times K}$, written as $M_{K\times NK} = B_{1\times N} \otimes I_{K\times K}$. The matrix $I_{K\times K}$ is an identity and $B_{1\times N} = [b_{11}, b_{12}, ..., b_{1N}]$ corresponds to the transmittance of a coded aperture pattern. The matrix is illustrated in **Fig. 5**.

Next, we consider the positional multiplexing of the light field to project $Z_{K\times 1}$ to further lower dimension. The input is $Z_{K\times 1}$, and all the elements in $Z_{K\times 1}$ are linearly multiplexed to be one integration with different weights, and the

**Table 1**. Illustration of the unified framework for different types of compressive light field acquisition

| Type | Sensing Matrix | 1st Stage | 2nd Stage | Compressive Acquisition | Sensing Ratio |
|---|---|---|---|---|---|
| Coded Aperture | $\Psi_{QK\times NK}$ | $M_{Q\times N}$ | $I_{K\times K}$ | Directional | Q/N |
| PLM | $\Psi_{NP\times NK}$ | $I_{N\times N}$ | $D_{P\times K}$ | Positional | P/K |
| Proposal | $\Psi_{PQ\times NK}$ | $M_{Q\times N}$ | $D_{P\times K}$ | Both Directional and Positional | PQ/NK |



(a) Directional multiplexing (b) Positional multiplexing (c) Joint multiplexing (Proposal)

**Fig. 6**. The illustration and comparison of different compressive sensing matrices

measurement is repeated for $K$ times with different weight patterns. Therefore, the positional multiplexing matrix is provided by $D_{P\times K}$. It becomes more explicit for the compressive light field acquisition by using matrix multiplication, $Y_{P\times 1} = D_{P\times K}Z_{K\times 1} = D_{P\times K}M_{K\times NK}R_{NK\times 1}$, and $Y_{P\times 1}$ is the final acquisition.

We have discussed one acquisition of directional multiplexing followed by multiple measurements of positional multiplexing. Since there are multiple directional multiplexing, the final sensing matrix $\Psi_{PQ\times NK}$ can be written as

$$\Psi_{PQ\times NK} = M_{Q\times N} \otimes D_{P\times K}, \tag{4}$$

where $M_{Q\times N}$ and $D_{P\times K}$ are the directional and positional multiplexing matrix, and $Q$ and $P$ are the number of measurements in the first and second acquisition stages, respectively.

For an identity matrix $D$ with $P = K$, $\Psi_{PQ\times NK}$ becomes actually the sensing matrix for coded aperture in [10] as shown in **Fig. 2**. Meanwhile, when $M$ is an identity with $Q = N$, $\Psi_{PQ\times NK}$ becomes the sensing matrix for PLM in [13] as shown in **Fig. 3**. Moreover, if both of $D$ and $M$ are identity matrices with $P = K$ and $Q = N$, $\Psi_{PQ\times NK}$ is the sensing matrix for multiview image acquisition. Finally, when both of $D$ and $M$ are random matrices with $P < K$ and $Q < N$, $\Psi_{PQ\times NK}$ is the joint sensing matrix in our proposal. Except for multiview acquisition, the remaining three types of compressive acquisitions are compared in **Table 1**, and the example visualizations of sensing matrix are also illustrated in **Fig. 6**.

### 3.3. Generation of Compressed matrix

Compressed sensing enables one signal to be sampled by a sensing matrix $\Psi$ below the Nyquist-sampling rate. The signal can be reconstructed from the sub-sampled measurements by adopting optimization method if the signal is sparse by itself or has a sparse representation in other domain.

Due to the high correlation in the positional domain, the 2D spatial structure of each angular image can be sparsely represented in frequency domain by using 2-D DCT. In addition, a light field can be regarded as a stack of several 2D angular images, and the directional structure of the light field is often smooth or piecewise smooth due to the small disparities among pixels from different perspectives. Therefore, we adopt another 2D-DCT matrix to sparsely represent the 2D directional structure of the light field. Similar to the generation of sensing matrix, Kronecker product is adopted in the generation of compressed matrix, represented as

$$\Phi^{4D} = (\Phi^{1D}_{dir} \otimes \Phi^{1D}_{dir}) \otimes (\Phi^{1D}_{pos} \otimes \Phi^{1D}_{pos}). \tag{5}$$

### 3.4. Model for Reconstruction

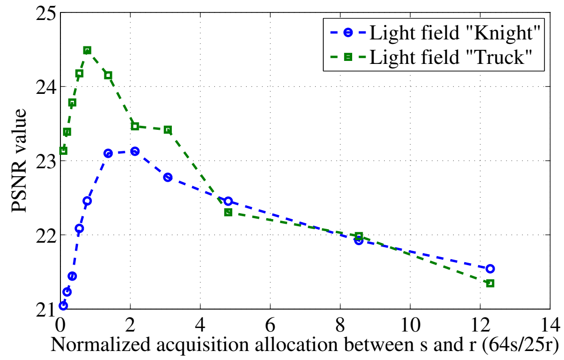
Basically, in the reconstruction part, the sparse solution $\hat{\theta}$ of the light field is estimated, and it can be represented as

$$\begin{aligned} \hat{\theta} &= \arg\min_{\theta} \parallel \theta_{NK\times 1} \parallel_{l_1}, \\ \text{s.t. } Y_{QP\times 1} &= \Psi_{QP\times NK}\Phi_{NK\times NK}\theta_{NK\times 1}. \end{aligned} \tag{6}$$

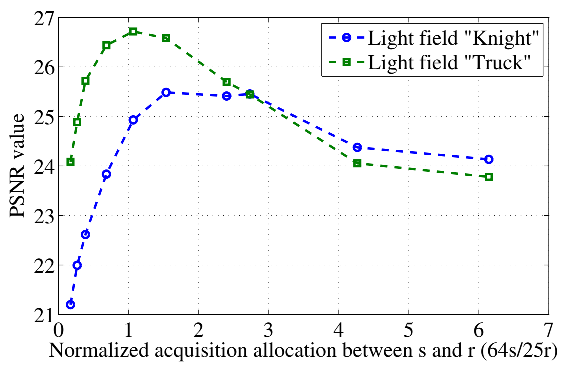
The optimization can be solved by linear programming and the reconstructed light field is finally obtained by $\hat{R}_{NK\times 1} = \Phi_{NK\times NK}\hat{\theta}_{NK\times 1}$.

## 4. EXPERIMENTAL RESULTS

The configuration of the experiment is set as follows. We adopt two light fields, "Knight" and "Truck", from Stanford archive [15] with the spatial and angular resolutions of $640 \times 480$ and $5 \times 5$. In order to reduce computation burden, the light field is partitioned into $8 \times 8 \times 5 \times 5$ non-overlapped atoms which are vectorized and processed individually. The whole acquisition and reconstruction is simulated in Matlab 2014a, where $l_1$ magic package [16] is adopted for sparse estimation. As for the compressed matrix, a 4D-DCT matrix is thoroughly adopted in the whole experiment. Furthermore, three acquisition models including coded aperture, PLM and our proposal are adopted respectively in acquisition, and binary random matrices are adopted in the generation of sensing matrix. In addition, symbols $s$ and $r$ represent the numbers of acquisitions in directional multiplexing and positional multiplexing respectively. The ranges for these parameters are $1 \leq s \leq 25$ and $1 \leq r \leq 64$, and the final sensing ratio $t$ is defined as $t = s \cdot r/1600$.




(a) $s \times r = 120$




(b) $s \times r = 240$

**Fig. 7**. The distortion analysis for different combinations of $\hat{s}$ and $\hat{r}$ as the given total number of acquisitions is fixed.

In our proposal, as the total number of acquisition is given, there are multiple combinations for the measured numbers in directional multiplexing and positional multiplexing. It is necessary to explore which combination achieves the optimal reconstruction. Therefore, the distortion analysis is conducted for different combinations at a fixed number of total acquisitions. The total numbers of acquisition are set as $s \times r = 120$ and $s \times r = 240$ for the two light fields. The distortion analysis is illustrated in **Fig. 7**, where the horizontal axis ($\hat{s}/\hat{r}$) is the ratio between the normalized numbers for directional ($\hat{s} = s/25$) and positional ($\hat{r} = r/64$) multiplexing. The vertical axis is the reconstruction quality in PSNR. The two light fields are clipped as $320 \times 240 \times 5 \times 5$. Acquisition and reconstruction are repeated for 10 times and the averaged results are presented. It is clear from the graph that more balanced allocation achieves better reconstruction quality, which proves the effectiveness of our proposal.

The comparisons of reconstruction images are provided in **Figs. 8, 9**. In order to save pages, we adopt $t = 0.08$ and $t = 0.2$ in the acquisition of "Knight " and "Truck" light field, respectively. Furthermore, the mean value and standard deviation of reconstruction error evaluated by RMSE in each light field are also presented in **Table 2**. Both of the subjective and objective evaluations show that our proposal achieves the best performance in terms of reconstruction quality.

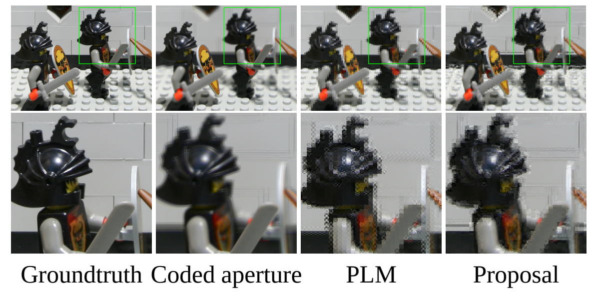



**Fig. 8**. Reconstructed light field "Knight" at $t = 0.08$ (Extreme-Low). Angular images (top) and closeups (bottom).

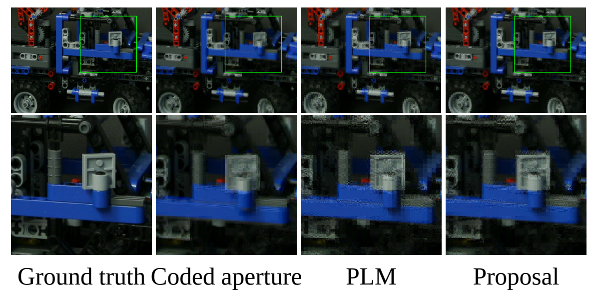



**Fig. 9**. Reconstructed light field "Truck" at $t = 0.2$ (Low). Angular images (top) and closeups (bottom).

**Table 2**. The mean value and std of reconstruction distortion at different sensing ratios by different acquisition methods

| Light field | Method | Ave | Std |
|---|---|---|---|
| "Knight" ($t = 0.08$) | Coded aperture | 22.090 | 4.299 |
| | PLM | 19.494 | **0.441** |
| | Proposal | **18.261** | 2.422 |
| "Truck" ($t = 0.2$) | Coded aperture | 11.629 | 3.724 |
| | PLM | 10.866 | 0.375 |
| | Proposal | **9.509** | **0.364** |

## 5. CONCLUSION AND FUTURE WORK

In this paper, we proposed a novel and unified framework to compressively acquire a light field. The proposed framework exploited both angular and spatial correlations of a light field, and also unified the previous frameworks of light field acquisition. Experimental results showed that the proposed balanced multiplexing exhibited better reconstruction quality than unbalanced multiplexing as the number of total measurements are the same, and the joint multiplexing method achieved better result than the other light field acquisition methods. In the future, the reconstruction quality of light field in our framework is expected to be further improved by adopting other sophisticated basis or dictionary as the compressed matrix.

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