Combining Sparsity with Rank-Deficiency for Energy Efficient EEG Sensing and Transmission over Wireless Body Area Network

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ABSTRACT

In Wireless Body Area Networks (WBAN) the energy consumption is dominated by sensing and communication. Previous techniques exploited the sparsity of the signal (in transform domains) to reduce communication costs for EEG transmission. For the first time, in this work, we propose to jointly exploit sparsity and rank-deficiency of the multi-channel signal ensemble in order to reduce both sensing and communication power consumptions. We test our method with state-of-the-art recovery techniques and find that the reconstruction accuracy from our method is considerably better and that too at lower energy consumption.

***Index Terms*—** EEG, WBAN, Compressed Sensing, Matrix Completion.

1. INTRODUCTION

In this work, we are particularly interested in telemonitoring of EEG signals using Wireless Body Area Network (WBAN). In WBAN, the signal is encoded at the nodes where computational power is limited while the decoding is at the base station where computational power is not at a premium. Compressed Sensing (CS) based techniques is a good solution to this problem because it requires a low-complexity encoder and sophisticated decoder. Traditional transform coding based methods are not suitable for such a scenario.

Previous CS based techniques [1-4] have been able to reduce the energy consumption required for processing and communication. It samples the full signal (thus does not reduce the sensing cost); projects the sampled signal onto a lower dimension thereby compressing it; and finally transmits the compressed signal - thereby reducing the communication cost. Since the projection is linear, the processing cost is low. CS requires a smart decoder since it has to solve an optimization problem to recover the EEG signal from the lower dimensional projections.

There is only a single work which was able to reduce the sampling power by randomly sub-sampling the EEG signals and recovering the full multi-channel signal ensemble by exploiting its rank deficiency [5, 6]. The basic assumption was that, since the EEG channels are correlated with each other, the multi-channel signal ensemble will have a low-rank. It was shown in [5], that the energy savings from the under-sampling scheme can save upto 50% of the total energy in the WBAN. Owing to limitations in space, we cannot show the power analysis in this work. The interested reader can peruse [5].

In this work, the problem remains the same, i.e. we propose to randomly sub-sample the EEG signals from multiple channels. But instead of using only rank-deficiency [5], we will also exploit the sparsity of the signal ensemble in a transform domain for its recovery.

The rest of the paper is organized into several sections. The contributions from prior works will be discussed in section 2. The proposed formulation is described in section 3. We discuss the experimental results in section 4. Finally the conclusions of this work are discussed in section 5.

2. LITERATURE REVIEW

One of the earliest works that applied CS for EEG signal compression and transmission is [1]. It projected the EEG signal onto an i.i.d Gaussian basis for compression and used CS to recover the EEG signal by exploiting the signal's sparsity in the Gabor domain. The work [1] employed a synthesis prior formulation for sparse signal recovery using Gabor as a sparsifying basis. This paper uses a Gaussian matrix for compression; for practical reasons, a Gaussian compression basis is not very suitable; since the matrix is dense and therefore is neither memory efficient nor easy to be operated with. In [2], different sparsifying transforms were compared – wavelets, Gabor, splines; it was reported that Gabor yielded the best reconstruction results.

The possibility of exploiting inter-channel correlation in order to improve EEG signal reconstruction was mentioned in [1], but there was no concrete idea regarding how to model it. This problem is partially addressed in [3]. They do not explicitly model the inter-channel correlation, but frame a joint reconstruction problem where the signals from all the channels are reconstructed simultaneously. This work uses wavelets as the sparsifying basis and a binary matrix of randomly positioned 1's and 0's as the compression basis. Such a binary compression matrix is easy to store and operated with.

A recent study assumes a block structure of the EEG signals [4] in a transform domain (DCT or wavelet). There is no theoretical or physical intuition behind this assumption; however it was shown in [4] that a Block Sparse Bayesian Learning (BSBL) algorithm yields good recovery results.

It must be remembered that although these techniques are loosely termed as 'Compressed Sensing' – in reality they are not. The philosophy of CS is to sample less than the Nyquist rate. In EEG, this has not been done – the signal is fully sampled; it is only compressed later on. Such a sample-and-compress paradigm does not adhere to the CS philosophy.

3. PROPOSED APPROACH

The studies discussed so far aimed at reducing only the communication costs; they sampled the full signal (therefore does not reduce sensing cost) and compress it by projecting onto a lower dimension by Gaussian / Binary matrices. The schematic diagram of the process is shown in Fig. 1.

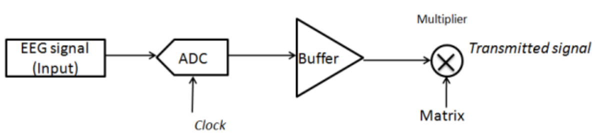


Fig. 1. Information Processing Pipeline for previous methods

In [5, 6] random sub-sampling of the EEG signals is proposed; this reduces sensing energy. The schematic is shown in Fig. 2.

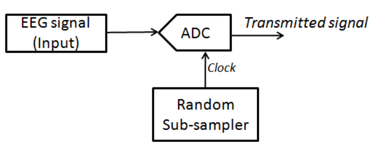


Fig. 2. Information Processing Pipeline for random under-sampling

In this scenario, the acquired signal is inherently compressed. Thus, we do not need to expend any energy in processing (for compression). The other benefit of this scheme is that we do not need a storage buffer and a multiplier. This helps in reducing the hardware footprint and promotes miniaturization.

3.1. Sparsity

We are measuring the signal in time domain, i.e. in Dirac basis. EEG signals are not sparse but have a sparse representation in a transform domain like wavelet [3] or DCT [3, 4]. Unfortunately if the signal is measured in the Dirac basis, sparsifying transforms like DCT and wavelet are not a good choice; this is because CS requires the sparsity basis to be maximally incoherent with the measurement basis [7]. The DCT and wavelet transforms are not too incoherent with the Dirac measurement basis; hence one cannot expect good recovery results from such sparsifying transforms.

Furthermore, previous CS based recovery techniques [1-4] operated on the EEG signals on a piecemeal fashion. They sparsified each of the channels separately and did not account for inter-channel coherence. In this work we propose to exploit temporal correlations and inter-channel correlations.

We can express the multi-channel data acquisition problem as follows:

$$y_i = R_i x_i + \eta, \forall i \quad (1)$$

It can be arranged as:

$$vec(Y) = Rvec(X) + \eta \quad (2)$$

where $Y = [y_1 | ... | y_C]$, $X = [x_1 | ... | x_C]$ and R is the block diagonal matrix with R_i's along diagonals. The 'vec' has the usual connotation.

The task is to recover *X*. The columns of *X* are the EEG signals for each channel and the rows are the EEG samples of each instant across all the channels. In this work, we propose to jointly exploit the sparsity of *X* along the columns and rows by applying a 2D Fourier transform. The Fourier basis is maximally incoherent with the Dirac basis and hence is a suitable choice for sparsiying the signal. However, a 1D Fourier transform along the columns of *X* does not yield a very sparse representation as it does not account for inter-chanel correlations. But the 2D Fourier transform, accounts for correlation across the channels and yields a very sparse representation. This is evident from Fig. 3.

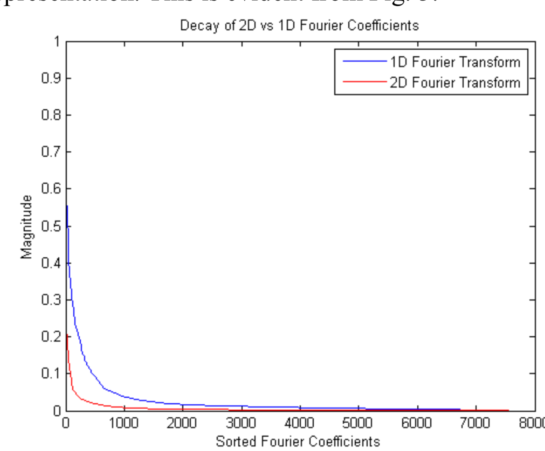


Fig. 3. Decay of 1D vs 2D Fourier Coefficients for EEG signal ensemble

Mathematically the operation is expressed as $\alpha = F_C X F_R = F_{2D} vec(X)$ will be sparse; F_C and F_R are 1D Fourier transforms operating on the columns and the rows respectively and F_{2D} represents the 2D Fourier transform.

One can recover the signal by exploiting the transform domain sparsity via l_1-norm minimization.

$$\min_X \|vec(Y) - Rvec(X)\|_2^2 + \lambda \|F_{2D} vec(X)\|_1 \quad (3)$$

3.2. Combining Sparsity with Rank Deficiency

In [5, 6] it was argued that since the EEG signals from multiple channels are correlated with each other the columns of X are not independent. Therefore it is possible to recover X from the partially sampled entries Y by exploiting its rank-deficiency. Basically, it turns out to be a low-rank matrix completion problem which can be recovered by:

$$\min_X \|vec(Y) - Rvec(X)\|_2^2 + \lambda \|X\|_* \quad (4)$$

In this work, we propose to combine sparse recovery (3) and low-rank recovery (4). In the past, studies in dynamic MRI reconstruction [8-11] showed that better recovery results can be obtained when sparsity based techniques are combined with low-rank recovery techniques. Our work is motivated by these studies. We expect similar improvements over [5, 6]. The optimization problem that needs to be solved is:

$$\min_X \|vec(Y) - Rvec(X)\|_2^2 + \lambda_1 \|F_{2D} vec(X)\|_1 + \lambda_2 \|X\|_* \quad (5)$$

The algorithm for solving (5) is derived based on the Split Bregman approach.

3.3. Algorithm Derivation

The task is to solve an optimization problem of the following form. We change the notations for the sake of convenience.

$$\min_X \|y - Ax\|_2^2 + \lambda_1 \|Dx\|_1 + \lambda_2 \|X\|_* \quad (6)$$

We solve (6) by Bregman type variable splitting with Alternating Directions Method of Multipliers (ADMM) [12]. We introduce two proxy variables - $p=vec(P)$ and $q=vec(Q)$ for the two penalty functions respectively. We add terms relaxing the equality constraints of each quantity and its proxy, and in order to enforce equality at convergence, we introduce Bregman relaxation variables B_1 and B_2. The new objective function is:

$$\min_{X,P,Q} \|y - Ax\|_2^2 + \lambda_1 \|Dp\|_1 + \lambda_2 \|Q\|_* + \gamma_1 \|P - X - B_1\|_F^2 + \gamma_2 \|Q - X - B_2\|_F^2 \quad (7)$$

This allows the problem (7) to be split into an alternating minimization of the following (easier) subproblems:

$$\min_X \|y - Ax\|_2^2 + \gamma_1 \|P - X - B_1\|_F^2 + \gamma_2 \|Q - X - B_2\|_F^2 \quad (8)$$

$$\min_P \lambda_1 \|Dp\|_1 + \gamma_1 \|P - X - B_1\|_F^2 \quad (9)$$

$$\min_Q \lambda_2 \|Q\|_* + \gamma_2 \|Q - X - B_2\|_F^2 \quad (10)$$

The subproblem (8) is easy to solve; it is just a least squares minimization problem that can be solved efficiently using any conjugate gradient algorithm. The subproblem (9) is an analysis prior denoising problem. The technique to solve this is borrowed from [13]:

$$z = \left(\frac{\gamma_1}{\lambda_1}\Lambda^{-1} + cI\right)^{-1}\left(cz + D(vec(X + B_1) - D^T z)\right)$$

$$p \leftarrow vec(X + B_1) - D^T z$$

where $\Lambda = diag(|D\, vec(X + B_1)|^{-1})$ and c is the maximum eigenvalue of $D^T D$.

The subproblem (10) is a nuclear norm minimization. The algorithm to solve this was derived in [14]. The method is called singular value shrinkage.

$$USV^T = X + B_2$$

$$\Sigma \leftarrow Soft(S, \lambda_2 / \gamma_2)$$

$$Q = U\Sigma V^T$$

Soft-thresholding is applied on the singular values of the matrix $X+B_2$; Q is updated by recomposing the matrix using the singular vectors and the thresholded singular values.

This concludes the derivation for solving (6).

4. EXPERIMENTAL EVALUATION

We compare our method with two CS based recovery techniques - sparse recovery [3] and BSBL recovery [4]. We also compare our method with the low-rank recovery technique proposed in [5, 6].

The experiments are carried out on the EEGLab Dataset [16]. We tested the recovery results for two different sub-sampling / compression ratios - 50% (2:1) and 25% (4:1). The term 'sub-sampling ratio' pertains to our proposed method and the low-rank technique. Prior CS studies do not sub-sample in the time. These methods [3, 4] sample the full signal and then compress it, hence the term 'compression ratio'.

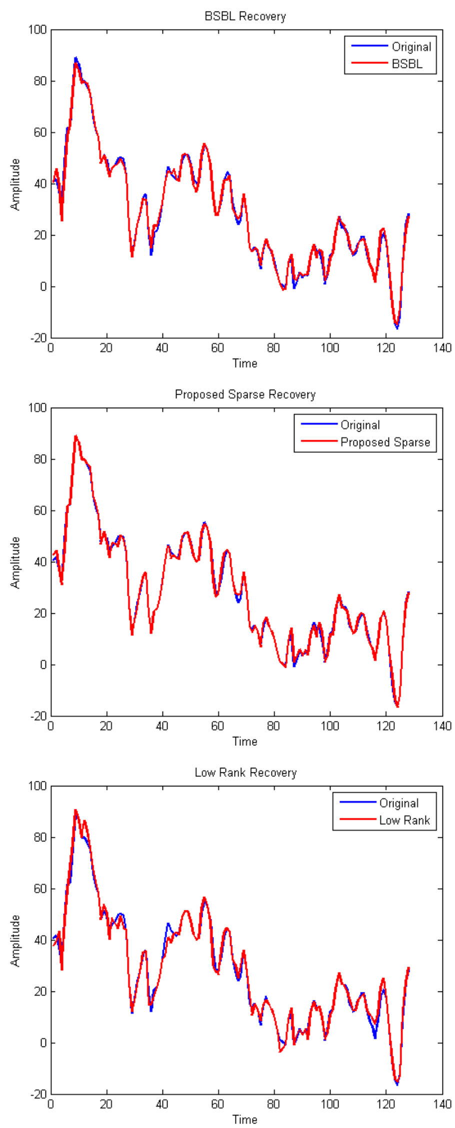
Our algorithm required specification of λ_1 and λ_2. To find λ_1, we put λ_2=0, and find λ_1 by the L-curve method [15]. For this value of λ_1, we find the value of λ_2 by the L-curve method. Using this technique we found that λ_1=1 and $\lambda_2=10^{-2}$ yields the best results.

The metric used for evaluation is the Normalized Mean Squared Error defined as $NMSE = \frac{\|original - reconstructed\|_2}{\|original\|_2}$. NMSE and its close counterpart SNR are standard metrics used for quantitative reconstruction performance [3-6]. The reconstruction results are shown in Table 1. For each signal ensemble and for each configuration, the random sampling matrix (for our proposed method and low-rank method [5]) and the compression matrix (for [3, 4]) have been simulated 100 times. The mean and standard deviations (of NMSE's) for all the EEG signals in the dataset are reported.

Table 1. Comparative Reconstruction Results (NMSE) on EEGLab

Method	Compression Ratio	
	2:1 (mean, std)	4:1 (mean, std)
BSBL [4]	0.080, ±0.046	0.178, ±0.148
Sparse Reconstruction [3]	0.160, ±0.084	0.328, ±0.186
Proposed Sparse	0.072, ±0.032	0.166, ±0.102
Low-rank [5]	0.084, ±0.044	0.186, ±0.088
Proposed Combined	0.060, ±0.028	0.154, ±0.056

Our proposed method yields better reconstruction results than all previously known techniques [3-6] for both 2:1 and 4:1 under-sampling. For qualitative results we show sections of original and reconstructed EEG signal in Fig. 4. These signals correspond to a sampling / compression ratio of 2:1.



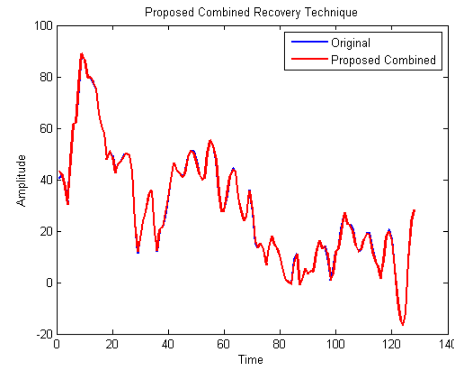


Fig. 7. Overlayed Original and Reconstructed Signals from Different Techniques - (Top to Bottom) BSBL, Proposed Sparse Low Rank and Proposed Combined reconstructed signals.

From Fig. 4. it can be seen that BSBL and Low-rank recoveryy yields significant artifacts. The reconstruction artifacts are less pronounced in our Sparse Recovery technique. But the artifacts virtualy vanish in our proposed combined (low-rank and sparse) method. The reconstructed and the original signals are almost indistinguishable.

Our proposed method yields reconstruction results which are better than state-of-the-art techniques. But the main advantage of our method is that, we achieve this reconstruction accuracy from partially sampled EEG signals thereby saving on sensing and data processing energy consumption.

5. CONCLUSION

In this work the task is to reconstruct the signal from its sub-sampled measurements. This helps in reducing – a) the power consumption at the sensor nodes, and b) reducing complexity of the digital front-end.

Previously matrix completion techniques were applied to solve the recovery problem [5, 6]. However, in this work we proposed an alternative approach. First, we showed that the multi-channel signal ensemble has a sparse representation in the 2D Fourier domain; therefore it is possible to apply compressed sensing based techniques for reconstruction. But we do not stop here, we show that even better recovery is achieved when sparsity and low-rank recovery is combined.

The reconstruction results from our proposed method is compared with state-of-the-art techniques in CS based and low-rank recovery based EEG signal reconstruction. Reconstruction results from our method is considerably better than the ones we compared against.

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