STABILITY ANALYSIS OF THE FBANC SYSTEM HAVING DELAY ERROR IN THE ESTIMATED SECONDARY PATH MODEL

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ABSTRACT

The Feedback active noise control (FBANC) scheme is widely used in portable ANC applications. But the FBANC has un-stability problem caused by the modeling error of the electro-acoustic path in its feedback mechanism. To analyze the stability problem, we propose a new stability analysis method utilizing the magnitude component of the open loop frequency response of the FBANC. With the proposed method, a stability bound equation is obtained in terms of the length of delay error of secondary path, the ANC filter length and the center frequency of primary noise. The stability bounds of the proposed method are verified by comparing with both the original Nyquist condition and the simulation results.

Index Terms— Feedback ANC, Stability, Secondary Path, Closed-form equation, Nyquist stability criterion

1. INTRODUCTION

Active noise control (ANC) is intended to suppress an external noise in an active way that an anti-noise with opposite phase and same magnitude cancels the external noise [1-2]. To generate such anti-noise, in the feed-forward ANC (FFANC) scheme, which is the typical ANC scheme, a reference signal is captured at the noise source and used for input of the ANC adaptive algorithm. In contrast to the FFANC, the feedback active noise control (FBANC) internally generates a reference signal by utilizing a feedback mechanism. Because of the convenience that a microphone or sensor for the reference signal doesn't need to be installed at the noise source, the FBANC scheme is widely applied in portable ANC applications, e.g., ANC headphones [3,4].

One of the difficulties in designing the FBANC applications is un-stability problem caused by its feedback mechanism. In the feedback path of FBANC, there is an estimated model block of secondary path which is modeled in a form of an FIR filter. The secondary path is an electro-acoustic path connecting a loud speaker, a microphone, acoustic propagation path and AD/DA converters. Theoretically, if the secondary path model is perfectly estimated, the FBANC scheme has no stability problem [1-2]. Since it is very difficult to accurately estimate because of its nonlinear and time-varying properties, the FBANC always has the potential un-stability problem [3-4].

In this paper, we analyze the un-stability problem of the FBANC system caused by the estimation error of delay component in the secondary path model. A closed-form equation of the stability bound of the delay error was analytically derived in terms of the ANC filter length and the noise's frequency. One of the most common methods to determine the stability of feedback systems is analyzing pole/zero of a system's characteristic equation. But this method is generally difficult to apply to the feedback ANC system because the feedback ANC system's characteristic equation is generally very high order. Specially, when the secondary path model has some delay error, the order equation becomes even higher and very complicated to solve. In [5], to overcome the complicated nature of the FBANC, the system's convergence condition is analyzed using transfer function domain analysis technique. But the effect of the delay error of the estimated secondary path model is excluded in the analysis.

In order to obtain the stability condition equation without solving the high-order pole/zero equation, a new stability analysis method was proposed. The basic concept of the proposed method is based on the Nyquist stability criterion. In the Nyquist stability criterion, a feedback system's stability is guaranteed when the polar plot of the open loop frequency response does not enclose the Nyquist point (-1,0) for any frequency [6]. Basically, the Nyquist technique is considered as a trial-and-error based approach. Where, many drawings of polar-plots are needed to find the stability bounds.

Instead of this original Nyquist definition, we introduce a tighter stability condition such that the open loop frequency response's polar plot must lie in the unit-circle, equivalently, the magnitude response of the polar plot must be less than 1.0 for all frequency. From the tighter stability condition, the stability bounds of the system parameters are derived in a closed-form equation. One of the significant advantages compared to the Nyquist technique is that the closed-form equation can provide the fast computation of parameter bounds. Real-time computation of stability bounds will be very useful for the design and operation of the FBANC system. The stability condition of the proposed approach provides narrower parameter bounds compared to those of the original Nyquist stability condition. Considering the practical operating ranges of the FBANC system, such as the range of noise frequency, the dimension of secondary path and the level of noise, etc., the lost parameter range is small enough to neglect in many applications.

2. THE STABILITY PROBLEM OF THE FEEDBACK ANC SYSTEM

In this section, we investigate the stability problem caused by the secondary path modeling error of the FBANC system. A typical structure of the adaptive feedback ANC system is shown in Fig. 1. The FBANC system suppresses the primary noise $d(n)$ by generating the anti-noise $y(n)$. The signal $e(n)$ denotes the residual. The reference signal $x(n)$ is generated through the feedback path as expressed in (1).

$$x(n) = \hat{y}'(n) + e(n) = y(n) * s(n) + e(n) = \hat{d}(n) . \qquad (1)$$

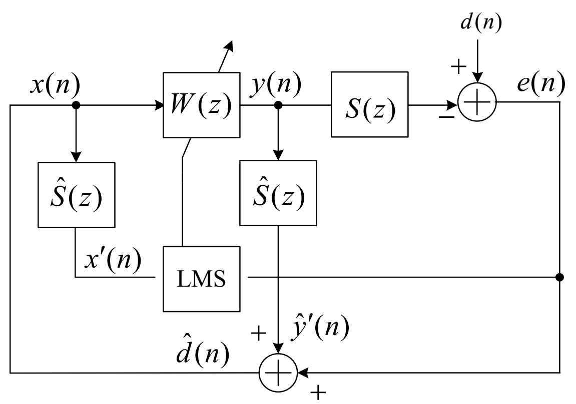

Fig. 1. Block diagram of a typical adaptive feedback ANC system.

The followings are the rest of definitions.

$W(z)$: the N-tap adaptive filter,

$S(z)$: The secondary path, the electro-acoustic coupling path from the loudspeaker to the error microphone,

$\hat{S}(z)$: The internal estimated model of $S(z)$,

$\hat{d}(n)$: the estimated primary noise.

Stability of a linear feedback system can be determined by examining its overall transfer function's pole/zero locations. The FBANC system's closed-loop transfer function from $d(n)$ to $e(n)$ can be obtained as,

$$H(z) = \frac{E(z)}{D(z)} = \frac{1 - W(z)\hat{S}(z)}{1 + W(z)[S(z) - \hat{S}(z)]} \qquad (2)$$

In order to examine the effect of the delay error explicitly, the transfer functions of the secondary path and its estimated model are modeled as simple delay components as $S(z) = z^{-\Delta}$ and $\hat{S}(z) = z^{-\hat{\Delta}}$, respectively. By plugging the simple delay models to (2), the transfer function changes as

$$H(z) = \frac{1 - W(z) z^{-\hat{\Delta}}}{1 + W(z)[z^{-\Delta} - z^{-\hat{\Delta}}]} \qquad (3)$$

When there is no delay error, i.e., $\hat{S}(z) = S(z)$, the denominator of (3) becomes one and the overall transfer function $H(z)$ becomes

$$H(z) = 1 - W(z)\hat{S}(z) = 1 - W(z)S(z) .$$

Here, the FBANC system has no poles and becomes an all-zero system which guarantees its stability.

However, when the estimated secondary path $\hat{S}(z)$ has a delay error, the denominator of (3) becomes a high order equation. This is because the adaptive filter $W(z)$ is generally tens to hundreds order, and the secondary path $S(z)$ and its estimation $\hat{S}(z)$ also contain a very long delay because of the acoustical propagation path.

In order to make sure that such system is stable, all the poles should exist in the unit-circle of the z-plane. However, it is difficult to obtain the roots of the denominator of (3) because the equation is a very high order. Moreover, the adaptive filter $W(z)$ is time-varying and determined depending on characteristics of the primary noise $d(n)$.

3. NYQUIST PLOT-BASED STABILITY ANALYSIS OF THE FEEDBACK ANC SYSTEM

A new stability analysis method is proposed to obtain the stability condition of the feedback ANC system as a form of a closed-form equation. In the proposed method, the complicated stability problem of obtaining roots of the high-order pole-zero equation is simplified as a simple inequation problem consisting of several sine functions. The closed-form stability condition is derived in terms of length of delay error of secondary path, ANC filter length and center frequency of primary noise.

The basic concept of the proposed method is based on the Nyquist stability criterion [6]. The Nyquist stability definition is that the polar plot of the open loop frequency response must not enclose the Nyquist point (-1,0) for all frequency. Instead of this original Nyquist definition, we introduce a tighter stability definition that *the frequency response* $W(e^{j\omega})[e^{-j\omega\Delta} - e^{-j\omega\hat{\Delta}}]$ *must lie in the unit-circle.* This proposed condition can be expressed as the magnitude of the frequency response for $0 \le \omega < 2\pi$ is less than 1.0 as shown in the following,

$$|W(e^{j\omega})| \cdot |e^{-j\omega\Delta} - e^{-j\omega\hat{\Delta}}| < 1 \quad \text{for} \quad 0 \le \omega < 2\pi \qquad (4)$$

In (4), by investigating the magnitude of $\left|W(e^{j\omega})\right|$ and $|e^{-j\omega\Delta} - e^{-j\omega\hat{\Delta}}|$, an equation of the stability bounds can be obtained. The first term $\left|W(e^{j\omega})\right|$ is a magnitude response of the adaptive filter. $|e^{-j\omega\Delta} - e^{-j\omega\hat{\Delta}}|$ is a length of base of an isosceles triangle having unit length legs and a vertex angle ω as shown in Fig. 2, and can be expressed as a sine function as shown in (4).

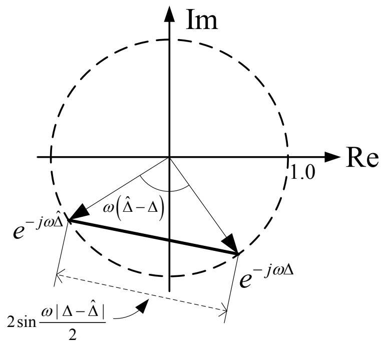

Fig.2. The magnitude of $e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}$ illustrated in the complex plane.

$$|e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}| = 2\sin\frac{\omega\left|\Delta-\hat{\Delta}\right|}{2} \tag{5}$$

The magnitude response of the adaptive filter $\left|W(e^{j\omega})\right|$ can be obtained as a simple formation when the primary noise $d(n)$ is assumed to be a single-tone sinusoid $d(n)=A\cos\left(\omega_0 n+\phi\right)$. ω_0 is the center frequency of the noise and ϕ is the phase. The coefficients of the ANC filter $W(z)$ are assumed to be at the optimum state. The optimum filter response for the single tone noise $d(n)$ can be obtained from (3) as

$$W_o\left(e^{j\omega}\right)\Big|_{\omega=\omega_0} = e^{j\omega_0\hat{\Delta}} \tag{6}$$

which makes the overall transfer function $H\left(e^{j\omega}\right)\Big|_{\omega=\omega_0}=0$.

The N-tap FIR ANC filter for the optimum response in (6) can be designed as

$$w_{o,i} = \begin{cases} \frac{2}{N}\cos[\omega_0(i+\hat{\Delta})] & for\ i=0,1,\cdots,N-1 \\ 0 & elsewhere \end{cases}$$
$$= \cos[\omega_0(i+\hat{\Delta})]\cdot w_{R.i} \tag{7}$$

where $w_{R.i}$ is an N-tap rectangular window.

The frequency response of the ANC filter $W_o\left(e^{j\omega}\right)$ can be expressed as the convolution of the discrete-time Fourier transform of the cosine function and that of the rectangular window as (8).

$$W_o\left(e^{j\omega}\right) = \mathbb{F}\left[\cos[\omega_0(n+\hat{\Delta})]\right]*\mathbb{F}\left[w_{R.i}\right]$$
$$= \frac{e^{j\omega_0\hat{\Delta}}}{N}\cdot\frac{\sin\left[\frac{N}{2}\left(\omega-\omega_0\right)\right]}{\sin\left[\frac{1}{2}\left(\omega-\omega_0\right)\right]} + \frac{e^{-j\omega_0\Delta}}{N}\cdot\frac{\sin\left[\frac{N}{2}\left(\omega+\omega_0\right)\right]}{\sin\left[\frac{1}{2}\left(\omega+\omega_0\right)\right]} \tag{8}$$

where $\mathbb{F}[\cdot]$ is the Fourier transform operation and $*$ is the convolution operator.

From (8), the magnitude response $\left|W_o\left(e^{j\omega}\right)\right|$ can be viewed as the sinc functions shifted onto $\omega=\omega_0$ and $\omega=-\omega_0$, respectively. When we assume that the filter length is long enough to accommodate a single period of the primary noise, i.e., $N \gg 2\pi/\omega_0$, the magnitude response of the optimum ANC filter $\left|W_o\left(e^{j\omega}\right)\right|$ can be expressed as

$$\left|W_o\left(e^{j\omega}\right)\right| \simeq \frac{1}{N}\cdot\frac{\sin\left[\frac{N}{2}\left(\omega-\omega_0\right)\right]}{\sin\left[\frac{1}{2}\left(\omega-\omega_0\right)\right]} \tag{9}$$

in the frequency range $0\le\omega<\pi$. The magnitude response of the optimum ANC filter $\left|W_o\left(e^{j\omega}\right)\right|$ is illustrated in Fig.3.

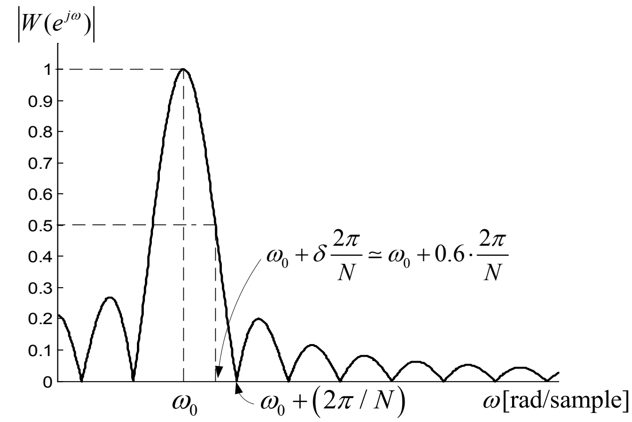

Fig.3. The magnitude response of the optimum ANC filter, $\left|W_o\left(e^{j\omega}\right)\right|$ for single-tone sinusoidal primary noise.

To obtain the stability bound of delay error of secondary path model of FBANC $\left|\Delta-\hat{\Delta}\right|$, put (5) and (9) into (4).

$$|W(e^{j\omega})|\cdot|e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}|$$
$$\cong\left[\frac{1}{N}\cdot\frac{\sin\left[\frac{N}{2}\left(\omega-\omega_0\right)\right]}{\sin\left[\frac{1}{2}\left(\omega-\omega_0\right)\right]}\right]\cdot 2\sin\frac{\omega\left|\Delta-\hat{\Delta}\right|}{2}<1 \tag{10}$$

In (10), the range of the second term $2\sin\left(\omega\left|\Delta-\hat{\Delta}\right|/2\right)$ is divided into two parts as following,

$$0\le|e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}|\le 1 \qquad \text{for } 0\le\omega\le\frac{\pi}{3|\Delta-\hat{\Delta}|}$$

$$1<|e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}|\le 2 \qquad \text{for } \omega<\frac{\pi}{3|\Delta-\hat{\Delta}|}$$

To make the product $|W(e^{j\omega})|\cdot|e^{-j\omega\Delta}-e^{-j\omega\hat{\Delta}}|$ less than 1.0, the magnitude of $|W(e^{j\omega})|$ should be less than 0.5 for the range of $\omega < \pi/\left(3|\Delta-\hat{\Delta}|\right)$. As shown in Fig. 3, $|W(e^{j\omega})|$ is less than 0.5 in the range of $\omega_0-\delta\frac{f_s}{N}\le\omega\le\omega_0+\delta\frac{f_s}{N}$. Therefore, the stability condition (4) is always satisfied in the range of

$$\left(\omega_0+\delta\frac{2\pi}{N}\right)<\frac{\pi}{3\left|\Delta-\hat{\Delta}\right|}. \qquad (11)$$

From (11), the stability bound of $|\Delta-\hat{\Delta}|$ can be obtained as

$$\left|\Delta-\hat{\Delta}\right|<\frac{\pi}{3}/\left(\omega_0+0.6\frac{2\pi}{N}\right). \qquad (12)$$

Here, the parameter δ is approximately 0.6.

This simple inequation of the stability bound provides the fast computation of parameter bounds among the delay error bound, the noise center frequency and the ANC filter length. In the next section, the proposed method's validity was proved by comparing the results obtained with (12) with the simulation results and the original Nyquist stability conditions.

4. VERIFICATION WITH SIMULATIONS

In order to verify the stability analysis performed in the previous section, the FBANC system is simulated with delay error in the estimated secondary path model. For the simulation, the sampling frequency is set to 8 kHz and the delay length of the secondary path is set to 10 msec, equivalently, 80 samples. The ANC filter length N is set to 400 samples. The tested primary noises are single-tone sinusoidal noises whose center frequencies are 20-400 Hz.

In Fig.4, the stability delay error bounds obtained with the proposed methods in (4) and (12) are compared with the simulation results and the original Nyquist stability bound. The Nyquist stability bound (Blue) is obtained by drawing the frequency response of $W(z)[z^{-\Delta}-z^{-\hat{\Delta}}]$ repeatedly on the complex-plane until it encloses the Nyquist point (-1, 0).

The proposed stability condition of (12) agrees well with both the original Nyquist condition and the simulation results showing only little differences less than 0.375 msec, equivalently, 3 samples for the primary noises having center frequencies over 100 Hz.

In Fig.4, it is also shown that the delay error bound for stability increases as the center frequency of the primary noise decreases. For example, the stability bounds of delay error are about 15 samples and 7 samples, equivalently, 1.9 msec and 0.9 msec, respectively, when the FBANC system is operated for 100Hz and 200Hz single tone noises.

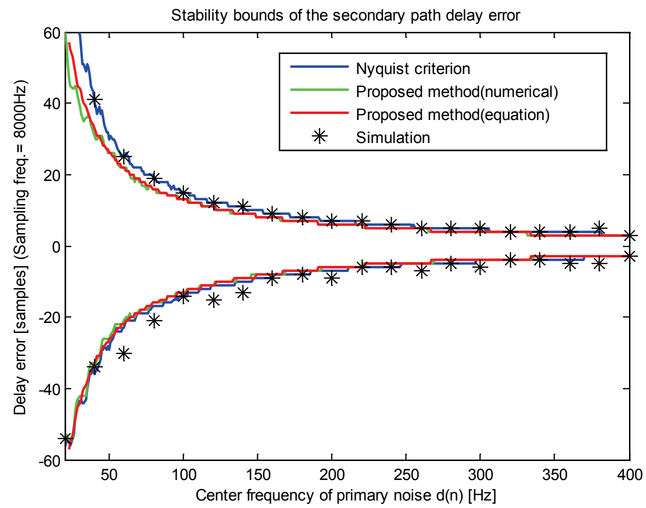

Fig.4. Comparison of stability bounds obtained with different methods. Stability bound obtained by Nyquist criterion(Blue), Numerically obtained bound of condition (4) (green), Bound obtained by equation (12) (red), and Simulation results (stars).

5. CONCLUSIONS

In this paper, the closed-form equation of stability conditions of the feedback ANC system is obtained in terms of the length of delay error of secondary path, ANC filter length and center frequency of primary noise. To obtain the equation, a tighter stability condition that the open loop frequency response's polar plot must lie in the unit-circle is introduced. The stability bounds of the proposed method agrees well with both the original Nyquist condition and the simulation results showing less than 0.375 msec difference of delay error for the primary noises having center frequencies over 100 Hz. The fast computation of parameter bounds provided by the proposed approach will be very useful for the design and real-time operation of the FBANC system.

6. REFERENCES

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