OPTIMIZATION FOR RANDOMLY DESCRIBED ARRAYS BASED ON GEOMETRY DESCRIPTORS

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ABSTRACT

The large degrees of freedom of irregular microphone placements render them difficult to analyze and optimize. This paper examines beamformer performance based on statistical descriptions of irregular geometries. Four geometry descriptors are developed to capture the properties of microphone distributions showing critical impact on array performance. Based on the relations of descriptors on performance matrices, a heuristic searching and a direct cluster design method are applied to obtain optimal arrays for speech applications. Results show significant SNR enhancements for optimized arrays over comparable regular arrays. Objective functions of heuristic searching result in rapid convergence to Monte Carlo optimizations, suggesting a strong correlation between performance and proposed descriptors. By clustering mics in the hyperbola areas to generate rich entropy, arrays with superior SNR performance are directly build according to the prior knowledge of acoustic scene. The feasibility of these optimization methods has also been demonstrated in real design cases.

Index Terms—Geometry descriptor, irregular array, optimization, cluster design

1. INTRODUCTION

Microphone arrays use spatial diversity to capture acoustic signals and suppress interference and noise based on source locations. It is widely used in applications such as speech enhancement, talker tracking, and acoustic surveillance system [1][2]. Regular arrays, whose elements are uniformly spaced are well studied in previous research. Their performance improvements were typically bounded by array geometry limitations , such as spatial aliasing, focal area resolution, and consistent performance over spectral rage of expected sources [1-5].

Irregular arrays have the the potential to outperform regular ones especially for broadband speech signals in immersive environments; however, the large degrees of freedom of microphone placements render them difficult to analyze and optimize [1-5]. It is not clear which geometric properties are critical for performance (such as aperture and element spacing for regular geometries). A direct approach to optimize an distribution of microphones for a specified acoustic scene involves evaluating the performance of randomly generated candidates via Monte Carlo simulation [2], where for each run the spatial gains are computed over the space containing the sources of interest. This method, however, is time-consuming for large spaces and complex acoustic scenes. This limits their feasibility for applications where rapid deployment is required, such as in the case of mobile platforms with changing acoustic scenes and surveillance applications.

Therefore, this paper examines beamformer performance based on statistical characteristics of microphone geometries. Effective geometry descriptors are identified and analyzed, showing significant impacts on key performance matrices for speech signal applications. Two optimization algorithms for irregular arrays are presented, including a Genetic Algorithm (GA) and a Hyperbola Cluster Design (HC). Efficient objective functions based on array geometry descriptors and flexible acoustic scene descriptions are applied to circumvent the need to compute the spatial gains, and more directly relates geometry to performance. In addition, experimental results of optimized arrays in terms of SNR are compared to comparable arrays with regular spacings, as well as those determined through Monte Carlo simulations.

2. PROBLEM FORMULATION

Consider arrays and sound sources distributed in space. Signals received by the p^{th} microphone can be expressed as:

$$v_p(t;\boldsymbol{r}_s,\boldsymbol{r}_p)=u(t;\boldsymbol{r}_s)*h(t;\boldsymbol{r}_s,\boldsymbol{r}_p) \quad , \tag{1}$$

where * denotes the convolution operation, $u(t;\boldsymbol{r}_s)$ is the sound wave from source located at $\boldsymbol{r}_s$, and $h(\cdot)$ is the impulse response for the propagation path from $\boldsymbol{r}_s$ to $\boldsymbol{r}_p$ given by:

$$h(t;\boldsymbol{r}_s,\boldsymbol{r}_p)=\sum_{n=0}^{\infty} a_{spn}(t-\tau_{spn}) \quad , \tag{2}$$

where $a_{spn}(t)$ is the n^{th} path propagation response, and τ_{spn} is the corresponding time delay.

To be consistent for all microphone distributions, a flexible delay-and-sum beamformer with inverse distance weighting is applied for all cases considered in this paper. The resulting power leaked between the focal point and other spatial regions (i.e. sidelobes) can be expressed in the frequency domain as:

$$S(\boldsymbol{r}_i,\boldsymbol{r}_s)=\int \sum_{p=1}^{P}\sum_{q=1}^{P} B_{ip}B_{iq}\hat{V}_p(\omega;\boldsymbol{r}_s,\boldsymbol{r}_p)\hat{V}_q^*(\omega;\boldsymbol{r}_s,\boldsymbol{r}_q) \exp(j\omega(\tau_{ip}-\tau_{iq}))\,d\omega \quad , \tag{3}$$

where $\boldsymbol{r}_i$ is the beamformer focal point, $\boldsymbol{r}_s$ is the location of the sound source, $\hat{V}_p$ is the Fourier transform of v_p, P is the number of microphones, and beam steering parameters B_{ip} and τ_{ip} are set according to target position $\boldsymbol{r}_i$ and microphone position $\boldsymbol{r}_p$, given by $B_{ip}=1/d_{ip}$ and $\tau_{ip}=d_{ip}/c$, where d_{ip} denotes the distance from $\boldsymbol{r}_i$ to $\boldsymbol{r}_p$ and c is the speed of sound. For scenes containing multiple sources, the total output power is computed from the superposition of all sources. In the following analysis, B_{ip} and τ_{ip} are treated as random variables, resulting directly from the array geometry to be optimized, and the distribution of differential path lengths for all microphone pairs in the array systems is directly related to the array's ability to suppress sources away from the focal point [5-7].

The relationship between beamformer output power for sources at and away from the focal point can be partially observed by applying the expected value operator to Eq. (3) to isolate the exponential sum and describe its ability to suppress interferences based on the microphone distribution properties. In cases when the interfering source is not in the mainlobe, the attenuation factors can be assumed uncorrelated with pairwise path differences for microphone pairs. With this assumption and considering only direct path propagation, the expected value over all microphone pairs, generated by the double summation in Eq. (3), results in:

$$\mathrm{E}\left[S(\boldsymbol{r}_i,\boldsymbol{r}_s)\right]=P^2\int\langle|\hat{U}(\omega;\boldsymbol{r}_s)|^2\rangle\mathrm{E}\left[B_{ip}B_{iq}A_{sp}A_{sq}^{*}\right]\mathrm{E}\left[\exp\left(j\,2\pi\left(\frac{d_{sq}-d_{sp}}{\lambda}+\frac{d_{ip}-d_{iq}}{\lambda}\right)\right)\right]d\omega \quad , \quad (4)$$

where the angular brackets denote the average power in the case of the source [6-8]. When a source is located at the focal point, $\boldsymbol{r}_s=\boldsymbol{r}_i$, the complex exponential becomes 1, and the signal at $\boldsymbol{r}_s$ is enhanced by the coherent addition independent of array geometry, while interfering sources have weaker average power due to incoherent phases of exponential terms. The degree to which this summation approaches zero depends on the Differential-path Distance (DPD) distribution of all microphone pairs to the interfering sources and focal point relative to the signal wavelengths. The level of incoherence can be directly related to the DPDs derived from microphone positions and expected source distribution in the acoustic scene. A limited range of DPD values, relative to the wavelength, results in a strong partial coherence for signals received from non-target positions, while a wide DPD range with a uniform distribution of phase terms from $-\pi$ to π results in an incoherence with a near zero power gain for non-target source positions [6-8]. Unfortunately, the statistics of the DPDs do not directly lead to geometries that can be easily visualized. However, they can be easily computed from microphone positions and used as an efficient alternative to compute the array gain pattern. Statistics based on these quantities will be referred to geometric descriptors, and used to both characterize arrays and assess performance for the optimization.

3. GEOMETRY DESCRIPTORS

In order to further characterize the relationship between array geometries and beamformer performance, four statistical descriptors of microphone distributions are proposed in this section, that have been correlated with key performance matrices. Important conditions for array performance are its distance from the focal point and the spread of its distribution. The array centroid represents the array position, and can be use to compute its distance to the focal point, and spatial dispersion or spread. It has a direct impact on the main lobe resolution and shape. For a distribution of P microphones, the array centroid is given by:

$$\boldsymbol{r}_0=(x_0,y_0,z_0)=\left(\frac{1}{P}\sum_{p=1}^{P}x_p,\frac{1}{P}\sum_{p=1}^{P}y_p,\frac{1}{P}\sum_{p=1}^{P}z_p\right) \quad , \quad (5)$$

where $\boldsymbol{r}_p=[x_p,y_p,z_p]$ is the position of the p^{th} microphone. The array distance or offset from the focal point is given by:

$$L=\sqrt{(x_0-x_i)^2+(y_0-y_i)^2+(z_0-z_i)^2} \quad , \quad (6)$$

where $\boldsymbol{r}_i=(x_i,y_i,z_i)$ denotes focal point. Array dispersion about the centroid is computed by:

$$a=\sqrt{\frac{1}{P}\sum_{p=1}^{P}(x_p-x_0)^2+(y_p-y_0)^2+(z_p-z_0)^2} \quad , \quad (7)$$

which is analogous to the array aperture typically used for analyzing regular geometries.

When comparing the performance of various arrays, the P, L, and a will be kept constant. These parameters are typically given as the environmental constraints (i.e. allowable positions for a fixed number of microphones) and directly impact resolution, the type of wavefront that can be exploited (near or far field), and the possible range of DPDs. These parameters will be used to characterize both regular and irregular arrays and ensure that similar classes of arrays are compared. So for classes of arrays with fixed P, L and a, descriptors involving the DPD diversity from interfering sources to the array elements are applied as descriptors related to performance [6]. Given a microphone pair (p, q) and 2 spatial positions, a DPD is explicitly defined as:

$$\Delta_{pq}(\boldsymbol{r}_i,\boldsymbol{r}_s)=(d_{sq}-d_{sp})+(d_{ip}-d_{iq}) \quad , \quad (8)$$

which is taken from the exponent argument in Eq.(4). This definition can be applied in the cases with both overlapping and distinct noise and target spaces. Since the DPD distribution spread is important for achieving incoherence and suppressing source points away from the focal point, the standard deviation is considered as a measure affecting performance, given by:

$$\sigma(\boldsymbol{r}_i,\boldsymbol{r}_s)=\sqrt{\frac{1}{P^2}\sum_{p=1}^{P}\sum_{q=1}^{P}\left(\Delta_{pq}(\boldsymbol{r}_i,\boldsymbol{r}_s)\right)^2} \quad , \quad (9)$$

From Eq.(9), it can be seen that DPDs will be symmetric about zero in the double summation, therefore DPDs will always be zero mean. Because the DPD standard deviation does not necessarily indicate the level of DPD diversity over its range, Pielou's Evenness Index, which is a normalized Shannon entropy, is applied to numerically assess the DPD diversity over the range [6] [11]. It is computed as:

$$J(\boldsymbol{r}_i,\boldsymbol{r}_s)=\frac{H(\boldsymbol{r}_i,\boldsymbol{r}_s)}{H_{max}(\boldsymbol{r}_i,\boldsymbol{r}_s)}=\frac{-\sum_{k=1}^{K}(p_k\ln p_k)}{-\sum_{k=1}^{K}(\frac{1}{K}\ln(\frac{1}{K}))}=\frac{-\sum_{k=1}^{K}(p_k\ln p_k)}{\ln K} \quad ,(10)$$

where K is the total number of DPD bins for the histogram estimate and p_k is the percentage of DPDs within the k^{th} bin. The value of K can be determined empirically to result in reasonably smooth histograms of the DPDs. For the results in this paper 10 to 20 bins over the dominate signal wavelength were sufficient for consistent performance. $H(\boldsymbol{r}_i,\boldsymbol{r}_s)$ is the Shannon entropy, and $H_{max}(\boldsymbol{r}_i,\boldsymbol{r}_s)$ is the maximum possible entropy for the given number of bins. This normalization avoids the variation brought by different ranges of DPD distributions and different numbers of microphones.

Therefore, four geometry descriptors {L, a, σ, J} are proposed to characterize both regular and irregular microphone distributions. Their significant impacts on array beamforming performance in specified environments have been demonstrated and summarized in paper [5-7]. For example, arrays with high DPD entropy and wide DPD spread correspond to arrays with better noise suppression ability in near-field applications, such as in immersive environments. These relationships between geometry descriptors and performance matrices can be applied to predict the array SNR performance in given acoustic environments, and further act as the objective functions in the optimization procedure to search for the optimal microphone distributions. In addition, since the DPD statistics so far do not have simple geometric interpretations and must be computed based on all the microphone positions and desired focal points, a cluster design method is developed to directly generate optimal arrays with good values of proposed geometry descriptors or guide *adhoc* microphone placements.

4. OPTIMATION ALGORITHMS

4.1. Heuristic searching

In contrast to a direct searching method via Monte Carlo simulations which randomly selects new distributions of microphones for each sample run, the GA approach exploits the history of previous fitness values in creating subsequent generation of arrays with better fitness, along with perturbations to include diversity in each generation. Fitness for each microphone distribution candidate is assessed by the objective function derived from the relationships between proposed geometry descriptors and relevant beamformer performance. Then, the next generation is selected based on parent fitness values. Various levels of diversity are achieved through crossover and mutation in the next generation. The evolution procedure continues until reaching acceptable fitness or iteration limit.

The GA objective function applied to predict the beamformer performance via geometry descriptors can be shown as:

$$F(\boldsymbol{G})=\int_{\boldsymbol{r}_i\in\, target space}\left\{\int_{\boldsymbol{r}_s\in\,\text{noise space}}\Phi(\boldsymbol{G},\boldsymbol{r}_i,\boldsymbol{r}_s)\,p(\boldsymbol{r}_s|\boldsymbol{r}_i)\,d\,\boldsymbol{r}_s\right\}p(\boldsymbol{r}_i)\,d\,\boldsymbol{r}_i \tag{11}$$

where G represents a particular array geometry with a set of geometric descriptors {*L, a,* σ *, J*}. $\Phi(\boldsymbol{G},\boldsymbol{r}_i,\boldsymbol{r}_s)$ is the relationship functions between geometric descriptors and performance matrices through nonlinear regression analysis and Monte Carlo simulations [14]. For a given focal point $\boldsymbol{r}_i$ and noise source at $\boldsymbol{r}_s$,

$$\begin{aligned}\Phi(\boldsymbol{G},\boldsymbol{r}_i,\boldsymbol{r}_s)=&-\hat{\Gamma}(L,a,\sigma,J,\boldsymbol{r}_i,\boldsymbol{r}_s)\\&+\lambda\cdot\max\left[\hat{B}_{3\text{dB}}(L,a,\sigma,J,\boldsymbol{r}_i)\text{-}\,\eta,0\right]\end{aligned}\quad, \tag{12}$$

where $\hat{B}_{3\text{dB}}$ and $\hat{\Gamma}$ represent the interested performance matrices, Mainlobe Width (MLW) and the Mainlobe-to-peak-sidelobe Ratio (MPSR). Considering the trade-off between MLW and MPSR, a penalty is enforced when the MLW exceeds a threshold while maximizing MPSR. η represents the limit on the maximum MLW. The maximum operation enforces the penalty with weight λ, only when MLW exceeds this limit.

For a specified acoustic scene, $p(\boldsymbol{r}_i)$ and $p(\boldsymbol{r}_s|\boldsymbol{r}_i)$ are the probability density functions representing the likelihood of positions for the desired target and possible noise source locations. Normally, there are related to the behavior patterns of sound sources, such as usual moving tracks and possibilities to make sound. In the case when no prior knowledge is available, these can simply be set to uniform distributions. Therefore, the criterion to search for optimal array geometry is represented by:

$$\boldsymbol{G}_{opt}=\underset{\text{G}\in\text{mic space}}{\operatorname{argmin}}\langle F(\boldsymbol{G})\rangle\quad, \tag{13}$$

A successful GA has to maintain the balance between inheritance and exploration, which means the tradeoff between searching diversity to ensure global optimum (related with computing complexity) and convergence rate. In our experiments, the size of initial population, ratio of elites selection, ratio of mutation and crossover, and standard deviation of perturbation in mutation are the relevant factors which need to be adjusted to maintain robust performance of optimization procedure in specified scenes. The details of GA setting and objective functions have been provided in paper [5-7,14].

4.2. Hyperbola cluster design

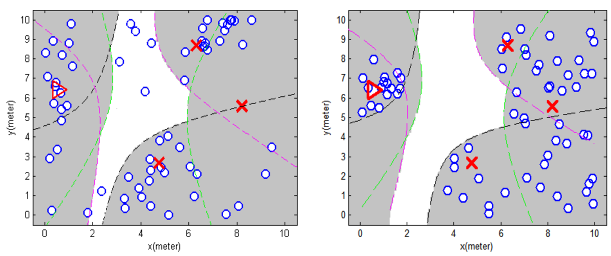
It has been demonstrated that arrays with high DPD entropy and wide DPD spread correspond to arrays with better noise suppression ability [6]. Although the DPD statistics do not have simple geometric interpretations and must be computed based on all the microphone positions and desired focal points, by defining the hyperbola areas, HC can directly design an optimal array with good values of proposed geometry descriptors or guide *ad hoc* microphone placements.

As defined in Eq.(8), the DPD is explained as the difference of the differential distances from each mic to two spatial positions $\{\boldsymbol{r}_i,\boldsymbol{r}_s\}$ in FOV. It can be rewritten as:

$$\Delta_{pq}(\boldsymbol{r}_i,\boldsymbol{r}_s)=(d_{sq}-d_{iq})-(d_{sp}-d_{ip})\quad, \tag{14}$$

Since a hyperbola curve can be defined equivalently as the locus of points where the absolute value of the difference of the distances to the two focuses is a constant (equal to the distance between its two vertices), it can be applied in here to distinguish microphones with different values of $(d_{sq}-d_{iq})$. As shown in Figure 1, each pair of $\{\boldsymbol{r}_i,\boldsymbol{r}_s\}$ are considered as the focuses. Microphones located on the same hyperbola curve (marked as dashed lines with same color) have identical values of $(d_{sq}-d_{iq})$, while microphones located inside the hyperbola curve show larger absolute values of differential distance. Therefore, with specified $\{\boldsymbol{r}_i,\boldsymbol{r}_s\}$, in order to obtain large spread and uniform distribution of DPDs over all microphone pairs, microphones should be clustered in both hyperbola areas (the grey areas in Figure 1) to generate a set of DPDs with the possible largest spread. The DPD values between the spread range are obtained by the nearby microphone pairs located in the same area, providing a smooth entropy. Note that there is no need to put microphones in the middle area of $\{\boldsymbol{r}_i,\boldsymbol{r}_s\}$.

Therefore, it concluded that for one pair of target and noise positions, the largest spread DPDs will be generated if microphones are clustered inside the hyperbola areas with target and noise positions as focuses. For example, Figure 1(a) gives a irregular array with top SNR performance resulted from GA optimization, where one target and three interferences are considered for this scene. The hyperbola areas for each target-noise pair are marked in dashed line with different color. It can be seen that most of microphones in the GA-optimized array are clustered in these hyperbole areas (grey areas). In order to demonstrate this conclusion, Figure 1(b) provides a example for the irregular array clustered based on the hyperbola theory in the same scene. Microphones are divided into four clusters uniformly distributed in these four hyperbola areas. Simulations are performed with human speech signals in corresponding acoustic scenes. The SNR results as in Table 1 demonstrate that the HC arrays have comparable or even better SNR results than GA optimized arrays, while great SNR improvements are observed in both of these geometries compared with corresponding regular arrays.



(a) (b)

Figure 1: Top view of GA-optimized irregular array and hyperbola clustered array. Blue circles represent microphones. Red crosses represent the possible noise space. Red triangle is the desired target space. (a) GA-optimized irregular array. (b) Hyperbola clustered array.

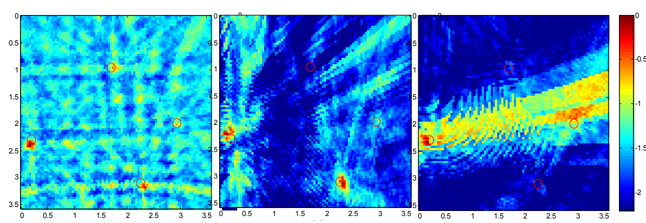
4.3. Experiments

Numerical simulations for three different source distributions (acoustic scenes) are performed. All the experiments are applied in a 10×10×2 m audio cage (the field of view) to simulate an typical indoor environment for audio surveillance applications. Simulated signals consist of colored noise generated by the band importance function from the speech intelligibility index (SII) model, which emphasizes the frequency bands most important to speech intelligibility [12]. As shown in Table 1, the SNR results of optimal geometries from GA and HC are compared to randomly generated irregular geometries from the first generation of GA (to see impact of applying objective functions), as well as comparable regular arrays with the same centroid and dispersion. By comparisons, significant SNR improvements are observed as a result of the objective function rules in GA. Through GA optimization with proposed geometric criteria as the objective function, the superior arrays are sorted out that perform better than the regular arrays in all the cases. Moreover, as a direct design method without time-consuming searching, HC arrays show comparable or even better SNR results than GA optimized arrays, while great SNR improvements are also observed compared with corresponding regular arrays.

In addition, five separate real recordings with different signal power levels were performed for the GA-optimized, HC and regular planar arrays with 9 microphones over the ceiling of the aluminum cage. Colored noise generated by the band importance function from the SII model are played through the speakers as the sound sources and varied for each recording. In order to simulate human speech, the probability for each source to make sound is set to 2/3. Figure 2 shows the top-view gain patterns derived from a 20ms frame of recorded signals, when targeting at the left source. The resulted average SNRs overall time slots demonstrated the superior performance of GA-optimized irregular array and HC array, when comparing with the regular one.

Table 1: SNR (dB) comparison of GA-optimized and HC arrays

Acoustic Scenes	GA set (50 arrays)		HC set (50 arrays)		50 random arrays		Reg. array
	Top 3 SNR	Avg. SNR	Top 3 SNR	Avg. SNR	Top 3 SNR	Avg. SNR	SNR
64 mics with continuous noise space	9.03 8.99 8.98	8.45	8.92 8.87 8.71	6.81	6.47 5.67 5.49	3.83	3.40
64 mics with discrete noise sources	26.33 25.78 25.63	23.83	28.04 27.38 27.03	24.76	22.61 22.18 22.09	17.96	17.28
9 mics with discrete noise sources	18.24 17.71 17.65	16.56	21.83 21.07 20.69	17.93	17.78 17.62 17.20	9.01	8.89



(a) (b) (c)

Figure 2: Top-view gain patterns. The red circles represent source positions. (a) Regular planar array. (b) GA-optimized array. (c) HC arrays.

5. CONCLUSION

This paper examines the spatial gain patterns of irregular arrays based on statistical descriptions of array geometries. Based on the relations of identified geometry descriptors on beamformer performance, a heuristic searching and a direct cluster design method are applied to obtain optimal arrays for speech signals in specified immersive environments. It has been demonstrated that these methods effectively sorts out these superior arrays showing significant SNR improvements when comparing with randomly generated arrays and regular arrays. Objective functions for the GA based on the statistical array descriptors result in rapid convergence relative to Monte Carlo optimizations, which suggests a strong correlation between beamformer performance and proposed geometry descriptors. In addition, by clustering mics in the hyperbola areas to generate rich DPD entropy, HC arrays can be directly built according to the prior knowledge of acoustic scene, and provide comparable SNR performance with the GA-optimized arrays. It is an easy and feasible optimization method for the ad hoc (not computer aided) microphone array design.

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