ON DIRECTIVITY FACTOR OF THE FIRST-ORDER STEERABLE DIFFERENTIAL MICROPHONE ARRAY

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ABSTRACT

Directivity factor (DF) is a fundamental performance measure of microphone arrays. This paper studies the DF of the first-order steerable differential array (FOSDA) whose response is constructed by a linear combination of monopole and two orthogonal dipoles using a four-element square array. The DF of the ideal FOSDA and the DFs of the FOSDA with microphone gain/phase errors are derived, all in closed forms. Based on the theoretical analysis, several findings regarding the properties of the DF of the FOSDA are obtained.

Index Terms— Differential microphone array, superdirective beamforming, directivity factor.

1. INTRODUCTION

In the past decades, microphone arrays have attracted great interest from researchers in the field of audio and speech processing [1,2]. The existing design approaches for microphone arrays can be classified generally into two categories [3]. One is traditional additive arrays, such as the delay-and-sum arrays [2], and the other one is differential arrays, whose response is related to spatial derivative of an acoustic pressure field [4]. Since differential microphone arrays have some advantages over their additive counterparts, such as higher directivity and frequency-invariant array response [3,4], recent years have seen much effort dedicated to the design of differential arrays [3–13, 15–17, 19].

Among the proposed differential arrays, the steerable differential arrays whose mainlobe orientation can be electronically adjusted to any desired direction are particularly useful when positions of sound targets may vary over a large extent [6, 7, 11]. It is known that at least three microphones in a circular geometry are required to construct a steerable first-order differential array [7]. In [11], a first-order steerable differential array (FOSDA) has been proposed, whose response is constructed by a linear combination of a monopole and two orthogonal dipoles using a four-element square array. In contrast, the FOSDA has the advantage that it can form a frequency-invariant equi-shaped beampattern for the orthogonal dipoles [11]. Some theoretical analysis on the sensitivity of the FOSDA for microphone gain/phase errors has been discussed [11]. More recently, the mainlobe misorientation of the FOSDA in the presence of microphone gain/phase errors is studied [20]. In array signal processing field, the directivity factor (DF) is a well-established performance measure [21]. However, it is noted that the effects of microphone mismatches on the DF of the FOSDA have not been studied yet. Moreover, the DF of the ideal FOSDA, i.e., without microphone mismatches, has been merely analyzed numerically [11], and a theoretical analysis is to be conducted. In this paper, the DF of the ideal FOSDA, and the DFs of the FOSDA with microphone gain/phase errors are presented, all in closed forms. Based on the theoretical results, some properties of the DF of the FOSDA are revealed.

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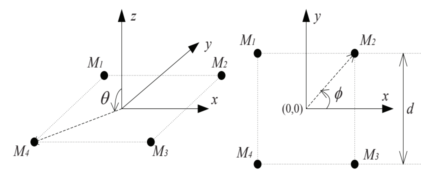


Fig. 1. The configuration of the FOSDA.

2. BACKGROUND

2.1. Configuration of the FOSDA

The FOSDA consists of four microphones in a square on the x-y plane [11], as shown in Fig. 1. The distance between two nondiagonal microphones is d. For a unit-amplitude incident plane wave with frequency f and incident angle (θ, ϕ) ($\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$ denote the elevation and azimuth angles, respectively), the ith microphone signal is given by

$$E_i = \exp\left[j\omega t + j\omega \sin\theta \left(p_{x_i} \cos\phi + p_{y_i} \sin\phi\right)/c\right] \quad (1)$$

where $\omega = 2\pi f$, t denotes the time, p_{x_i} and p_{y_i} refer to the x- and y- coordinates of the ith microphone, c is the speed of sound, and $j = \sqrt{-1}$.

2.2. Array Response of the FOSDA

The ith microphone signal with microphone gain and phase errors can be expressed as

$$E_i^{(g,p)} = \eta_i E_i e^{j\psi_i} \tag{2}$$

where $\eta_i = 1 - \epsilon_i$, and ϵ_i, ψ_i are the gain and phase errors.

The normalized array response of the FOSDA with its mainlobe oriented toward $\phi = \varphi_s$ is

$$\overline{E}_{s(\alpha)}^{(g,p),\varphi_s}(\theta,\phi) = \alpha\overline{E}_m^{(g,p)}(\theta,\phi) + (1-\alpha)\overline{E}_d^{(g,p),\varphi_s}(\theta,\phi) \tag{3}$$

where $\alpha \in (0,1)$ is the directivity controlling parameter, $\overline{E}_m^{(g,p)}(\theta,\phi)$ and $\overline{E}_d^{(g,p),\varphi_s}(\theta,\phi)$ represent the normalized responses of the monopole and the steered dipole, respectively. In (3), the high-pass frequency response and $\pi/2$ phase shift have been compensated out of the dipole responses.

The normalized response of the monopole is

$$\begin{aligned}\overline{E}_m^{(g,p)}(\theta,\phi) &= \frac{1}{4}\sum_{i=1}^{4} E_i^{(g,p)} \\ &= \frac{1}{4}\left\{e^{j\upsilon_{24}}\left[\xi_{24}\cos\left(\Theta_{24}+\tau_{24}\right) + j\eta_{24}\sin\left(\Theta_{24}+\tau_{24}\right)\right]\right. \\ &\quad \left. + e^{j\upsilon_{31}}\left[\xi_{31}\cos\left(\Theta_{31}+\tau_{31}\right) + j\eta_{31}\sin\left(\Theta_{31}+\tau_{31}\right)\right]\right\}\end{aligned}$$

where $\upsilon_{24} = (\psi_2+\psi_4)/2$, $\upsilon_{31} = (\psi_3+\psi_1)/2$, $\tau_{24} = (\psi_2 - \psi_4)/2$, $\tau_{31} = (\psi_3 - \psi_1)/2$, $\xi_{24} = \eta_2 + \eta_4$, $\xi_{31} = \eta_3 + \eta_1$, $\eta_{24} = \eta_2 - \eta_4$, $\eta_{31} = \eta_3 - \eta_1$, $\Omega = \omega d/(2c)$, $\Theta_{24} = \sqrt{2}\Omega\sin\theta\cos(\phi - \pi/4)$, $\Theta_{31} = \sqrt{2}\Omega\sin\theta\cos(\phi+\pi/4)$.

The steered dipole is constructed by two orthogonal dipoles oriented toward $\pm\pi/4$, i.e., $E_d^{(g,p),-\pi/4} = E_3^{(g,p)} - E_1^{(g,p)}$, and $E_d^{(g,p),\pi/4} = E_2^{(g,p)} - E_4^{(g,p)}$. The normalized response of the steered dipole for small microphone gain and phase errors can be expressed as

$$\begin{aligned}&\overline{E}_d^{(g,p),\varphi_s}(\theta,\phi) \\ &\approx \frac{e^{j\upsilon_{31}}}{2\sqrt{2}}\cos\left(\varphi_s + \frac{\pi}{4}\right)\left[\frac{\xi_{31}}{\Omega}\sin\left(\Theta_{31}+\tau_{31}\right) - \frac{j\eta_{31}}{\Omega}\right] \\ &\quad + \frac{e^{j\upsilon_{24}}}{2\sqrt{2}}\sin\left(\varphi_s + \frac{\pi}{4}\right)\left[\frac{\xi_{24}}{\Omega}\sin\left(\Theta_{24}+\tau_{24}\right) - \frac{j\eta_{24}}{\Omega}\right].\end{aligned}$$

3. DF OF THE IDEAL FOSDA

In the following, we assume $\Omega \ll 1$. According to [11, 21], the DF of the ideal FOSDA for a spherical sound field, i.e., without any microphone mismatches, can be represented as

$$Q(\Omega,\alpha) = \frac{4\pi\left|\overline{E}_{s(\alpha)}^{\varphi_s}(\frac{\pi}{2},\varphi_s)\right|^2}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left|\overline{E}_{s(\alpha)}^{\varphi_s}(\theta,\phi)\right|^2\sin\theta \mathrm{d}\theta\mathrm{d}\phi} \tag{4}$$

where $\overline{E}_{s(\alpha)}^{\varphi_s}(\theta,\phi)$ is the normalized array response of ideal FOSDA, which equals (3) with $\epsilon_i = 0$, $\psi_i = 0$. Like [11], herein we have assumed that the steering direction of the array is within the x-y plane.

Setting $\epsilon_i = 0$, $\psi_i = 0$, and substituting (3) into (4) yields

$$\begin{aligned}Q(\alpha) &\approx 3\{\alpha - (\alpha-1)[\cos(\varphi_s - \pi/4)\sin(\varphi_s+\pi/4) \\ &\quad + \cos^2(\varphi_s+\pi/4)]\}^2/(4\alpha^2 - 2\alpha + 1) \\ &= \frac{3}{4\alpha^2 - 2\alpha + 1}.\end{aligned} \tag{5}$$

Remark 1: As shown in (5), the DF of the ideal FOSDA is nearly frequency-invariant (note that it holds under the condition $\Omega \ll 1$). Moreover, the DF of the ideal FOSDA is also nearly independent on the steering direction of the mainlobe φ_s. When $\alpha = 0.25$, i.e., corresponding to a hypercardioid response, the DF attains its maximal value $Q \approx 4$.

4. EFFECTS OF MICROPHONE MISMATCHES

In the presence of microphone gain/phase errors, the DF of the FOSDA for a spherical sound field is given by [11]

$$Q^{(g,p)}(\Omega,\alpha) = \frac{4\pi\left|\overline{E}_{s(\alpha)}^{(g,p),\varphi_s}(\frac{\pi}{2},\varphi_s)\right|^2}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\left|\overline{E}_{s(\alpha)}^{(g,p),\varphi_s}(\theta,\phi)\right|^2\sin\theta \mathrm{d}\theta\mathrm{d}\phi}. \tag{6}$$

4.1. Effect of Microphone Gain Errors

From (3), for small microphone gain errors, the normalized array response of the FOSDA can be reduced to

$$\begin{aligned}&\overline{E}_{s(\alpha)}^{(g),\varphi_s}(\theta,\phi) \approx \frac{\alpha}{4}\left(\xi_{24}+\xi_{31}\right) \\ &+ \frac{(1-\alpha)\sin\theta}{2}\left[\cos\left(\varphi_s+\frac{\pi}{4}\right)\xi_{31}\cos\left(\phi+\frac{\pi}{4}\right)\right. \\ &\left. + \sin\left(\varphi_s+\frac{\pi}{4}\right)\xi_{24}\cos\left(\phi-\frac{\pi}{4}\right)\right] \\ &+ \frac{j\Omega\alpha\sin\theta}{2\sqrt{2}}\left[\eta_{31}\cos\left(\phi+\frac{\pi}{4}\right) + \eta_{24}\cos\left(\phi-\frac{\pi}{4}\right)\right] \\ &- \frac{j(1-\alpha)}{2\sqrt{2}\Omega}\left[\eta_{31}\cos\left(\varphi_s+\frac{\pi}{4}\right) + \eta_{24}\sin\left(\varphi_s+\frac{\pi}{4}\right)\right].\end{aligned} \tag{7}$$

By substituting (7) into (6), the DF of the FOSDA in the presence of microphone gain errors can be derived as

$$\begin{aligned}&Q^{(g)}(\Omega,\alpha) \\ &\approx \frac{\left[\frac{1}{2}\alpha\left(\xi_{24}+\xi_{31}\right) + (1-\alpha)\rho\right]^2 + \frac{1}{2\Omega^2}(1-\alpha)^2\zeta}{\frac{\alpha^2}{4}\left(\xi_{24}+\xi_{31}\right)^2 + \frac{(1-\alpha)^2}{3}\lambda + \frac{1}{2\Omega^2}(1-\alpha)^2\zeta}\end{aligned} \tag{8}$$

where $\rho = \xi_{31}\cos^2(\varphi_s + \pi/4) + \xi_{24}\sin^2(\varphi_s+\pi/4)$, $\lambda = \xi_{31}^2\cos^2(\varphi_s+\pi/4) + \xi_{24}^2\sin^2(\varphi_s+\pi/4)$, and $\zeta = [\eta_{31}\cos(\varphi_s+\pi/4) + \eta_{24}\sin(\varphi_s+\pi/4)]^2$.

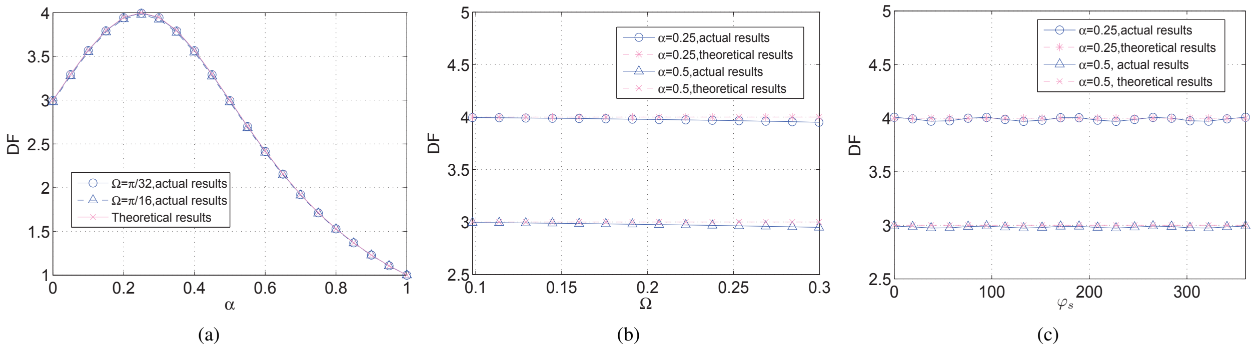


Fig. 2. The DF of the ideal FOSDA. (a) DF versus α with $\varphi_s = 60°$. (b) DF versus Ω with $\varphi_s = 60°$. (c) DF versus φ_s with $\Omega = \pi/16$.

Specifically, when there are no microphone gain errors, it follows that (8) will degenerate to (5).

Remark 2: Unlike the case without microphone mismatches, the DF of FOSDA will be frequency-variant in the presence of microphone gain errors. In particular, it can be proved from (8) that, the DF will increase with increasing frequency, and the DF will always be greater than or at least equal to 1 (note that these conclusions hold regardless of the value of α). Also unlike the DF without microphone mismatches, the DF with microphone gain errors will depend on the mainlobe steering direction.

4.2. Effect of Microphone Phase Errors

For small microphone phase errors, using (3) the normalized array response of the FOSDA is given by

$$\left|\overline{E}^{(p),\varphi_s}_{s(\alpha)}(\theta,\phi)\right| \approx \left|\alpha + \frac{1-\alpha}{\sqrt{2}\Omega}\left[\cos\left(\varphi_s+\frac{\pi}{4}\right)\tau_{31} + \sin\left(\varphi_s+\frac{\pi}{4}\right)\tau_{24}\right] + (1-\alpha)\sin\theta\left[\cos\left(\varphi_s+\frac{\pi}{4}\right) \times \cos\left(\phi+\frac{\pi}{4}\right) + \sin\left(\varphi_s+\frac{\pi}{4}\right)\cos\left(\phi-\frac{\pi}{4}\right)\right]\right|. \quad (9)$$

Using (9) and (6), we can derive the DF of the FOSDA with microphone phase errors

$$Q^{(p)}(\Omega,\alpha) \approx \frac{3\left(2\Omega+\sqrt{2}\chi-\sqrt{2}\alpha\chi\right)^2}{6\left[\sqrt{2}\alpha\Omega+(1-\alpha)\,\chi\right]^2+4\Omega^2(1-\alpha)^2} \quad (10)$$

where $\chi = \tau_{31}\cos(\varphi_s+\pi/4) + \tau_{24}\sin(\varphi_s+\pi/4)$. Note that (10) will also degenerate to (5) when there are no microphone phase errors.

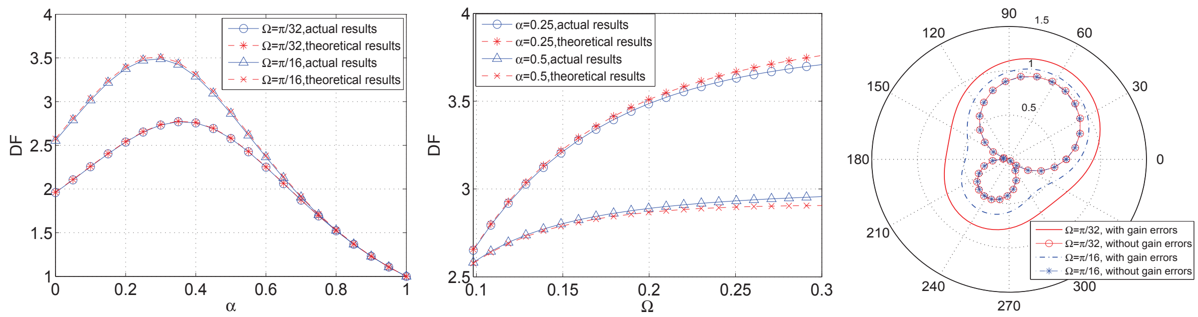
Remark 3: With microphone phase errors, the DF of the FOSDA is frequency-variant and is a function of mainlobe steering direction. Unlike the effect with microphone gain errors, depending on the directivity controlling parameter α, the DF with microphone phase errors may increase or decrease with increasing frequency. In particular, if α is chosen too small, mainlobe orientation reversal may occur [20], which leads to a poor DF. Moreover, it can be proved from (10) that, the maximal DF achievable may remain the same as that of the ideal FOSDA, i.e. $Q \approx 4$, if α is chosen appropriately.

5. NUMERICAL EVALUATION

In this section, several simulation results are presented to verify the above theoretical analysis.

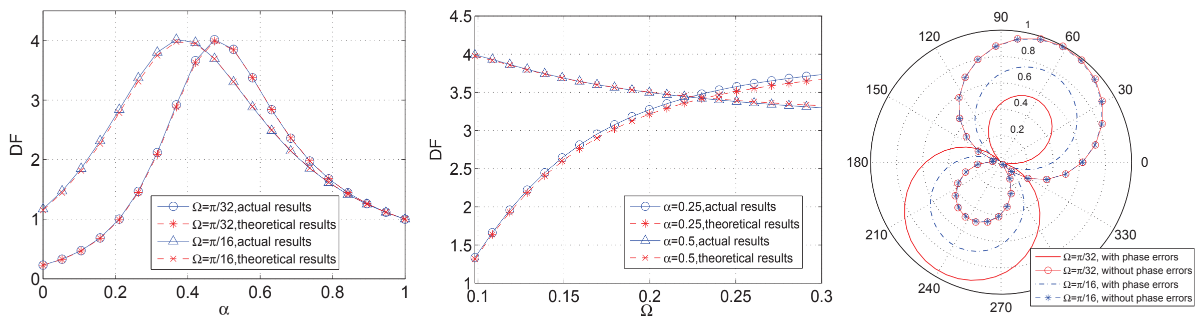
As a first example, we consider the DF of the ideal FOSDA. Fig. 2(a) shows the DF of the ideal FOSDA as a function of the directivity controlling parameter α with $\Omega = \pi/32$ and $\pi/16$, $\varphi_s = 60°$, where the theoretical results using (5) are also shown for comparison (for the sake of clarity, the actual DF values obtained directly from (4) are denoted as "actual results"). As can be seen, the actual results are consistent well with the theoretical results, and the DF achieves its maximal value $Q \approx 4$ when $\alpha = 0.25$ (a hypercardioid response). Figs. 2(b) and 2(c) show the DF of the ideal FOSDA for different Ω and φ_s, respectively, with $\alpha = 0.25$ and $\alpha = 0.5$ (a cardioid response). Herein, $\varphi_s = 60°$ in Fig. 2(b) and $\Omega = \pi/16$ in Fig. 2(c). From Figs. 2(b) and 2(c), we can see that the DF of the ideal FOSDA is nearly independent on Ω (i.e., frequency), and also independent on the steering direction φ_s.

Next we study the effect of microphone gain errors on the DF of the FOSDA. Here we set $\eta_1 = \eta_2 = 0.9$, $\eta_3 = 1$, $\eta_4 = 1.04$, and $\varphi_s = 60°$. Fig. 3(a) shows the DF versus α with $\Omega = \pi/32$ and $\pi/16$, where the actual results are obtained from (6), and the theoretical results from (8). In contrast with the DF of ideal FOSDA, i.e., Fig. 2(a), it is shown that the maximal DF is no longer fixed at $\alpha = 0.25$, due to the presence of microphone gain errors. Moreover, the maximal DF achievable is less than that of the ideal FOSDA. For example, the maximal DF for $\Omega = \pi/32$ is 2.78 with $\alpha = 0.36$, while the maximal DF for $\Omega = \pi/16$ is 3.52 with $\alpha = 0.29$. In Fig. 3(b), the DF versus Ω is plotted with $\alpha = 0.25$ and 0.5. As can be seen, the DF will increase with increasing fre-



(a) (b) (c)

Fig. 3. The DF of the FOSDA with microphone gain errors $\eta_1 = \eta_2 = 0.9$, $\eta_3 = 1$, $\eta_4 = 1.04$, and $\varphi_s = 60°$. (a) DF versus α. (b) DF versus Ω. (c) Array responses with $\alpha = 0.25$.



(a) (b) (c)

Fig. 4. The DF of the FOSDA with microphone phase errors $\psi_1 = \psi_4 = 0.1$ radians, $\psi_2 = -0.1$ radians, $\psi_3 = 0$, and $\varphi_s = 60°$. (a) DF versus α. (b) DF versus Ω. (c) Array responses with $\alpha = 0.25$.

quency, which agrees well with our theoretical analysis. This can be clearly understood with the help of Fig. 3(c), where the array responses with and without microphone gain errors are shown with $\alpha = 0.25$. In the presence of microphone gain errors, the array response of the FOSDA will tends to be omnidirectional, thus leads to a poor DF.

In our last example, we consider the effect of microphone phase errors on the DF of the FOSDA. Suppose that $\psi_1 = \psi_4 = 0.1$ radians, $\psi_2 = -0.1$ radians, $\psi_3 = 0$, and $\varphi_s = 60°$. Fig. 4(a) shows the DF versus α with $\Omega = \pi/32$ and $\pi/16$, where the actual results are obtained from (6), and the theoretical results from (10). Unlike the ideal FOSDA, the value of α where the DF of the FOSDA with microphone phase errors is maximized is not always fixed at 0.25. For $\Omega = \pi/32$ and $\pi/16$, the DF is maximized at $\alpha = 0.48$ and 0.39, respectively. It is interesting to note that the maximal DF achievable remains the same as that of the ideal FOSDA, i.e., $Q \approx 4$, which is different from the effect of microphone gain errors. The DF versus Ω with $\alpha = 0.25$ and 0.5 is shown in Fig. 4(b). Compared with the case with microphone gain errors, the DF versus Ω is no longer monotonic. It depends on the value of α. For $\alpha = 0.25$, the DF is a monotonically increasing function of frequency, while for $\alpha = 0.5$ it is a monotonically deceasing function of frequency. Recall the fact that the presence of microphone phase errors may lead to reversal of mainlobe orientation of the FOSDA if α is set to be less than the lower bound as stated by Proposition 2.2 in [20]. Therefore, for a smaller α, its DF tends to decrease, especially at low frequencies. To illustrate, Fig. 4(c) plots the array responses of the FOSDA for $\alpha = 0.25$ with and without microphone phase errors, where $\Omega = \pi/32$ and $\pi/16$. For $\Omega = \pi/32 \approx 0.098$, we can obtain that the lower bound of α is 0.376 [20]. Therefore, for the design with $\alpha = 0.25$, the FOSDA will suffer from mainlobe orientation reversal and thus has a low DF 1.35.

6. CONCLUSIONS

The DF of the FOSDA has been studied theoretically in this paper. In particular, the DF of the ideal FOSDA and the DFs of the FOSDA with microphone gain/phase errors are derived, all in closed forms. Several interesting findings have been presented, which are helpful to better understand the characteristics of the FOSDA. The theoretical results have been further verified by numerical examples.

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