# MAX-MIN NETWORK FLOW AND RESOURCE ALLOCATION FOR BACKHAUL CONSTRAINED HETEROGENEOUS WIRELESS NETWORKS

*Wei-Cheng Liao, Mingyi Hong, and Zhi-Quan Luo*

Department of Electrical and Computer Engineering
University of Minnesota, Minneapolis, MN 55455, USA

## ABSTRACT

We consider a heterogenous network (HetNet) consisting of a number of base stations (BSs) and network routers connected via a backhaul network. The optimal provision of such networks requires proper resource allocation across the radio access links in conjunction with appropriate traffic engineering within the backhaul network. In this paper we propose an efficient distributed algorithm for the joint resource allocation across the wireless links and the flow control within the backhaul network. The proposed algorithm, which maximizes the minimum rate among all the users and/or flows, is based on a decomposition approach that leverages both the Alternating Direction Method of Multipliers (ADMM) and the WMMSE algorithm, and is shown to be globally convergent to a stationary solution of the joint flow control and resource allocation problem. Moreover, this algorithm is easily parallelizable and can be extended to the multi-antenna scenario.

***Index Terms***— Heterogeneous Networks, ADMM Algorithm, Software Defined Network, Cross-layer Optimization

## 1. INTRODUCTION

With the advent of cloud computing technologies and the mass deployment of low power base stations (BSs), the next generation cellular radio access networks (RAN) has undergone a major structural change. The traditional single-hop access mode between a serving BS and its users is being replaced by a mesh network consisting of a large number of wireless access points connected by either wireline or wireless backhaul links with finite bandwidth [1]. Unfortunately, the multi-hop nature of this architecture renders the existing single-hop interference management techniques [2, 3] ineffective. New network management methods must be developed for joint wireless resource allocation and traffic engineering within the multi-hop backhaul networks. Furthermore, with cloud centers viewed as special nodes in the backhaul network, this joint network provision problem is also a key component of the newly proposed software defined networking (SDN) concept [4] which advocates centralized joint network provisioning by cloud centers.

For a wireline network with given end-to-end flow (or commodities) demands, the traditional traffic engineering problem aims to optimally route the traffic from the source nodes (e.g., the cloud centers) to the destination nodes (e.g., the users) while satisfying the link capacity constraints. As such, it can be formulated as a linear programming problem [5, 6] and efficiently solved. However, the joint resource allocation and traffic engineering problem is much more challenging due to the multiuser interference of wireless transmission. In particular, for each wireless link, the capacity is a nonconvex function of the transmit power, and is not known *a priori*. Power control is an integral part of the system optimization.

The impact of the finite bandwidth of backhaul networks on wireless resource allocation has been studied recently in the context of joint processing between BSs, e.g., [7, 8, 9, 10]. However, these works do not consider the existence of the multi-hop routes between the source and the destination nodes. The joint optimization problem considered here has also been the focus of cross-layer network utility maximization (NUM) framework, see e.g. [11, 12, 13] and a tutorial paper [14]. In particular, reference [11] considered only the orthogonal wireless links without multiuser interference, thus leading to tractable convex capacity functions. In [12, 13], the multiuser interference factor is considered in a fast fading environment for which the Lagrange duality gaps can be shown to be zero. A similar joint optimization problem is investigated by [15] in a sensor network scenario. However, this approach assumes single-antenna sensors and requires the utility function to be strongly convex.

For large-scale networks, it is crucial that the considered joint optimization problem can be implemented distributedly and/or in parallel. Most of the existing distributed NUM algorithms is based on the primal/dual decomposition method with subgradient update [11], which unfortunately can exhibit slow convergence. In contrast, we propose to leverage the Alternating Direction Method of Multipliers (ADMM) [16, 17] to tackle the joint resource allocation and traffic engineering problem. The resulting algorithm is significantly more efficient than the subgradient-based methods. Notice that the ADMM technique has also been successfully applied in other contexts of digital communication [18, 19, 20, 21].

The main contribution of this paper is the development of an efficient algorithm that can solve the joint wireless resource allocation and the backhaul traffic engineering problem to a locally optimal solution. The proposed algorithm is amenable to distributed and parallel implementation, and is therefore suitable for large-scale networks. Moreover, the algorithm can be easily extended to the multi-antenna case.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider resource management and network optimization for a heterogenous network (HetNet). The nodes of this HetNet, $\mathcal{V}$, consist of the set of network routers $\mathcal{N}$, the set of BSs $\mathcal{B}$, and the set of mobile users $\mathcal{U}$. The set of directed links that connect the nodes of $\mathcal{V}$ is denoted as $\mathcal{L}$. In addition, we assume that there are $M$ source-destination pairs, denoted by $\{(S(m), D(m))\}_{m=1}^{M}$. For each $m = 1, ..., M$, a data flow of rate $r(m) \geq 0$ is to be sent from the source node $S(m)$ to the destination node $D(m)$ over the network.

The set of directed links $\mathcal{L}$ consists of both wired and wireless links. The wired links connect routers in $\mathcal{N}$ and BSs in $\mathcal{B}$, and is denoted as $\mathcal{L}^w \triangleq \{(s,d) \mid (s,d) \in \mathcal{L},\ \forall\ s,d \in \mathcal{N} \bigcup \mathcal{B}\}$. Here

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$(s, d)$ denotes the directed link from node $s$ to node $d$. Assume each wired link $l \in \mathcal{L}^w$ has a fixed capacity, $C_l$. Then the total flow rate on link $l$ is constrained by

$$\sum_{m=1}^{M} r_l(m) \leq C_l, \ \forall \, l \in \mathcal{L}^w, \tag{1}$$

where $r_l(m) \geq 0$ denotes the nonnegative flow rate on link $l$ for commodity $m$.

The wireless links provide single-hop connections between the BSs to the mobile users. For each wireless link, we assume there are $K$ orthogonal frequency subchannels. Thus, the set of wireless links can be represented as $\mathcal{L}^{wl} \triangleq \{(s, d, k) \mid (s, d, k) \in \mathcal{L}, \, \forall \, s \in \mathcal{B}, \, \forall \, d \in \mathcal{U}, \, k = 1 \sim K\}$ with $(s, d, k)$ being the wireless link from node $s$ to node $d$ on subchannel $k$. For subchannel $k$, BS $s \in \mathcal{B}$ applies the linear scalar precoder $p^k_{ds} \in \mathbb{C}$ to the transmitted complex unit-norm symbol of mobile user $d \in \mathcal{U}$, so each mobile user can be served by multiple BSs. Assuming that each mobile user treats the interference from other BSs as noise, the total flow rate constraint on wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$ is expressed as

$$\sum_{m=1}^{M} r_l(m) \leq \bar{r}_l(\mathbf{p})$$
$$\triangleq \log\left(1 + \frac{|h^k_{ds}|^2 |p^k_{ds}|^2}{\sum\limits_{\substack{(s',d',k)\in I(l) \\ (s',d',k)\neq l}} |h^k_{ds'}|^2 |p^k_{d's'}|^2 + \sigma_d^2}\right) \tag{2}$$

where $\mathbf{p} \triangleq \{p^k_{ds} \mid \forall \, (s, d, k) \in \mathcal{L}^{wl}\}$; $h^k_{ds} \in \mathbb{C}$ is the channel tap for the wireless link $l = (s, d, k)$; $\sigma^2_d$ is the variance of AWGN noise at mobile user $d$; $I(l) \subseteq \mathcal{L}^{wl}$ is the set of interfering links defined as $I(l) \triangleq \{(s', d', k) \mid h^k_{ds'} \neq 0, (s, d, k) = l\}$. Each BS $s \in \mathcal{B}$ has a total power budget $\bar{p}_s \geq 0$, satisfying

$$\sum_{k=1}^{K} \sum_{d:(s,d,k)\in\mathcal{L}^{wl}} |p^k_{ds}|^2 \leq \bar{p}_s, \ \forall \, s \in \mathcal{B}. \tag{3}$$

Each node in the network should also follow the flow conservation constraint, i.e., the total incoming flows of node $v \in \mathcal{V}$ equals the total outgoing flow of that node,

$$\sum_{l\in \mathrm{In}(v)} r_l(m) + 1_{\{S(m)\}}(v) r_m$$
$$= \sum_{l\in \mathrm{Out}(v)} r_l(m) + 1_{\{D(m)\}}(v) r_m, m = 1 \sim M, \ \forall \, v \in \mathcal{V} \tag{4}$$

where $\mathrm{In}(v)$ and $\mathrm{Out}(v)$ are, respectively, the set of links that goes into node $v$ and the set of links that comes out of node $v$.

In this paper, we are interested in maximizing the minimum flow rate of all commodities, while jointly performing the following tasks *1)*: route $M$ commodities from node $S(m)$ to node $D(m)$, $m = 1 \sim M$; and *2)* design the linear precoder at each BS. This problem can be formulated as

$$\begin{aligned} \max_{\mathbf{p},\mathbf{r}} \quad & r \\ \text{s.t.} \quad & r \geq 0, \ r_m \geq r, \ r_l(m) \geq 0, \ m = 1 \sim M, \ \forall \, l \in \mathcal{L} \\ & (1), \ (2), \ (3), \ \text{and} \ (4), \end{aligned} \tag{5}$$

where $\mathbf{r} \triangleq \{r, \ r_l(m), \ r_m \mid \forall \, l \in \mathcal{L}, \, m = 1 \sim M\}$. Problem (5) is difficult to solve because of the following reasons: (i) it is a nonconvex problem due to the rate constraints on wireless links; (ii) the bisection procedure for solving the max-min rate power allocation (beamformer) design [22] cannot be applied here, due to the existence of conservation constraints and the presence of multiple subchannels; (iii) the size of the problem can be huge. In the following, we propose an efficient distributed algorithm to compute a stationary solution of the problem (5).

## 3. A DISTRIBUTED ALGORITHM FOR THE JOINT POWER ALLOCATION AND ROUTING PROBLEM

In this section, we will propose a distributed algorithm that solves problem (5) to a stationary solution. The proposed algorithm is the combination of two algorithms: 1) the max-min WMMSE algorithm developed in [23] for minimum rate maximization in $M$-pair interference channel; 2) the ADMM algorithm that is used to distributively solve the multi-commodity routing problem. To achieve this goal, we first exploit the following rate-MSE relationship

**Lemma 1** *[23] For a given $l = (s, d, k) \in \mathcal{L}^{wl}$, $\bar{r}_l(\mathbf{p})$ can be equivalently expressed as*

$$\bar{r}_l(\mathbf{p}) = \max_{u_l, w_l} c_{1,l} + c_{2,l} p^k_{ds} - \sum_{n=(s',d',k)\in I(l)} c_{3,ln} |p^k_{d's'}|^2 \tag{6}$$

*where $(c_{1,l}, c_{2,l}, c_{3,ln})$ are given by $c_{1,l} = 1 + \log(w_l) - w_l(1 + \sigma^2_d |u_l|^2)$, $c_{2,l} = 2w_l \mathrm{Re}\{u^*_l h^k_{ds}\}$, and $c_{3,ln} = w_l |u_l|^2 |h^k_{ds'}|^2$.*

Note that Lemma 1 reformulates $\bar{r}_l(\mathbf{p})$ by introducing two extra sets of variables $\mathbf{u} \triangleq \{u_l \mid l \in \mathcal{L}^{wl}\}$ and $\mathbf{w} \triangleq \{w_l \mid l \in \mathcal{L}^{wl}\}$. The term inside the maximization operator is the MSE for estimating the message transmitted on link $l$. Given Lemma 1, we reformulate problem (5) by replacing $\bar{r}_l(\mathbf{p})$ in (5) with its MSE. We call such new constraint a *rate-MSE constraint*. Then, the following problem with two extra optimization variable sets $\mathbf{u}$ and $\mathbf{w}$ is solved instead:

$$\begin{aligned} \max \ & r \\ \text{s.t.} \ & r \geq 0, \ r_m \geq r, \ r_l(m) \geq 0, \ m = 1 \sim M, \ \forall \, l \in \mathcal{L}, \\ & (1), \ (3), \ \text{and} \ (4), \end{aligned} \tag{7}$$

$$\sum_{m=1}^{M} r_l(m) \leq c_{1,l} + c_{2,l} p^k_{ds} - \sum_{n=(s',d',k)\in I(l)} c_{3,ln} |p^k_{d's'}|^2, \quad \forall \, l \in \mathcal{L}^{wl}. \tag{8}$$

We can observe that for any given $\{\mathbf{r}, \mathbf{p}\}$, the optimal $\mathbf{u}$ (resp. $\mathbf{w}$) for (6) can be obtained while $\mathbf{w}$ (resp. $\mathbf{u}$) is held fixed. Moreover, these optimal solutions can be expressed in closed form for any $l \in \mathcal{L}^{wl}$:

$$u_l = \left(\sum_{(s',d',k)\in I(s,d,k)} | \, h^k_{ds'}|^2 |p^k_{d's'}|^2 + \sigma^2_d\right)^{-1} h^k_{ds} p^k_{ds}, \tag{9}$$

$$w_l = \left(1 - (h^k_{ds} p^k_{ds})^* u_l\right)^{-1}. \tag{10}$$

These expressions suggest that the set of variables $\mathbf{u}$ and $\mathbf{w}$ can be separately updated locally at each mobile user if the interference plus noise and local channel state information are locally known. On the other hand, when fixing $\mathbf{u}$ and $\mathbf{w}$, the problem for updating $\{\mathbf{r}, \mathbf{p}\}$ is convex. Therefore it can be solved in polynomial time. Therefore, we apply the alternating optimization technique to solve problem (7); see the N-MaxMin Algorithm in Table 1 for detailed steps. The following result states that the iterates $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by this algorithm converge to the stationary solutions of problem (5).

**Network Max-Min WMMSE (N-MaxMin) Algorithm**:

1: **Initialization** Generate a feasible set of variables $\{\mathbf{r}, \mathbf{p}\}$, and let $t = 1$.
2: **Repeat**
3: $\quad \mathbf{u}^{(t)}$ is updated by (9)
4: $\quad \mathbf{w}^{(t)}$ is updated by (10)
5: $\quad \{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ is updated by solving (14) using Algorithm 1.
6: $\quad t = t + 1$.
7: **Until** Desired stopping criteria is met

**Table 1**. Network Max-Min WMMSE (N-MaxMin) Algorithm

**Theorem 1** *[24] The sequence $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by N-MaxMin Algorithm converges to a stationary solution of problem* (5)*. Moreover, every global optimal solution of problem* (5) *corresponds to a global optimal solution of the reformulated problem* (7)*, and they achieve the same objective value.*

We note that our algorithm in Table 1 and the convergence analysis in Theorem 1 carry over easily to the multi-antenna case as well.

### 3.1. An ADMM Approach for Updating $\{\mathbf{r}, \mathbf{p}\}$

Unlike the computation of $\mathbf{u}$ and $\mathbf{w}$, the updates for $\{\mathbf{r}, \mathbf{p}\}$ requires solving convex optimization subproblems. We can use off-the-shelve toolboxes, but this is not the most efficient. In the sequel, we tailor the steps for updating $\{\mathbf{r}, \mathbf{p}\}$ to make the entire algorithm more scalable and suitable for distributed implementation. The key is to decompose the problem into easy subproblems, each of which is relatively easy to solve. The main difficulty here is to decouple the flow constrains (4) for the nodes and the rate-MSE constraints (8) for the wireless links.

To be more specific, the variables in the coupling constraints (4) and (8) can be decomposed by introducing a few slack variables. Consider the following slack variables: i) $\hat{r}_m^{S(m)} = r_m$ and $\hat{r}_m^{D(m)} = r_m$, $m = 1 \sim M$; ii) $\hat{r}_l^s(m) = r_l(m)$ and $\hat{r}_l^d(m) = r_l(m)$, $\forall\, l = (s, d) \in \mathcal{L}^w$(or $l = (s, d, k) \in \mathcal{L}^{wl}$). The flow rate conservation constraints on node $v \in \mathcal{V}$ can then be rewritten as

$$\sum_{l \in \mathrm{In}(v)} \hat{r}_l^v(m) + 1_{\{S(m)\}}(v)\hat{r}_m^v = \sum_{l \in \mathrm{Out}(v)} \hat{r}_l^v(m) + 1_{\{D(m)\}}(v)\hat{r}_m^v,\ m = 1 \sim M. \quad (11)$$

In addition, for the rate-MSE constraint, we introduce several copies of the transmit precoder on a given wireless link $l = (s, d, k) \in \mathcal{L}^{wl}$, i.e. $p_{d's',ds}^k = p_{ds}^k$, $l \in I(s', d', k)$. Intuitively, by doing such variable splitting, each variable $p_{d's',ds}^k$ will only appear in *a single* rate-MSE constraint. For a given link $l = (s, d, k) \in \mathcal{L}^{wl}$, its rate-MSE constraint only depends on the set of precoders $\{p_{ds,d's'}^k \mid \forall\, (s', d', k) \in I(l)\}$ as

$$\sum_{m=1}^{M} r_l(m) \le c_{1,l} + c_{2,l} p_{ds,ds}^k - \sum_{n=(s',d',k) \in I(l)} c_{3,ln} |p_{ds,d's'}^k|^2. \quad (12)$$

Moreover, for the analysis of the convergence result, another slack variable $\hat{r}$ is introduced such that $r = \hat{r}$. For notational simplicity, these equality relationship can be compactly expressed as

$$\hat{\mathbf{r}} = \tilde{\mathbf{r}},\ \hat{\mathbf{p}} = \tilde{\mathbf{p}}, \quad (13)$$

where $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{p}}$ are concatenated vectors obtained from the original flow rate and precoder variables, respectively; $\hat{\mathbf{r}}$ and $\hat{\mathbf{p}}$ are concatenated vectors of the slack variables.

**Algorithm 1: ADMM for** (14):

1: **Initialize** all primal variables $\mathbf{r}^{(0)}, \hat{\mathbf{r}}^{(0)}, \mathbf{p}^{(0)}, \hat{\mathbf{p}}^{(0)}$ (not necessarily to be a feasible solution for problem (14)); Initialize all dual variables $\boldsymbol{\delta}^{(0)}, \boldsymbol{\theta}^{(0)}$; set $t = 0$
2: **Repeat**
3: $\quad$ Solve the following problem and obtain $\mathbf{r}^{(t+1)}, \hat{\mathbf{p}}^{(t+1)}$:

$$\begin{aligned} &\max_{\mathbf{r}, \hat{\mathbf{p}}}\ L_{\rho_1,\rho_2}(\mathbf{r}, \hat{\mathbf{p}}, \hat{\mathbf{r}}^{(t)}, \mathbf{p}^{(t)}; \boldsymbol{\delta}^{(t)}, \boldsymbol{\theta}^{(t)}) \\ &\quad \text{s.t. } r \ge 0, r_m \ge r,\ r_l(m) \ge 0,\ m = 1 \sim M,\ l \in \mathcal{L}, \\ &\qquad \text{(1) and (12)} \end{aligned} \quad (15)$$

$\quad$ This step can be *solved in parallel across all links*.
4: $\quad$ Solve the following problem and obtain $\hat{\mathbf{r}}^{(t+1)}, \mathbf{p}^{(t+1)}$:

$$\begin{aligned} &\max_{\hat{\mathbf{r}}, \mathbf{p}}\ L_{\rho_1,\rho_2}(\mathbf{r}^{(t+1)}, \hat{\mathbf{p}}^{(t+1)}, \hat{\mathbf{r}}, \mathbf{p}; \boldsymbol{\delta}^{(t)}, \boldsymbol{\theta}^{(t)}) \\ &\quad \text{s.t. (11) and (3)} \end{aligned} \quad (16)$$

$\quad$ This problem can be *solved in parallel across all nodes*.
5: $\quad$ Update the Lagrange dual multipliers $\boldsymbol{\delta}^{(t+1)}$ and $\boldsymbol{\theta}^{(t+1)}$ by

$$\begin{aligned} \boldsymbol{\delta}^{(t+1)} &= \boldsymbol{\delta}^{(t)} - \rho_1(\hat{\mathbf{r}}^{(t+1)} - \tilde{\mathbf{r}}^{(t+1)}), \\ \boldsymbol{\theta}^{(t+1)} &= \boldsymbol{\theta}^{(t)} - \rho_2(\tilde{\mathbf{p}}^{(t+1)} - \hat{\mathbf{p}}^{(t+1)}). \end{aligned} \quad (17)$$

6: $\quad t = t + 1$
7: **Until** Desired stopping criteria is met

**Table 2**. Summary of the proposed Algorithm 1

Hence, the updating step for $\{\mathbf{r}, \mathbf{p}\}$ is equivalently expressed as

$$\begin{aligned} &\max\ (r + \hat{r})/2 \\ &\quad \text{s.t. } r \ge 0, r_m \ge r,\ r_l(m) \ge 0,\ m = 1 \sim M,\ l \in \mathcal{L} \\ &\qquad \text{(1), (12), (11), (3), and (13).} \end{aligned} \quad (14)$$

It is important to note that the constraints of problem (14) (except the linear equality constraints (13)) are now separable between two optimization variable sets *i)* the tuple $\{\mathbf{r}, \hat{\mathbf{p}}\}$ and *ii)* the tuple $\{\hat{\mathbf{r}}, \mathbf{p}\}$. Additionally, the objective function is linear and separable over $\mathbf{r}$ and $\hat{\mathbf{r}}$. Therefore the ADMM algorithm can be used to solve problem (14). The resulting algorithm, listed in Table 2, is referred to as Algorithm 1. Note that the partial augmented Lagrange function for problem (14) is given by

$$\begin{aligned} L_{\rho_1,\rho_2}(\mathbf{r}, \hat{\mathbf{p}}, \hat{\mathbf{r}}, \mathbf{p}; \boldsymbol{\delta}, \boldsymbol{\theta}) = r &+ \left[\boldsymbol{\delta}^T(\hat{\mathbf{r}} - \tilde{\mathbf{r}}) - \frac{\rho_1}{2}\|\hat{\mathbf{r}} - \tilde{\mathbf{r}}\|^2\right] \\ &+ \left[\boldsymbol{\theta}^H(\tilde{\mathbf{p}} - \hat{\mathbf{p}}) - \frac{\rho_2}{2}\|\tilde{\mathbf{p}} - \hat{\mathbf{p}}\|^2\right], \end{aligned}$$

where $\rho_1 > 0$ and $\rho_2 > 0$ are, respectively, some constant coefficients for each of the linear equality constraints (13); $\boldsymbol{\delta}$ and $\boldsymbol{\theta}$ are the Lagrange dual variables for (13). Moreover, by appealing to the standard analysis for ADMM algorithm [16, 17], we can show that Algorithm 1 converges to the optimal solutions of problem (14).

**Theorem 2** *[24] Every limit point of the sequence $\{\mathbf{r}^{(t)}, \mathbf{p}^{(t)}\}$ generated by Algorithm 1 is an optimal solution of problem* (14)*.*

It is important to note that steps (3)–(5) are all separable over the nodes/links of the network, and each of them can be updated in (semi)closed-form (see the journal version [24] for details). Moreover, when the following assumptions are made, the entire algorithm can be carried out in a distributed manner: *i)* each mobile user has local channel state information from all interfering BSs; *ii)* there exists

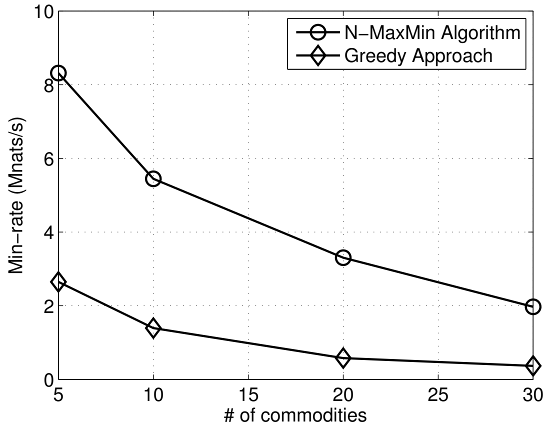


**Fig. 1**. The minimum rate achieved by N-MaxMin algorithm and the greedy heuristic algorithm for different number of commodities. We have $\bar{p} = 20$dB.

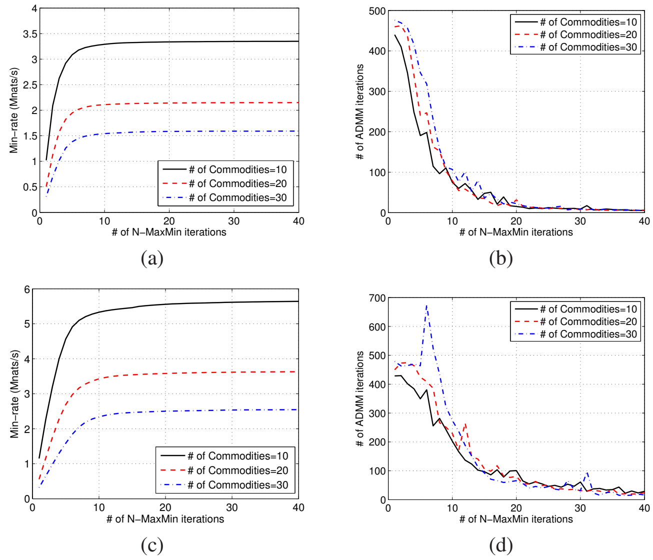


**Fig. 2**. The minimum rate performance and the required number of iterations for the proposed N-MaxMin algorithm. In [(a)(b)] $\bar{p} = 10$dB and in [(c)(d)] $\bar{p} = 20$dB. In [(a)(c)], the obtained minimum rate versus the iterations of N-MaxMin is plotted. In [(b)(d)], the required number of inner ADMM iterations is plotted versus the iteration for the outer N-MaxMin algorithm.

a master node which can communicate with the soure/destination node of each commodity; and *iii)* the destination node of each link performs the updating step for that link and communicates the result to the neighboring nodes. Condition *i)* allows $\mathbf{u}$ and $\boldsymbol{w}$ to be updated at the mobile users, while conditions *ii)-iii)* allow for distributed implementation of steps 3)–5) in Table 2.

## 4. SIMULATION RESULTS

In this section, we report numerical results on the performance of the proposed algorithms when applied to a HetNet with 57 BSs and 11 network routers within the area of $1200m \times 1600m$. The capacity of each wired link is given in the range between 2Mnats/s to 1Gnats/s for both directions. The number of subchannels is $K = 3$ and each subchannel has 1 MHz bandwidth. The power budget for each BS is chosen equally by $\bar{p} = p_s, \forall\ s \in \mathcal{B}$, and $\sigma_d^2 = 1, \forall\ d \in \mathcal{U}$. The wireless links follow the Rayleigh distribution with $CN(0, (200/dist)^3)$, where $dist$ is the distance between BS and the corresponding user. The source (destination) node of each commodity is randomly selected from network routers (mobile users), and all simulation results are averaged over 100 randomly selected end-to-end commodity pairs. Below we refer to one round of the N-MaxMin iteration as an *outer iteration*, and one round of Algorithm 1 for solving $(\mathbf{r}, \mathbf{p})$ as an *inner iteration*.

In the first experiment, we assume that all mobile users are served by BSs within 300 meters and are interfered by all BSs. For this problem, the parameters of N-MaxMin algorithm are set to be $\rho_1 = 0.1$ and $\rho_2 = 0.001$; the termination criterion is $\frac{(r^{(t+1)}+\hat{r}^{(t+1)})-(r^{(t)}+\hat{r}^{(t)})}{r^{(t)}+\hat{r}^{(t)}} < 10^{-3}$ and $\max\{\|\tilde{\mathbf{r}}^{(t)} - \hat{\mathbf{r}}^{(t)}\|_\infty, \|(\tilde{\mathbf{p}}^{(t)})^2 - (\hat{\mathbf{p}}^{(t)})^2\|_\infty\}\} < 5 \times 10^{-4}$, where $(\tilde{\mathbf{p}}^{(t)})^2$ represents elementwise square operation. We compare the performance of the proposed algorithm with a greedy heuristic approach, in which each mobile user $u \in \mathcal{U}$ is served by a *single* BS via a *single* subchannel. Such serving BS and the subchannel is chosen greedily as the one with the strongest channel strength among all BSs and all channels for user $u \in \mathcal{U}$. After BS-user association is decided, each BS uniformly allocates its power budget to all tones, and subsequently to all users occupying the same tone. With the obtained power allocation and BS-user association, the capacities of all wireless links are known and fixed, so the minimum rate of the commodities can be optimized by solving a standard multi-commodity routing problem with known channel capacities. In Fig. 1, we show the minimum rate performance of different algorithms when $\bar{p} = 20$dB and $M = 5 \sim 30$. We observe that the minimum rate achieved by the N-MaxMin algorithm is more than twice that of the heuristic algorithm. This suggests that proper power allocation and BS-user association is needed for problem (5).

In the second set of numerical experiments, we evaluate the proposed N-MaxMin algorithm using different number of commodity pairs and different power budgets at the BSs. Here we use the same settings as in the previous experiment, except that all mobile users are interfered by the BSs within a distance of 800 meters, and that we set $\rho_2 = 0.005$ (resp. $\rho_2 = 0.001$) when $\bar{p} = 10$ dB (resp. $\bar{p} = 20$ dB). The minimum rate performance for the N-MaxMin algorithm and the required number of inner iterations are plotted in Fig. 2. Due to the fact that the obtained $\{\mathbf{r}, \mathbf{p}\}$ is far from the stationary solution in the first few outer iterations, there is no need to complete Algorithm 1 at the very beginning. Hence, we limit the number of inner iterations to be no more than 500 for the first 5 outer iterations. After the early termination of the inner Algorithm 1, we use the obtained $\mathbf{p}$ to update $\mathbf{u}$ and $\mathbf{w}$ by (9) and (10), respectively.

In Fig. 2(a)–(b), we see that when $\bar{p} = 10$ dB, the minimum rate converges at about the 10th outer iteration when the number of commodities is up to 30, while less than 500 inner iterations are needed per outer iteration. Moreover, after the 10th outer iteration, the number of inner ADMM iterations reaches below 100. In Fig. 2(c)–(d), the case with $\bar{p} = 20$dB is considered. Clearly the required number of outer iterations is slightly more than that in the case of $\bar{p} = 10$dB, since the objective value and the feasible set are both larger. However, in all cases the algorithm still converges fairly quickly.

In summary, simulation results show that the proposed N-MaxMin algorithm significantly outperforms the greedy heuristic in term of the achieved min flow rate. Furthermore, all steps of Algorithm 1 have (semi)closed-form solutions, and they can be independently computed in parallel across all nodes/links of the considered network.

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