A REFINED TIME-FREQUENCY REASSIGNMENT TECHNIQUE APPLIED TO DOLPHIN ECHOLOCATION SIGNALS

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ABSTRACT

Existing time-frequency representations are usually too noise-sensitive or has too bad resolution for studying multiple transient signal components of biosonar signals. In this paper we calculate reassigned spectrograms using the first Hermite function as window. The optimal length Hermite function giving the perfectly localized reassignment for a Gaussian function is combined with a novel technique which includes the reassigned spectrogram using another length of the Hermite window function. For this window function the reassignment is rescaled to achieve perfect localization. The geometric mean of the two reassigned spectrograms is taken as the final estimate. The proposed method is evaluated for localization properties and noise reduction and is applied to synthetic echolocation data.

Index Terms— time-frequency, reassignment, localization, Hermite function, transient, biosonar, dolphin

1. INTRODUCTION

The *reassignment principle* was introduced in [1], but was not applied to a larger extent until it was reintroduced in [2]. The idea of reassignment is to keep the localization of a single component by reassigning mass to the center of gravity. For multi-component signals, the reassignment improves the readability as the cross-terms are reduced by a smoothing of the specific distribution and the reassignment then squeezes the signals terms. However, the reassignment technique can be sensitive to noise disturbances and reassigned multitaper spectrograms has also been proposed for noise reduction, [3]. Recently, the theoretical expressions for the reassigned Gabor spectrograms of Hermite functions have been derived in [4, 5]. Based on these, we propose a novel technique using a half-length optimal Hermite function window, where the reassignment procedure is rescaled to achieve the perfectly localized spectrogram. The geometric mean of the reassigned optimal window spectrogram and the rescaled reassigned half-length window spectrogram is taken as the the final estimate. The proposed method is robust for noise disturbances and has good resolution properties.

Previous studies have shown that time-frequency representations of transient biosonar signals are valuable tools when analyzing details in the signals generated by for instance bottlenose dolphins, [6], and that localization and time-frequency resolution are important factors to optimize. This is relevant in marine biosonar studies when comparing component azimuths of echolocation signals (clicks) recorded in different positions within the echolocation beam. Recent biosonar studies have also shown that the measured echolocation signals from bottlenose dolphins, (*Tursiops truncatus*) and beluga whales, (*Delphinapterus leucas*) may also contain more than one transient component, [7, 8]. Existing time-frequency representations are usually too noise-sensitive or has too bad resolution for studying these possibly multiple transient signal components. A better method would be useful when trying to establish whether certain echolocation signals originate from one or two sound sources, a current topic of debate in this field of research, e.g., [8, 9, 10].

In section 2, the novel technique of rescaled reassigned spectrogram of a Hermite function windowed Gaussian signal is presented. Section 3 evaluates the performance of the new method and Section 4 presents the results for an example of a synthetic two-component signal composed of short Gaussian windowed linear chirps that is constructed to be similar as a real data measured from a bottlenose dolphin. Section 5 concludes the paper.

2. REASSIGNED SPECTROGRAMS

A Gaussian windowed constant frequency signal

$$x(t) = g(t - t_0)e^{-i\omega_0 t}, \quad (1)$$

where the unit-energy Gaussian function is

$$g(t) = \pi^{-\frac{1}{4}} e^{-\frac{1}{2}t^2}, \quad -\infty < t < \infty \quad (2)$$

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is often used to model a short non-stationary signal. The quadratic class of distributions obey time-frequency shift-invariance $S_x(t-t_0,\omega-\omega_0) = S_g(t,\omega)$, meaning that the further analysis can be restricted to $x(t) = g(t)$. The magnitude of the short-time Fourier transform for the signal in Eq. (2) applying a Hermite function window is, [4],

$$M_g^{h_k}(t,\omega) = \frac{1}{\sqrt{2^{k-1}(k-1)!}}(t^2+\omega^2)^{\frac{(k-1)}{2}} e^{-\frac{1}{4}(t^2+\omega^2)}, \tag{3}$$

and the spectrogram is found as

$$SP_g^{h_k}(t,\omega) = |M_g^{h_k}(t,\omega)|^2. \tag{4}$$

The corresponding reassigned spectrogram is

$$ReSP_g^{h_k}(t,\omega) =$$

$$= \frac{1}{2\pi}\iint_{-\infty}^{\infty} SP_g^{h_k}(s,\xi)\delta(t-\hat{t}(s,\xi),\omega-\hat{\omega}(s,\xi))dsd\xi, \tag{5}$$

where for spectrograms based on the Hermite function windows of a Gaussian signal the more recent formulation, [5], can be used, i.e.,

$$\hat{t}(t,\omega) = t + \frac{\partial}{\partial t}\log M_g^{h_k}(t,\omega) \tag{6}$$

$$\hat{\omega}(t,\omega) = \omega + \frac{\partial}{\partial \omega}\log M_g^{h_k}(t,\omega). \tag{7}$$

Only the first Hermite function leads to perfect localization when the reassignment technique is applied. For all Hermite functions $k > 1$, the reassigned spectrograms will be circles, [4]. Therefore we restrict to the first Hermite function for further use in the reassignment procedure. For the first Hermite function

$$h_1(t) = \frac{1}{\sqrt{\sqrt{2\pi}}}e^{-\frac{t^2}{2}}$$

circular symmetry gives

$$SP_g^{h_1}(t,\omega) = e^{-\frac{1}{2}(t^2+\omega^2)} \tag{8}$$

and perfect localization using the reassignment operator, [4]. The reassignment procedure using the optimal Hermite function $h_1(t)$ is named (ReSP1).

2.1. A refined reassigned spectrogram

We illustrate the rescaling approach by using a window of half the length of the original one, i.e.,

$$h_1'(t) = \frac{1}{\sqrt{2\sqrt{2\pi}}}e^{-\frac{t^2}{8}},$$

and calculate the corresponding spectrogram of the Gaussian function as

$$\begin{aligned} SP_g^{h_1'}(t,\omega) &= \frac{1}{2\sqrt{2}\pi}\left|\int_{-\infty}^{\infty} e^{-\frac{s^2}{8}}e^{-\frac{(s-t)^2}{2}}e^{-i\omega s}ds\right|^2 \\ &= \frac{1}{2\sqrt{2}\pi}e^{-t^2}\left|\int_{-\infty}^{\infty} e^{-\frac{5s^2}{8}}e^{(t-i\omega)s}ds\right|^2 \\ &= \frac{1}{2\sqrt{2}\pi}\frac{8\pi}{5}e^{-t^2}e^{\frac{8t^2}{10}-\frac{8\omega^2}{10}} \\ &= \frac{8}{10\sqrt{2}}e^{-\frac{1}{2}(\frac{2}{5}t^2+\frac{8}{5}\omega^2)}. \end{aligned} \tag{9}$$

The scaling of the time-axis is $5/2$ and the frequency axis $5/8$. For the Gaussian function in Eq. (2), the reassignment of Eq. (5) will not be perfectly localized. But as we now have knowledge of the actual error in the time-frequency domain introduced by the scaled Hermite function, the reassignment procedure can be compensated accordingly. Compensating with these factors in the reassignment,

$$ReSP_g^{h_1'}(t,\omega) =$$

$$= \frac{1}{2\pi}\iint_{-\infty}^{\infty} SP_g^{h_1'}(s,\xi)\delta(t-\frac{5}{2}\hat{t}(s,\xi),\omega-\frac{5}{8}\hat{\omega}(s,\xi))dsd\xi, \tag{10}$$

will give us a another perfectly localized spectrum using the half-width window (ReSP2). The two window functions $h_1(t)$ and $h_1'(t)$ both give reassigned spectra that are perfectly localized. However, they have different properties regarding the resolution as well as reassignment of noise components. An old spectrogram approach presented in [11], suggests that the geometric mean of spectrograms increase the resolution when different lengths of the spectrogram windows are applied. Similarly, we suggest here a geometric mean of reassigned spectrograms with different *scaling* of the Hermite function window. Of course, a solution to Eq. (10) could be found for any other length and we could certainly combine more than two windows and in different ways for the final multiplied reassigned spectrograms. However, we limit the investigation in this paper to using the geometric combination of two reassigned spectrograms, i.e.

$$MReSP_g(t,\omega) = ReSP_g^{h_1}(t,\omega)\cdot ReSP_g^{h_1'}(t,\omega), \tag{11}$$

named as Multiple Reassigned Spectrograms (MReSP).

3. SIMULATIONS

We illustrate the performance of the proposed method for a time- and frequencyshifted Gaussian function with $t_0 = 256$ and $\omega_0 = 2\pi 0.125$. The corresponding optimal Hermite function is applied as the window $h_1(t)$ and the half-width Hermite function as the window $h_1'(t)$. Stationary white Gaussian noise giving an total SNR of 0 dB is added to the signal. In Figure 1, examples of the resulting time-frequency spectra are seen. The localization of all the reassigned spectrograms (ReSP1, ReSP2 and MReSP) are clearly seen compared to the usual spectrogram method using the window $h_1(t)$, (SP). The difference in localization of some of the noise components between ReSP1 and ReSP2 is seen and the combination of these two spectra to the MReSP keeps the localization from these methods but does also reduce the contributing noise.

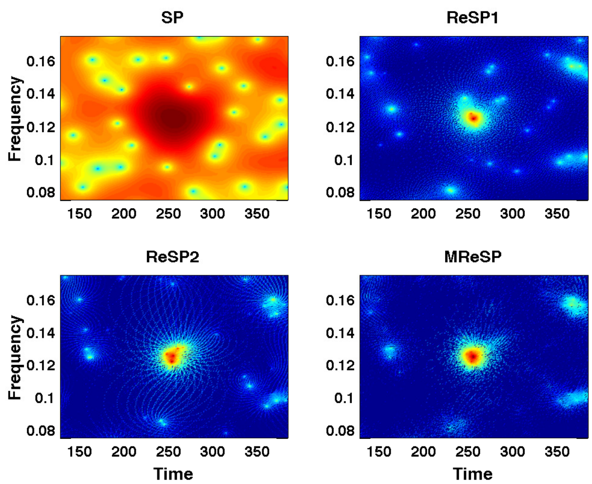

Fig. 1. Example of the performance of the different methods for a Gaussian signal disturbed by white Gaussian noise with SNR=0 dB.

We evaluate the performance of the different algorithms for different SNR. The focus is on concentration and the evaluation is made in the square limited by the axes seen in Figure 1, i.e., time-interval 128 to 384 and frequency interval $2\pi 0.075$ to $2\pi 0.175$ using the Rényi entropy of order α,

$$R_\alpha(P) = \frac{1}{1-\alpha} \log_2 \int_{t_0}^{t_1} \int_{\omega_0}^{\omega_1} (P(t,\omega))^\alpha dt d\omega, \qquad (12)$$

for $\alpha > 0$, and any energy normalized time-frequency distribution $P(t, \omega)$, [3, 12, 13] . The Rényi entropy is calculated for the often used $\alpha = 3$. We estimate the spectrograms for 100 different realizations of the signal disturbed by noise. The Rényi entropy is computed for each estimate and the average is depicted in Figure 2. The results clearly show that the best concentration is given from the MReSP, which also can be seen to be more robust for a higher noise disturbance (lower SNR) than the ReSP1 and ReSP2. The ReSP1 outperforms as expected the ReSP2 as the window function $h_1(t)$ is more optimal to the specific Gaussian signal than the window function $h_1'(t)$. The usual spectrogram (SP) has as expected a concentration that is not comparable to the reassigned techniques.

The optimal result in Figure 2 will be degraded if the model assumption of the Gaussian function is wrong compared to the actual signal. To evaluate this degradation, a simulation is performed where the resulting difference between the assumed Gaussian function of the methods and the actual one in the estimate is investigated. The performance is evaluated for the Gaussian function without disturbance. For the scaling factor $SC = 1$, the optimal Hermite function windows, $h_1(t)$ and $h_1'(t)$, are applied in the estimation procedure. All reassigned methods will give a Rényi entropy that is zero, i.e., all values are reassigned to a single point. Assuming the Gaussian signal to be half the actual length, (SC=0.5), or twice as wide, (SC=2), show that the performance degrades even for smaller deviations. However, the MReSP still gives a smaller value of the Rényi entropy than using the ReSP1 or ReSP2. The usual spectrogram (SP) is quite robust to the scaling which is natural as a narrow window in time contributes with additional widening of the window in frequency.

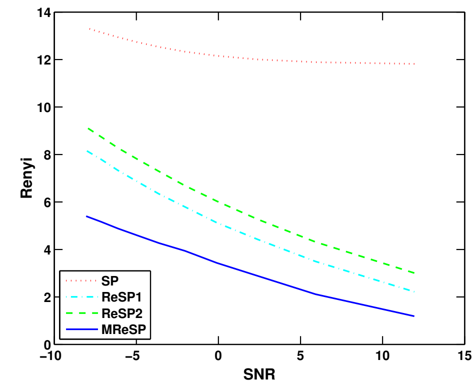

Fig. 2. The average Rényi entropy ($\alpha = 3$) from 100 simulations for the different methods and different SNR for a Gaussian function disturbed by white noise.

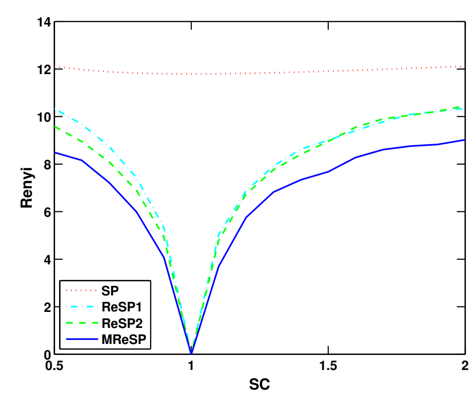

Fig. 3. The Rényi entropy of the different methods applied to the noise-free Gaussian function when the assumption of the signal length changes from half, (SC=0.5), to twice, (SC=2), the length that it actually has.

4. APPLICATION TO TRANSIENT BIOSONAR SIGNALS

The test signal illustrated in Figure 4 is a synthetic two-component signal composed of short Gaussian windowed linear chirps that is constructed to be similar as a real data measured from a bottlenose dolphin. We focus on to resolve the two components in a disturbance of white Gaussian noise. The results of the example signal are shown in Figure 5 for the different methods where also a comparison is made to the result of the Choi-Williams distribution (CW), [14], and the Reassigned Smoothed Pseudo Wigner-Ville Distribution (ReSPWVD), [2]. For all methods, the parameters giving the smallest Rényi entropy are chosen.

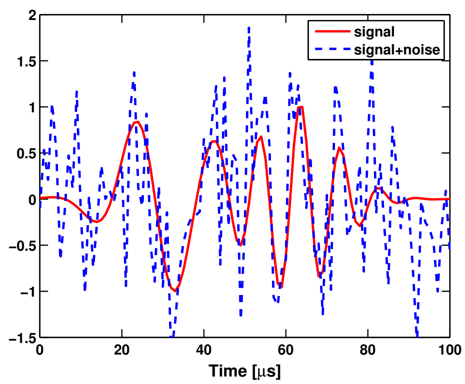

Fig. 4. A synthetic two-component signal composed of Gaussian short windowed linear chirps that is constructed to be similar as a real data measured from a bottlenose dolphin. A disturbance of SNR=-3 dB.

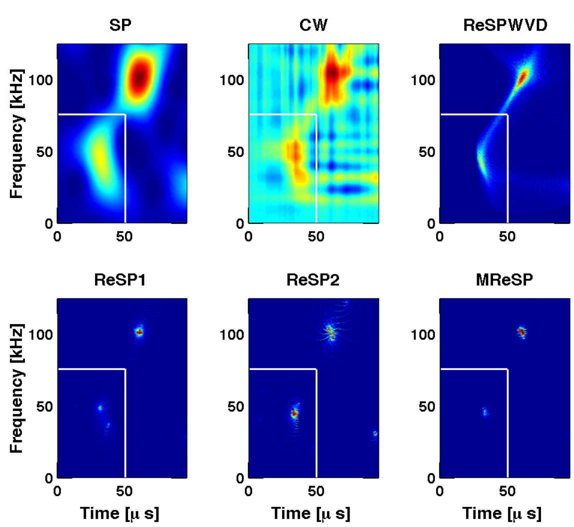

Fig. 5. Example of different time-frequency spectra for the signal in Figure 4.

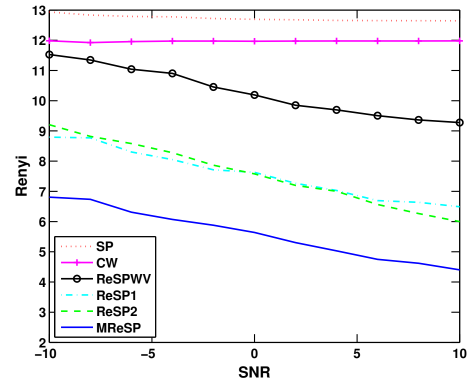

Fig. 6. The performance of the methods for the signal in Figure 4 for different SNRs. The Rényi entropys are averages from the results of 100 simulations.

The Rényi entropies of the example signal are measured for the low frequency component localized in the lower left corner of the time-frequency plots limited by the white lines in each figure and is evaluated for different amounts of white noise disturbance. We estimate the spectrograms for the signal disturbed by 100 different realizations of the noise and calculate the Rényi entropy for each simulation. The average is shown in Figure 6 for different SNR. The spectrogram (SP) and the Choi-Willams distribution (CW) both have bad performance followed by the ReSPWVD. Both ReSP1 and ReSP2 give similar Rényi entropy but the lowest values are in all cases given by MReSP, showing the advantage in localization as well as robustness against disturbances.

5. CONCLUSIONS

The results indicate that the refined reassignment technique, exemplified by the MReSP, can be used to precisely localize multiple signal components in both time and frequency in noisy environments. It thus holds great potential to be a valuable tool in e.g., biosonar research where small time and frequency differences between multiple echolocation signal component needs to be characterized. It will also be useful to incorporate this method in applications where voice tracking of individual dolphins is necessary, e.g. where the individual echolocation behavior is studied in large groups of animals, echolocating concurrently at closely spaced objects.

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