TOMOGRAPHIC TECHNIQUES FOR THE RETRIEVAL OF TROPOSPHERIC WATER VAPOUR FIELDS BY USING CO–ROTATING LEO SATELLITES

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ABSTRACT

This paper presents a study for the estimation of 2–D maps of atmospheric water vapour content from integrated water vapour measurements carried out by a constellation of co–rotating low earth orbit satellites. The proposed method uses the normalised differential spectral attenuation (NDSA) approach – able to achieve integrated water vapour content information from attenuation measurements over microwave links among the satellites – and tomographic techniques to solve the inverse problem of atmospheric water vapour field reconstruction. Simulation results demonstrate the feasibility of the proposed approach to retrieve a 2–D map of the atmospheric water vapour content. This study has been developed under the framework of the ANISAP project funded by the European Space Agency.

***Index Terms*—** Tropospheric water vapour content estimation, normalised differential spectral attenuation (NDSA), co-rotating LEO satellites, tomography, inverse problems.

1. INTRODUCTION

The retrieval of the water vapour (WV) content in the troposphere is an important problem in the field of remote sensing and meteorology. The WV content, along with temperature and pressure, influences the attenuation of a propagating electromagnetic wave, so that information about the WV content can be achieved from attenuation measurements along a link crossing the troposphere. A major problem for a direct use of attenuation measurements stems from the presence of the scintillation phenomenon, due to tropospheric turbulence, that may cause important random variations on the measured values. To overcome this problem, the normalised differential spectral attenuation (NDSA) method has been introduced and analysed in the Ku/K bands [1][2][3]. The method is based on the different attenuation encountered by two transmitted tones at closely spaced frequencies. It can be demonstrated that such measurements are strongly correlated with the integral water vapour (IWV) content along the path [4]. The method has been originally proposed for a couple of counter-rotating (COUNT–RO) low Earth orbit (LEO) satellites (one carrying a transmitter, the other a receiver) operating in the Ku/K bands [2]; the use of additional frequencies (179 and 182 GHz) has been proposed to estimate IWV up to 15 km [4]. A vertical profile of the WV can then be achieved from the vertical IWV profile.

In this paper, we analyse the possibility of applying the NDSA measurement approach to estimate a 2–D WV field by means of a constellation of co-rotating (CO–RO) satellites. The CO–RO scenario assumes that one (or more) transmitting (TX) satellites follow, in the same orbit and in the same direction, one (or more) receiving (RX) satellites. Fig. 1 depicts the scenario of a single transmitting and receiving satellite (circular orbits and Earth shape are assumed for the sake of simplicity). The positions of the satellites are chosen so that the link crosses the atmosphere at a given minimum altitude (h_m) from the Earth. The radio link (dashed line) crosses the troposphere yielding an integral attenuation measurement that is a function of the local WV content encountered along the path. During satellites movement (clock–wise in Fig. 1), the path spans an annular area in the orbital plane. The goal is estimating, from these IWV measurements, the 2–D WV field lying in the annular region of altitude from h_m to h_M, where h_M represents the maximum altitude of the Earth atmosphere. The single link scenario can be generalised to the case of more TX and/or RX satellites, which allows a denser atmosphere sounding to be achieved. In principle, if each point of the region of interest has been crossed at least once (that is, its contribution is present in at least one occurrence of the entire measurement set) the reconstruction of the 2–D WV field can be formulated as an inverse problem. A similar scenario is encountered in the tomographic reconstruction of images. In our case, however, unlike in classical tomographic methods used in biomedical imaging, powerful mathematical tools, such as the radon transform, cannot be applied.

The main difference between the CO–RO and COUNT–RO scenarios is that a 2–D, instead than a 1–D, map of WV can be achieved. Furthermore, it can be shown [5] that higher integration times can be used (and, then, higher SNRs can be

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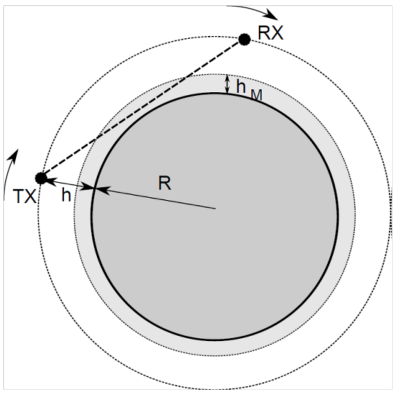

Fig. 1. CO–RO single link scenario: R is Earth's radius, h_M is the maximum atmosphere altitude, h is the altitude of satellites orbit.

achieved) to estimate the received power over the propagation path without affecting too much the spatial resolution of the retrieved map.

Some simulation results are presented in order to assess the performances of the proposed method. A synthetic WV field is constructed by means of interpolation of true radiosonde data. Using realistic models of attenuation and scintillation, the IWV measurements are simulated and then the retrieval of the actual WV map, formulated as an inverse problem, is performed. Simulation results demonstrate the feasibility of the method and highlight performance limits.

The paper is organised as follows. In Section 2, the NDSA method is reviewed. In Section 3, the signal model of the CO-RO acquisition system is described. Solutions for the reconstruction of the underlying 2–D WV field from IWV measurements are proposed in Section 4. Section 5 presents the experimental results and Section 6 draws some concluding remarks.

2. THE NDSA MEASUREMENT METHOD

The NDSA method requires the simultaneous measurement of the overall attenuation encountered by two electromagnetic waves propagating in the atmosphere at two frequencies spaced by Δf (spectral separation). Let S be the *spectral sensitivity* parameter defined by [2]

$$S = \frac{1}{\Delta f} \frac{P_2 - P_1}{P_2} \tag{1}$$

where P_1 and P_2 are the received powers deriving from two simultaneously transmitted tones, $s_1(t)$ and $s_2(t)$, with frequencies $f_1 = f_0 + \Delta f/2$ and $f_2 = f_0 - \Delta f/2$ respectively, where f_0 is the frequency which the spectral sensitivity is referred to. It can be shown [2] that S is linearly correlated with the IWV content along the radio link. Let $\hat{S}$ be an estimator of S defined by

$$\hat{S} = \frac{1}{\Delta f} \left(1 - \frac{\hat{P}_1}{\hat{P}_2}\right) \tag{2}$$

where $\hat{P}_i$, $i = 1, 2$, are estimators of the received power over the two radio channels. For these quantities, we use the following definition:

$$\hat{P}_i = \frac{r_i^2}{2} - \frac{\sigma_n^2}{2} \tag{3}$$

where r_i is the coherently detected amplitude of the signal received on the ith channel and σ_n^2 is the variance of thermal noise, as detailed below. The variable r_i is obtained by mixing the input signal with a synchronously generated tone and integrating or lowpass filtering. The quantity r_i can be expressed by [6]

$$r_i = A_i \int_0^{T_s} \chi_i(t) dt + n_i \tag{4}$$

where:

- $\chi_i(t)$ refers to the tropospheric scintillation disturbance, which is generated by diffraction effects related to fluctuations of the refraction index [7][8]. It is assumed that $\chi_1(t)$ and $\chi_2(t)$ are wide sense stationary processes, identically log-normal distributed and correlated, with log-amplitude variance defined as

$$\sigma_\chi^2 = \mathrm{Var}\left[20 \log_{10}\left(\frac{\chi_i(t)}{\mathrm{E}[\chi_i(t)]}\right)\right]. \tag{5}$$

  The scintillation phenomenon is a slowly time-varying process, with a maximum bandwidth B_χ of a few Hertz. Moreover, the zero-lag correlation coefficient ρ between $\chi_1(t)$ and $\chi_2(t)$ can be assumed close to unity [9];
- n_i is the additive thermal noise, assumed as zero-mean, white and Gaussian (AWGN), independent of the signal, uncorrelated among the channels and having variance σ_n^2;
- A_i is the attenuation coefficient (assuming, without loss of generality, unitary transmitted amplitude). It is due to free-space propagation loss, antenna gains, and other factors, among which the attenuation derived from the WV content is the important information we would like to extract from the S measurements;
- T_s is the receiver integration time.

Substituting (4) and (3) into (2) yields the estimator of the spectral sensitivity S. The IWV measurement is achieved from $\hat{S}$ after using a linear transformation [2].

3. CO-RO ACQUISITION SYSTEM MODEL

In order to retrieve the 2–D WV map from a set of IWV measurements, we need to model the acquisition system. For the sake of clarity, we will describe the case of a single link scenario, which can be easily extended to a general number of

TX and RX satellites. Furthermore, in this section, we will neglect the presence of scintillation.

Let $f(r, \theta)$ be the 2–D field, expressed in polar coordinates, representing the WV map to be reconstructed from integral measurements. In order to obtain a discrete model, hereafter we consider f on a sampled polar grid; moreover, we will use discrete computation of the integrals along the paths. Let $k = 0, 1, ...K - 1$, be the index over the IWV measurements set and let u_k be the variable spanning the points over the path relative to the kth acquisition. Then, the kth IWV measurement is given by

$$m_k = \int f(u_k) du_k + e_k \sim \sum_i f(u_{k,i}) \Delta u + e_k \quad (6)$$

where $u_{k,i}$ is the ith sample along u_k; Δu is the sampling step along the line; e_k represents the total error on the kth measurement.

Since, in general, $u_{k,i}$ does not belong to the sampled grid of f, the evaluation of $f(u_{k,i})$ has to be performed by means of interpolation. We have chosen linear interpolation based on the four samples surrounding $u_{k,i}$, as shown in Fig. 2. Hence, we have

$$f(u_{k,i}) = \sum_{p=0}^{1} \sum_{q=0}^{1} a_{k,i,p,q} f(r_{k,i} + p\Delta r, \theta_{k,i} + q\Delta\theta) \quad (7)$$

where $r_{k,i}$ and $\theta_{k,i}$ are points belonging to the discrete polar grid of f; Δr and $\Delta\theta$ are the radial and angular resolution of the grid, respectively; and the coefficients $a_{k,i,p,q}$ are the weights of the linear interpolation, which can be easily computed from the position of $u_{k,i}$ with respect to the grid points.

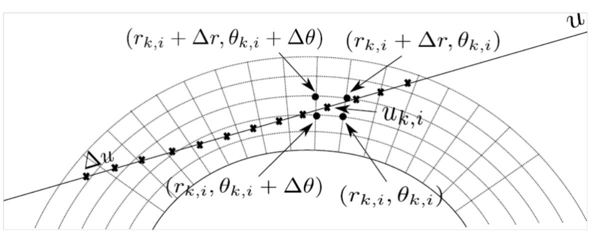

Fig. 2. Sampling scheme for a single measurement in the CO-RO scenario.

By substituting (7) into (6) and by considering the whole set of K IWV measurements expressed in matrix form, we have

$$\mathbf{m} = \mathbf{A}\mathbf{f} + \mathbf{e} \quad (8)$$

where $\mathbf{m}$ and $\mathbf{e}$ are $K \times 1$ vectors containing the IWV values m_k and the error terms e_k, respectively; $\mathbf{f}$ contains the WV values over the grid, organised in vector form ($\mathbf{f}$ is the unknown we would like to achieve); the system matrix $\mathbf{A}$ contains the weights of the interpolation.

4. INVERSION OF IWV MEASUREMENTS AND 2–D WV FIELD RECONSTRUCTION

Given the acquisition model in (8), a simple solution to estimate the vector $\mathbf{f}$ from the observation of the IWV measurements $\mathbf{m}$ is represented by the least square (LS) estimator, whose expression is

$$\hat{\mathbf{f}}_{\text{LS}} = \mathbf{A}^{+}\mathbf{m} \quad (9)$$

where $\mathbf{A}^{+}$ is the Moore–Penrose pseudo-inverse of $\mathbf{A}$. The estimator in (9) yields the minimum of the LS error function

$$\mathcal{E}_{\text{LS}} = \|\mathbf{A}\mathbf{f} - \mathbf{m}\|_2^2 \quad (10)$$

which, in general, does not guarantee any optimality in terms of bias and variance of the estimator; a rigorous approach to obtain optimal estimators - in some sense - would require the knowledge of the probability density function of the error term $\mathbf{e}$, which represents all the impairments affecting the IWV measurement, as reported in Section 2.

Since the system matrix $\mathbf{A}$ is usually ill-conditioned, its lowest singular values have to be zeroed in order to reduce the sensitivity to measurement errors of the LS estimator. The alternative estimator that we consider here is based on Tikhonov regularisation (TIK) and is given by

$$\hat{\mathbf{f}}_{\text{TIK}} = \left(\mathbf{A}^{\text{T}}\mathbf{A} + \lambda\mathbf{L}^{\text{T}}\mathbf{L}\right)^{+} \mathbf{A}^{\text{T}}\mathbf{m} \quad (11)$$

where $\mathbf{L}$ is a regularisation matrix which represents a prior knowledge on $\mathbf{f}$ and λ is a weighting factor. The estimator in (11) can be seen as a generalisation of the LS estimator, since it is the solution of the functional

$$\mathcal{E}_{\text{TIK}} = \|\mathbf{A}\mathbf{f} - \mathbf{m}\|_2^2 + \lambda\|\mathbf{L}\mathbf{f}\|_2^2. \quad (12)$$

In our simulations (see Section 5), the regularization matrix $\mathbf{L}$ has been assumed to be a highpass filter (a 4–th order highpass filter, applied separately across radial and angular coordinates, was used), whereas the weighting term λ has been empirically set to $0.1\|\mathbf{A}\|_{\text{F}}/\|\mathbf{L}\|_{\text{F}}$, where $\|\cdot\|_{\text{F}}$ denotes the Frobenius norm of a matrix.

5. EXPERIMENTAL RESULTS

In order to verify the feasibility of the CO–RO approach, a simulator of the proposed acquisition system has been implemented: such a tool permits to obtain a set of IWV measurements, given the 2–D WV map and the parameters of the satellite constellation. The reference 2–D WV map, denoted as $\mathbf{f}$ in Section 3, was obtained by interpolating true radiosonde data providing vertical profiles of atmosphere physics (WV, temperature, pressure). The radiosonde data were placed at regular angular positions, with a spacing of five degrees, and then linear interpolation was applied to each variable. The resolution of the reference map was set to

$\Delta r = 0.5$ km (radial direction) and $\Delta\theta = 1$ degree (angular direction).

As to the acquisition system and the satellite constellation geometry that allow the system matrix $\mathbf{A}$ to be computed, we have set the following parameters:

- satellites co–orbiting on a circular polar orbit at constant angular speed;
- Earth radius: 6378 km;
- satellites orbit radius: 6651 km (273 km from the Earth);
- annular region of interest: radial bounds 6380 km $\leq r \leq$ 6388 km (2 to 10 km from the Earth); angular bounds $0° \leq \theta < 360°$;
- integration time at the receiver: $T_s = 1.5$ s (equivalent to an angular resolution of $0.1°$);
- transmitted power: 3 dBW on each channel;
- TX and RX antenna gains: 26.4 dB;
- integration step along each link: 0.25 km.

Path losses due to atmospheric absorption, free-space and defocusing [3] were considered. As to the atmospheric attenuation, depending on WV, pressure and temperature, the Liebe model [10] was used.

The error vector $\mathbf{e}$ accounts for all the impairments affecting the IWV measurements. The disturbances and the relative parameters used in the simulations are the following:

- scintillation: the processes were generated as two correlated log-normal distributions, with variance σ_χ, defined in (5), which was set to 0.3. The zero-lag correlation between the two channel was set to $\rho = 0.85$, whereas time correlation (simulated as lowpass filtering) is dictated by the scintillation bandwidth B_χ, which was set to 5 Hz;
- thermal noise: the process was simulated as zero-mean AWGN with variance $k_B T_{\text{eq}}/T_s$, where k_B is the Boltzmann constant, T_s is the aforementioned integration time, and T_{eq} is the equivalent noise temperature, which was set to 25.3 dBK;
- model mismatches in the conversion from S to IWV [2][4].

Two configurations of satellites were considered: the case of 1 transmitter and 1 receiver (1TX-1RX) and the case of 1 transmitter and 3 receivers (1TX-3RX). The former represents the simplest solution, while the latter is expected to yield better reconstruction performances (both solutions, however, require the deployment of only one transmitter). The performances were evaluated in terms of the RMSE between the reconstructed WV map $\hat{f}$ and the true WV map f normalized to the peak-value, defined as

$$\text{NRMSE} = \frac{\sqrt{E[(\hat{f} - f)^2]}}{\max f} \times 100 \tag{13}$$

Table 1 shows the NRMSE obtained by using the LS and the TIK estimators for the two considered satellite configurations. The cases of *ideal* and *realistic* acquisitions, in the absence and in the presence of all the aforementioned disturbances, respectively, are distinguished. Visual results of the reconstructed maps in the realistic case are reported in Fig. 3.

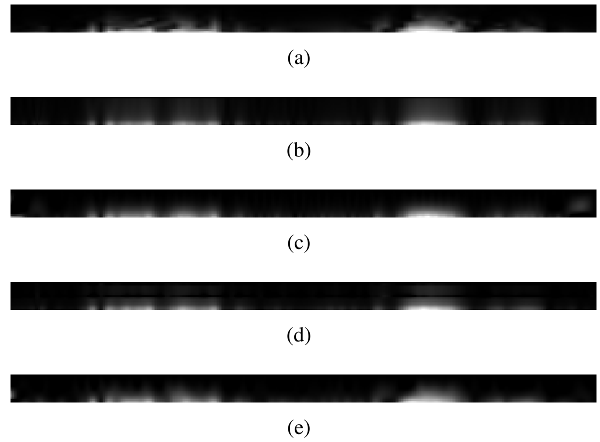

Fig. 3. Reconstructed 2-D WV maps by means of different satellite configurations and estimators in the realistic case: (a) original; (b) 1TX-1RX, LS estimator; (c) 1TX-1RX, TIK estimator; (d) 1TX-3RX, LS estimator; (e) 1TX-3RX, TIK estimator.

Table 1. NRMSE of LS and TIK estimators for the different satellite configurations, in the ideal and realistic cases.

	ideal case		realistic case	
	1TX-1RX	1TX-3RX	1TX-1RX	1TX-3RX
LS	5.3	2.0	5.8	4.4
TIK	3.6	3.2	5.0	3.8

The system matrix $\mathbf{A}$ is severely ill-conditioned and the results show a certain deviation from perfect reconstruction even in the ideal case. Nevertheless, the results of the reconstruction in the realistic case vary, depending on the satellite configuration and the used estimator, from 3.8% to 5.8%. Such figures can be considered as appealing in meteorological applications. The TIK estimator allows a lower NRMSE to be obtained in the realistic case with respect to the LS one, while it fails in the ideal case for the 1TX-3RX configuration. This fact might be mitigated by optimizing the choice of the regularization matrix $\mathbf{L}$ and the weighting factor λ.

6. CONCLUSIONS

In this paper, we have presented a study for the reconstruction of atmospheric water vapour fields by means of integrated water vapour measurements acquired by a constellation of co-rotating low Earth orbit satellites. By modelling the acquisition process as a linear system, the reconstruction of the water vapour field can be formulated as the solution of an inverse problem. A realistic simulator, which takes into account the phenomenon of scintillation and other disturbances impairing the measurements, has been implemented. Experimental results show the feasibility of the proposed approach and satisfactory reconstruction performances.

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