

# AN ADVANCED MULTI-CARRIER MODULATION FOR FUTURE RADIO SYSTEMS

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## ABSTRACT

Many alternative modulation schemes have been investigated to replace OFDM for radio systems. But they all have some weak points. In this paper, we present a novel modulation scheme, which minimizes the predecessors' drawbacks, while still keeping their advantages.

**Index Terms**— FBMC, FMT, GFDM, OFDM, OQAM.

## 1. INTRODUCTION

In today's mobile communication systems, cyclic-prefix-based orthogonal frequency division multiplexing (CP-OFDM) is widely adopted. Nevertheless, for moving towards future radio systems beyond 2020, it is time to ask whether the traditional fashion for Multi-Carrier Modulation (MCM) can effectively meet particular demands from emerging scenarios, e.g. extensive crowd communication and network heterogeneity. One major concern of the CP-OFDM is that it cannot provide any degree of freedom to use flexible waveforms. Considering network heterogeneity, doubly-dispersive channel, i.e. time and frequency, appears and the dominant parts can be varying [1]. Indeed, research analysis showed that the localization property in time and frequency domains both play an important role in the waveform design for addressing the two-dimensional fading [2]. Obviously the CP-OFDM with a fixed rectangular pulse shape cannot response to this requirement.

To address the waveform flexibility, people started to improve the OFDM with some other pulse shapes. However, the mathematicians proved that for a MCM system it cannot simultaneously employ flexible pulses, remain orthogonality and transmit at the Nquist rate, which was later known as the Balian-Low theorem (BLT) [3]. Thus, novel MCM transmission fashion is needed to breakthrough this bottleneck. During the past decades, two main MCM alternatives have been proposed, namely Filter-Bank-based Multi-Carrier with Offset QAM (FBMC/OQAM) and Filter Multi-Tone (FMT). The former shifts the orthogonality condition from complex field

to real field by transmitting data in a staggered manner [4]. While, the FMT consists in relaxing the Nquist rate transmission, which enables the waveform flexibility [5]. However, neither of them is robust against frequency selective fading. In addition, the FMT inherits a Spectral Efficiency (SE) loss. Besides these schemes, a new concept of MCM has recently appeared in the literature. It replaces the linear filtering with a circular filtering for pulse shaping. This idea was originally raised with the introduction of General Frequency Division Multiplexing (GFDM) [6]. With this way, the Power Spectrum Density (PSD) gets improved; but it cannot guarantee the orthogonality due to the BLT [7]. After GFDM, the circular filtering concept is further employed for the FMT, which gives the birth for the cyclic block FMT (CB-FMT) [8]. It successfully overcame the orthogonality issue. Nevertheless, as the modulation kernel remains FMT, the SE loss still exists. This paper is inspired by the question whether we can find one scheme that maximizes the advantages from the predecessors and gets rid of their drawbacks? To address this question, we investigated a new type of FBMC/OQAM, which also adopts the circular filtering to the FBMC/OQAM and we call it circular OQAM or COQAM in short. It turns out that the COQAM can indeed response to our needs. In this paper, we give an illustration and analysis on this novel MCM scheme.

The rest of the paper is organized as follows: in Sec. II, we give an analysis on the State-of-The-Art (SoTA) MCMs. In Sec. III, we present the concept of the COQAM modulation. In Sec. IV, an implementation algorithm is presented. In Sec. V, we detail the COQAM transmitter design in a radio transmission. In Sec. VI, we evaluate the COQAM efficiency. Some conclusions are drawn in Sec. VII.

## 2. ANALYSIS ON THE SOTA MCMS

In this section, we give an analysis for the aforementioned five SoTA schemes from three different aspects, i.e. spectral efficiency, orthogonality and waveform flexibility, respectively.

### 2.1. Spectral efficiency analysis

For a multi-carrier system, denoting  $F_0$  as the spacing between sub-carriers (in Hz) and  $T_0$  as the symbol duration (in second), the maximum SE at a Nquist rate yields  $T_0 \cdot F_0 = 1$ .

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Thus, either the symbol duration or the sub-carrier spacing gets increased, its product becomes greater than 1, which is deemed as a SE loss. The SE for the afore-mentioned MCMs are given in Tab. 1. For the notations, we denote  $T_{\text{CP}}$  the

MCM	Spectral Efficiency
CP-OFDM [9]	$\frac{T_0+T_{\text{CP}}}{T_0} > 1$
OQAM [4]	$T_0 \cdot F_0 = 1$
FMT [5]	$\frac{1+\alpha}{T_0} \cdot T_0 > 1$
GFDM [6]	$\frac{T_0+T_{\text{CP}}/K}{T_0} > 1$
CB-FMT [8]	$\frac{(T_0+T_{\text{CP}}/K)(1+\alpha)}{T_0} > 1$

**Table 1.** SE analysis based on the refs. given in the table.

increased symbol duration due to the CP insertion, and  $\alpha F_0$  the bordered subcarrier spacing due to the over-sampled nature of FMT. Note that for GFDM and CB-FMT, it is possible to group  $K$  modulated symbols to one block with one CP appending ahead so that one CP can be actually shared by the  $K$  modulated symbols, leading eventually to a reduced SE loss. In the worst case, i.e.  $K = 1$ , GFDM yields the same SE loss as CP-OFDM. While for CB-FMT, a two-dimensional overhead should be counted, resulting in the severest SE loss.

## 2.2. Orthogonality analysis

The orthogonality of a MCM is a crucial factor, because it directly relates to the communication quality, which, further, links to the receiver complexity. These features becomes still more critical when considering these schemes with space-time encoded MISO transmission, e.g., Alamouti coding [10]. Thus, more weights are often given to the orthogonality than the SE in a MCM evaluation. Here the orthogonality is analyzed in two cases, i.e. under distortion-free channel and under multi-path channel. The former shows whether the MCM forms a perfect reconstruction system. While the latter reveals the robustness against a frequency selective fading, which further hints on the complexity of a transceiver design. Tab. 2 summarizes the outcome of our analysis based on the given references.

MCM	distortion-free	multi-path
CP-OFDM [9]	Yes	Yes
OQAM [11]	Yes	No
FMT [12]	Yes	No
GFDM [7, 13]	No	No
CB-FMT [8]	Yes	Yes

**Table 2.** Orthogonality analysis.

## 2.3. Waveform flexibility analysis

As previously stated, the waveform flexibility plays an important role for future radio systems to address diverse applica-

tions. Here, we define the flexibility as the possibility to play with diverse criteria, e.g. time-frequency localization, while still insuring a perfect, or nearly perfect, orthogonality. Thus, CP-OFDM with its fixed rectangular pulse cannot enable any waveform flexibility. On the contrary, OQAM addresses full flexibility. Indeed, the embedded pulse shapes can be designed under many diverse criteria while keeping the orthogonality in a distortion-free channel [14]. The GFDM is able, in principle, to address the waveform flexibility. However, as shown in [7], the orthogonality loss is highly impacted by the employed pulse shape. The last scheme, CB-FMT, actually has some constraints on the waveform design. It is hinted in [8] that the pulse spectrum must not exceed a certain bandwidth. Then to avoid a significant SE loss a good frequency localization feature is mandatory, e.g. the CB-FMT implementation scheme in [8] is only valid for a roll-off of at most 1/9 in the case of a 9/8 oversampling factor. This prevents the use of prototype filter with good time localization.

## 3. COQAM: AN ADVANCED MCM

### 3.1. Motivation and Concept

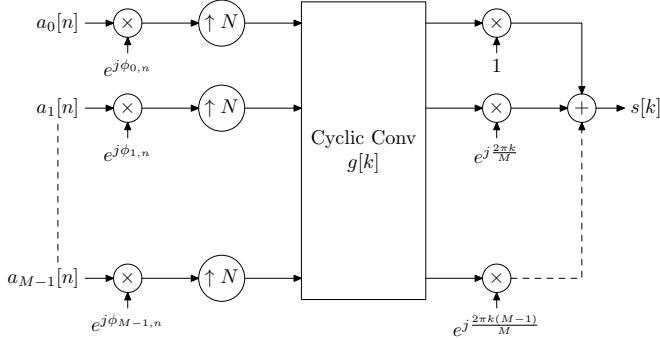
As our analysis showed, the CP-OFDM cannot address any flexibility. The OQAM and FMT schemes lose the orthogonality under multi-path channel, which needs a more complicated receiver design and low feasibility with MIMO transmission. Even more severe orthogonality issue is inherited in the GFDM scheme, which further impacts its waveform flexibility. The CB-FMT completely solved the orthogonality issue and managed to employ an improved pulse shape compared with CP-OFDM. However, it is still an open question how it can fully address the waveform flexibility. In addition, the CB-FMT has a two-dimensional SE loss, i.e. in time because of the CP insertion and in frequency due to the oversampling.

Knowing the shortcomings, an intuitive question is whether we can find an improved modulation that keeps most of the benefits from the predecessors and gets rid of their drawbacks. With this motivation, we investigated a new MCM scheme called COQAM, whose idea is to replace the linear convolution inherited in the OQAM with a circular convolution used in the GFDM and CB-FMT. By this way we get a modulation scheme that corresponds to a block transform. So that a CP can be easily added to enhance the orthogonality. For a discrete-time signal  $s[k]$  defined in a block interval such that  $k \in [0, MK - 1]$ , the baseband COQAM modulation is expressed as

$$s_{\text{COQAM}}[k] = \sum_{m=0}^{M-1} \sum_{n=0}^{K-1} a_m[n] \tilde{g}[k - nN] e^{j \frac{2\pi}{M} m(k - \frac{D}{2})} e^{j \phi_{m,n}},$$

with  $M$  the number of subcarriers;  $N = M/2$  an offset factor;  $K$  the number of real symbol slots per each block;  $\phi_{m,n}$  the

additional phase term at subcarrier  $m$  and slot index  $n$ , which can be expressed as  $\frac{\pi}{2}(n + m)$ . To implement a circular convolution with a prototype filter  $g$  of length  $L = KM = D+1$ , we introduce a pulse shaping filter denoted  $\tilde{g}$ , obtained by the periodic repetition of duration  $KM$  of the prototype filter  $g$ , i.e.  $\tilde{g}[k] = g[\text{mod}(k, MK)]$ . Note that the COQAM is an orthogonal system and the orthogonality condition for the filter design is in line with OQAM, i.e. all the designed prototype filter for OQAM can be reused in COQAM systems. The corresponding baseband COQAM modulation structure is depicted in Fig. 1.



**Fig. 1.** COQAM baseband modulation.

### 3.2. Practical implementation algorithm

A direct implementation of Fig. 1 cannot be envisioned in practice. Here we present an efficient algorithm, based on Inverse Fast Fourier Transform (IFFT). Only the modulator side is presented. Omitting some mathematical derivations, the baseband COQAM signal, for a block of size  $MK$ , can be expressed in a matrix form as

$$\mathbf{s}_{MK \times 1}^{\text{COQAM}} = \text{diag}\{\mathbf{G}_{MK \times K} (\mathbf{E}_{MK \times M} \mathbf{W}_{M \times M} \mathbf{A}_{M \times K})^T\},$$

where  $(\cdot)^T$  denotes the matrix transpose operation and  $\text{diag}\{\cdot\}$  corresponds to the extraction of the diagonal elements of a matrix.  $\mathbf{A}_{M \times K}$  is a matrix containing the pre-processed data for  $m \in [0, M - 1]$  and  $n \in [0, K - 1]$ , i.e.

$$\mathbf{A}_{M \times K} = \{a_m[n]e^{j\phi_{m,n}}e^{-\frac{j\pi m D}{M}}\}_{M \times K}. \quad (1)$$

The Fourier transform matrix  $\mathbf{W}_{M \times M}$ , with input indices  $(m, k) \in [0, M - 1]$ , is expressed by  $\{e^{\frac{j2\pi mk}{M}}\}_{M \times M}$ . The extended matrix  $\mathbf{E}_{MK \times M}$  corresponds to a  $K$  repetition of the identity matrix  $\mathbf{I}_{M \times M}$  of size  $M$  given by

$$\mathbf{E}_{MK \times M}^T = [\mathbf{I}_{M \times M} \ \cdots \ \mathbf{I}_{M \times M}]_{M \times MK}. \quad (2)$$

The polyphase circulant matrix  $\mathbf{G}_{MK \times K}$  reads as

$$\mathbf{G}_{MK \times K} = \{g^{k,n}\}_{MK \times K}, \quad (3)$$

with  $g^{k,n} = g[\text{mod}(k - nN, MK)]$ , for  $k \in [0, MK - 1]$  and  $n \in [0, K - 1]$ . A complete picture of our algorithm is displayed in Fig. 2, for processing of one block of  $\mathbf{A}_{M \times K}$ .

### 4. COQAM TRANSMITTER FOR A RADIO SYSTEM

In order to maintain a perfect orthogonality after a transmission through a multi-path channel, we introduce a CP to cancel the inter block interference. Moreover, as we want to prevent an alteration of the Power Spectral Density (PSD), resulting from a spectral leakage due to the block processing, a windowing is applied after CP insertion. The transmission system resulting from these two operations is named windowed CP-FBMC/COQAM (WCP-COQAM). Denoting the CP length by  $L_{\text{CP}}$ , we get  $L_{\text{CP}} = L_{\text{GI}} + L_{\text{RI}}$ , where  $L_{\text{GI}}$  is the CP part used to fight against the multi-path channel interference and  $L_{\text{RI}}$  is the portion devoted to windowing.

The  $l$ -th block of the WCP-COQAM signal  $s_{\text{WCP-COQAM}}[k]$ , for  $k = 0, \dots, KM + L_{\text{CP}} - 1$ , can be obtained from the  $l$ -th block of the COQAM signal, for  $k = 0, \dots, KM - 1$ , by

$$s_{\text{WCP-COQAM}}[k] = \sum_{r=l-1}^{l+1} s_{\text{COQAM}}[\text{mod}(k - L_{\text{CP}}, KM)] \times w[k - rQ], \quad (4)$$

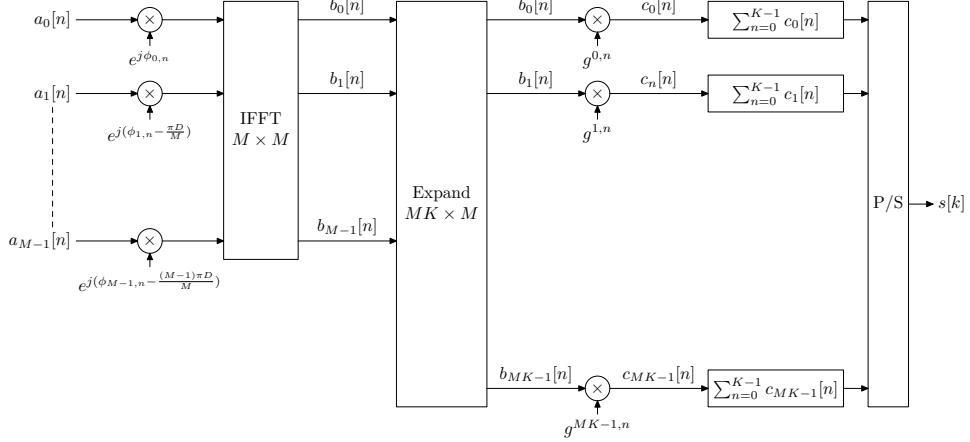
where  $Q = KM + L_{\text{GI}}$  and  $w[k]$ , defined in the  $k = 0, \dots, KM + L_{\text{CP}} - 1$  interval, is the window function computed as follows

$$w[k] = \begin{cases} \text{window coeffs.} & k \in [0, L_{\text{RI}} - 1] \\ 1 & k \in [L_{\text{RI}}, KM + L_{\text{GI}} - 1] \\ w[KM + L_{\text{CP}} - 1 - k] & \text{otherwise.} \end{cases}$$

Extensive works on the window design are given in [15]. In this paper, we simply use the Hamming window for this windowing process. Furthermore, it is worth noting that the additional part  $L_{\text{RI}}$  does not reduce the spectral efficiency as it falls only in the overlapped samples between successive blocks.

### 5. SIMULATION AND DISCUSSION

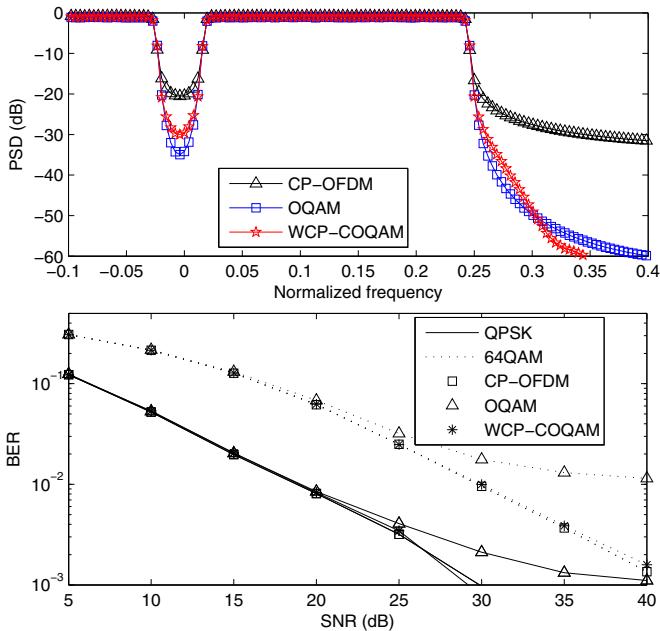
Due to the page limit, extensive simulation for the comparison between WCP-COQAM and the SoTA schemes cannot be reported. Here, we only take CP-OFDM and OQAM into account, which, nevertheless, can already provide a clear view of the improvement with the proposed scheme. The PSD and Bit Error Rate (BER) are evaluated to show that WCP-COQAM can remain a good spectrum shape and its orthogonality under a multi-path channel can be kept. In our simulation, the PSD is estimated using the Welch method [16]. The parameters used in our simulation are: the number of subcarriers is  $M = 256$  for all MCM schemes. IOTA prototype filter is used for OQAM, WCP-COQAM with a length of  $4M$  ( $K = 4$ ) [4].  $L_{\text{GI}}$  and  $L_{\text{RI}}$  for WCP-COQAM are 64 and 16 samples, respectively, while the CP length for CP-OFDM is 64 samples. The result of the PSD comparison is shown in the upper part of Fig. 3. The PSD of the WCP-COQAM can maintain a satisfactory low level of the out-of-band spectral



**Fig. 2.** Efficient implementation of the COQAM modulator: IFFT-based algorithm.

leakage. Moreover, it also provides a low spectral radiation in a notch band, which is a crucial feature for the solutions for dynamic spectrum access, e.g. cognitive radio.

The BER simulation results are depicted in the lower figure. We reuse the wireless dispersive fading channel in [17]. As for the receiver techniques, one-tap Zero-Forcing (ZF) equalization is used for all the MCM schemes. The result shows that even under a severe fading environment, the WCP-COQAM curve is always merged with the CP-OFDM. While for the OQAM a degradation by several decibels is displayed at BER of  $10^{-3}$  or even earlier in the high constellation case. This can clearly prove that the WCP-COQAM solves the orthogonality issue.



**Fig. 3.** Upper: PSD comparison with  $M = 256$ , IOTA filter [4] for OQAM and WCP-COQAM. Lower: BER comparison, QPSK and 64QAM under multi-path channel, 1-tap ZF equalizer.

Here we provide a short summary of the pros and cons of all the MCM schemes in Tab. 3. For the WCP-COQAM, our simulations have illustrated its good PSD property and its orthogonality advantage. Moreover, it can reuse all the prototype filters for the conventional OQAM, which further enables the waveform (WFM) flexibility. However, as it also uses CP, the SE loss exists in this system. The outcome of Sec. 2 is also reflected in this table. Note that the double-cross for CB-FMT at first column is due to that it comprises a two-dimensional SE loss. While the double-cross for CP-OFDM at the third column is owing to that its fixed rectangular pulse. From this table, we can conclude that the WCP-COQAM maximizes the benefits of the SoTA MCM schemes and minimizes the drawbacks.

MCM	Spec. Eff.	Orthogo.	WFM flex.
CP-OFDM	✗	✓	✗✗
OQAM	✓	✗	✓
FMT	✗	✗	✓
GFDM	✗	✗	✗
CB-FMT	✗✗	✓	✗
WCP-COQAM	✗	✓	✓

**Table 3.** MCMs Pros and Cons summary

## 6. CONCLUSION

In this paper, we analyzed the SoTA MCM schemes and pointed out their advantages and drawbacks. Further we proposed an improved FBMC/OQAM concept, which has been proved to be able to maximize the benefits and minimize the drawbacks. More complete comparison results among the SoTA MCMs as well as a detailed transceiver design of the COQAM system will be reported in the future [18].

## 7. REFERENCES

- [1] METIS deliverable D1.1, "Scenarios, requirements and KPIs for 5G mobile and wireless system," Tech. Rep. ICT-317669.
- [2] G. Matz, H. Bölskei, and F. Hlawatsch, "Time-frequency foundations of communications," *IEEE Signal Processing Magazine*, vol. 30, no. 6, pp. 87–96, Nov. 2013.
- [3] J. J. Benedetto, C. Heil, and D. F. Walnut, "Gabor systems and the Balian-Low theorem," in *Gabor analysis and algorithms*, pp. 85–122. Birkhäuser, 1998.
- [4] B. Le Floch, M. Alard, and C. Berrou, "Coded Orthogonal Frequency Division Multiplex," *Proceedings of the IEEE*, vol. 83, pp. 982–996, June 1995.
- [5] G. Cherubini, E. Eleftheriou, and S. Ölçer, "Filtered Multitone Modulation for Very High-Speed Subscriber Lines," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 5, pp. 1016 – 1028, 2002.
- [6] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM-Generalized Frequency Division Multiplexing," in *IEEE Vehicular Technology Conference (VTC Spring'09)*, Barcelona, Spain, Apr. 2009.
- [7] N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Bit Error Rate Performance of Generalized Frequency Division Multiplexing," in *IEEE Vehicular Technology Conference (VTC Fall'12)*, Quebec city, Canada, Sept. 2012.
- [8] A. M. Tonello, "A Novel Multi-carrier Scheme: Cyclic Block Filtered Multitone Modulation," in *Proc. of ICC 2013*, Budapest, June 2013.
- [9] W.Y. Zou and Y. Wu, "COFDM: An Overview," *IEEE Transactions on Broadcasting*, vol. 41, no. 1, pp. 1–8, Mar. 1995.
- [10] R. Zakaria and D. Le Ruyet, "On interference cancellation in Alamouti coding scheme for filter bank based multicarrier systems," in *ISWCS*, June 2013, pp. 1–5.
- [11] G. Ndo, H. Lin, and P. Siohan, "FBMC/OQAM equalization: Exploiting the imaginary interference," in *PIMRC'12*, Sydney, Australia, Sept. 2012.
- [12] N. Benvenuto, S. Tomasin, and L. Tomba, "Equalization methods in OFDM and FMT systems for broadband wireless communications," *IEEE Trans. on Communications*, vol. 50, no. 9, Sept. 2002.
- [13] N. Michailow, R. Datta, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized Frequency Division Multiplexing: A Flexible Multi-Carrier Modulation Scheme for 5th Generation Cellular Networks," in *Proceedings of the German Microwave Conference (GeMiC'12)*, Ilmenau, Germany, Mar. 2012.
- [14] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filterbank theory," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1170–1183, May 2002.
- [15] F.J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," *Proceedings of the IEEE*, vol. 66, no. 1, pp. 51 – 83, Jan. 1978.
- [16] P. D. Welch, "The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms," *IEEE Trans. Audio and Electroacoust.*, vol. AU-15, pp. 17–20, Jun. 1967.
- [17] T. Fusco, A. Petrella, and M. Tanda, "Data-Aided Symbol Timing and CFO Synchronization for Filter Bank Multicarrier Systems," *IEEE Trans. on Wireless Communications*, vol. 8, pp. 2705–2715, May 2009.
- [18] H. Lin and P. Siohan, "Multi-carrier modulation for future radio-systems: Analysis and proposal," Submitted, 2014.