A Low-Complexity Blind CFO Estimation for OFDM Systems

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***Abstract*— The estimation of carrier frequency offset (CFO) is an important issue in the study of the orthogonal frequency division multiplexing (OFDM) systems. In the past, many CFO estimation methods have been proposed. In this paper, we propose a new blind CFO estimation for OFDM systems based on the so-called remodulated received vectors [8]. The CFO estimate is given by a closed form formula. The proposed method has very low complexity and its performance is robust to different modulation symbols and the presence of virtual carriers.**

***Index Terms*—orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), blind estimation**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a popular modulation technique because of its spectral efficiency and robustness to frequency selective channels. It is well-known that OFDM systems are sensitive to the carrier frequency offset (CFO) [1]. CFO destroys the orthogonality between subcarriers and induces inter-carrier interference. Therefore, if the CFO is not corrected at the receiver, the performance of OFDM systems can degrade seriously. In the past, many methods have been proposed for CFO estimation [2]-[7]. These methods can be divided into two groups: data-aided and non data-aided (blind) schemes. This paper focuses on blind CFO estimation.

In [3], the authors utilize the frequency analysis (FA) for blind CFO estimation, but it works only for the constant modulus constellations. Other blind CFO estimation schemes for non constant modulus modulation have been reported in literatures. In [4], a kurtosis-based cost function was proposed for blind CFO estimation, and its complexity is quite low. Another simple method based on cyclic prefix (CP) was proposed in [5], but it suffers performance degradation in multipath environment. Several blind CFO estimation methods employing the null subcarriers or the virtual carriers (VC) have been developed. The multiple signal classification (MUSIC) method was proposed in [6], which requires a one-dimensional search and its complexity depends on the search resolution. In [7], the authors proposed an estimation of signal parameters via rotation invariance technique (ESPRIT) for blind CFO estimation. The ESPRIT-based scheme has a closed form solution, but its performance is highly dependent on the channel impulse response.

In this paper, we propose a new blind CFO estimation for OFDM systems based on the so-called remodulated received vectors [8]. A closed form formula is derived. The proposed method works for both constant and non constant modulation symbols, and for OFDM systems with or without VCs. Compared with existing methods, the proposed method achieves a better results at a fraction of their complexity.

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This paper is organized as follows. The system model is introduced in Section II. Section III describes the proposed algorithm with low complexity. Simulation results are presented in Section IV and concluding remarks are provided in Section V.

Notation: In this paper, $\mathrm{E}\{x\}$ means the statistical expectation of a random variable x. The symbols $\mathbf{A}^T$, $\mathbf{A}^*$, and $\mathbf{A}^\dagger$ denote the transpose, the complex conjugate, and the conjugate-transpose of matrix $\mathbf{A}$ respectively. The MATLAB notations for rows and columns are used. For example, $\mathbf{A}(:,m)$ represents the mth column of $\mathbf{A}$. The $m \times m$ diagonal matrix $\mathbf{D}_m(\theta)$ is defined as

$$\mathbf{D}_m(\theta) = \mathrm{diag}\left\{\left[\begin{array}{cccc} 1 & e^{j2\pi\theta/N} & \cdots & e^{j2\pi(m-1)\theta/N} \end{array}\right]\right\}.$$

II. SYSTEM MODEL

Consider an OFDM system. Let N and L be respectively the size of the discrete Fourier transform (DFT) and the length of the cyclic prefix (CP). Let $\mathbf{x}_k = [\begin{array}{cccc} x_k(0) & x_k(1) & \cdots & x_k(N-1) \end{array}]^T$ be the kth time domain vector at the output of the inverse DFT. The transmitter adds L CP samples $\mathbf{x}_{k,\mathrm{cp}} = [\begin{array}{cccc} x_k(N-L) & \cdots & x_k(N-2) & x_k(N-1) \end{array}]^T$ to form the $(N+L) \times 1$ cyclic prefixed vector $\mathbf{x}'_k = [\begin{array}{cc} \mathbf{x}_{k,\mathrm{cp}}^T & \mathbf{x}_k^T \end{array}]^T$. Assume that the channel order does not exceed L. Then at the receiver, the kth received vector of size $N+L$ is given by

$$\mathbf{r}'_k = e^{j2\pi\frac{k(N+L)}{N}\theta}\mathbf{D}_{N+L}(\theta)\mathbf{H}\left[\begin{array}{c} \mathbf{x}_{k-1,\mathrm{cp}} \\ \mathbf{x}'_k \end{array}\right] + \mathbf{n}_k, \quad (1)$$

where θ is the normalized CFO and $\mathbf{H}$ is a $(N+L)\times(N+2L)$ Toeplitz matrix expressed as

$$\mathbf{H} = \left[\begin{array}{ccccc} h(L) & \cdots & h(0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & h(L) & \cdots & h(0) \end{array}\right] \quad (2)$$

with $[\begin{array}{ccc} h(0) & \cdots & h(L) \end{array}]$ being the channel impulse response. The last term $\mathbf{n}_k$ in (1) is the kth noise block which is assumed to be an additive white Gaussian noise (AWGN) vector with variance σ_n^2.

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In this paper, we will show how to blindly estimate the normalized CFO θ from the received signal. It does not need to know the channel information and works for both constant modulus and non constant modulus modulation symbols.

III. Proposed method for CFO estimation

In [8], the authors introduced a remodulation of the received signal for blind channel estimation (in the absence of CFO). In this paper, we will show how the remodulated signal can be used for blind CFO estimation when there is CFO. Let us consider the remodulated vector:

$$\tilde{\mathbf{r}}_k \triangleq \begin{bmatrix} \mathbf{r}'_{k-1}(L:N+L-1) \\ \mathbf{r}'_k(0:L-1) \end{bmatrix}. \quad (3)$$

That is, $\tilde{\mathbf{r}}_k$ is a $(N+L)\times 1$ vector formed by the last N entries of $\mathbf{r}'_{k-1}$ and the first L entries of $\mathbf{r}'_k$. The vector $\tilde{\mathbf{r}}_k$ can be represented by

$$\tilde{\mathbf{r}}_k = e^{j2\pi\frac{k(N+L)-N}{N}\theta}\mathbf{D}_{N+L}(\theta)\mathbf{H}\begin{bmatrix} \mathbf{x}'_{k-1} \\ \mathbf{x}_{k,\text{cp}} \end{bmatrix} + \tilde{\mathbf{n}}_k, \quad (4)$$

where $\tilde{\mathbf{n}}_k = [\ \mathbf{n}_{k-1}(L:N+L-1)^T \quad \mathbf{n}_k(0:L-1)^T\ ]^T$. Now let us consider the following vector

$$\mathbf{y}_k(\xi) = \mathbf{r}'_k - e^{j2\pi\xi}\tilde{\mathbf{r}}_k. \quad (5)$$

Utilizing (1) and (4), we can express $\mathbf{y}_k(\xi)$ as

$$\begin{aligned}\mathbf{y}_k(\xi) =& e^{j2\pi\frac{k(N+L)}{N}\theta}\mathbf{D}_{N+L}(\theta)\mathbf{H}\left(\begin{bmatrix} \mathbf{x}_{k-1,\text{cp}} \\ \mathbf{x}'_k \end{bmatrix}\right. \\ &\left. -e^{j2\pi(\xi-\theta)}\begin{bmatrix} \mathbf{x}'_{k-1} \\ \mathbf{x}_{k,\text{cp}} \end{bmatrix}\right) + \underbrace{(\mathbf{n}_k - e^{j2\pi\xi}\tilde{\mathbf{n}}_k)}_{\boldsymbol{\eta}_k}. \end{aligned} \quad (6)$$

Computing the autocorrelation matrix of $\mathbf{y}_k(\xi)$, we have

$$\begin{aligned}\mathbf{R}_y(\xi) =& \mathrm{E}\{\mathbf{y}_k(\xi)\mathbf{y}_k^\dagger(\xi)\} \\ =& \mathrm{E}\{(\mathbf{r}'_k - e^{j2\pi\xi}\tilde{\mathbf{r}}_k)(\mathbf{r}'_k - e^{j2\pi\xi}\tilde{\mathbf{r}}_k)^\dagger\}. \end{aligned} \quad (7)$$

Assume that the noise and signal are uncorrelated and the $N\times 1$ transmitted vector $\mathbf{x}_k$ satisfies $\mathrm{E}\{\mathbf{x}_k\mathbf{x}_{k-i}^\dagger\} = \sigma_x^2\mathbf{I}_N\delta(i)$, where σ_x^2 is the average power of the transmitted data and $\delta(i)$ is the Kronecker delta function. Then it can be shown that the autocorrelation matrix is expressed as

$$\mathbf{R}_y(\xi) = \sigma_x^2\mathbf{D}_{N+L}(\theta)\mathbf{H}\boldsymbol{\Psi}\mathbf{H}^\dagger\mathbf{D}_{N+L}(-\theta) + \sigma_n^2\mathbf{R}_\eta(\xi), \quad (8)$$

where $\boldsymbol{\Psi}$ is a $(N+2L)\times(N+2L)$ matrix given by

$$\boldsymbol{\Psi} = \begin{bmatrix} 2(1-\epsilon_1)\mathbf{I}_L & \mathbf{0} & \mathbf{0} & \epsilon_2^*\mathbf{I}_L & \mathbf{0} \\ \mathbf{0} & 2\mathbf{I}_L & \mathbf{0} & \mathbf{0} & \epsilon_2^*\mathbf{I}_L \\ \mathbf{0} & \mathbf{0} & 2\mathbf{I}_{N-2L} & \mathbf{0} & \mathbf{0} \\ \epsilon_2\mathbf{I}_L & \mathbf{0} & \mathbf{0} & 2\mathbf{I}_L & \mathbf{0} \\ \mathbf{0} & \epsilon_2\mathbf{I}_L & \mathbf{0} & \mathbf{0} & 2(1-\epsilon_1)\mathbf{I}_L \end{bmatrix} \quad (9)$$

with $\epsilon_2 = 1 - e^{j2\pi(\xi-\theta)}$ and

$$\epsilon_1 = \cos(2\pi(\xi-\theta)),$$

and $\mathbf{R}_\eta(\xi)$ is a $(N+L)\times(N+L)$ matrix expressed as

$$\mathbf{R}_\eta(\xi) = \begin{bmatrix} 2\mathbf{I}_L & \mathbf{0} & -e^{-j2\pi\xi}\mathbf{I}_L \\ \mathbf{0} & 2\mathbf{I}_{N-L} & \mathbf{0} \\ -e^{j2\pi\xi}\mathbf{I}_L & \mathbf{0} & 2\mathbf{I}_L \end{bmatrix}. \quad (10)$$

From (8), it can be verified by dirrect computation that the diagonal entries are given by

$$[\mathbf{R}_y(\xi)]_{i,i} = \begin{cases} 2\sigma_x^2\left(\rho - \epsilon_1\sum_{l=i+1}^{L}|h(l)|^2\right) + 2\sigma_n^2 & \text{if } 0 \le i < L, \\ 2\sigma_x^2\left(\rho - \epsilon_1\sum_{l=0}^{i-N}|h(l)|^2\right) + 2\sigma_n^2 & \text{if } N \le i < N+L, \\ 2\sigma_x^2\rho + 2\sigma_n^2 & \text{otherwise,} \end{cases} \quad (11)$$

where $\rho = \sum_{l=0}^{L}|h(l)|^2$ is independent of ξ and θ. Note that only the first and last L diagonal entries are dependent on ξ. Taking a summation of these entries, we have

$$J(\xi) = \sum_{i=0}^{L-1}[\mathbf{R}_y(\xi)]_{i,i} + \sum_{i=N}^{N+L-1}[\mathbf{R}_y(\xi)]_{i,i}. \quad (12)$$

Utilizing (11), $J(\xi)$ can be written as

$$J(\xi) = 2L\rho\sigma_x^2(2 - \cos(2\pi(\xi-\theta))) + 4L\sigma_n^2. \quad (13)$$

It is clear that $J(\xi)$ has a minimum when $\xi = \theta$. Next, we will derive a closed form solution for the CFO estimate $\hat{\theta}$ in terms of the remodulated vectors $\mathbf{r}'_k$ and $\tilde{\mathbf{r}}_k$.

A. Coarse estimation

Assume that the receiver has collected K blocks. The autocorrelation matrix can be approximated as

$$\mathbf{R}_y(\xi) \approx \frac{1}{K-1}\sum_{k=1}^{K-1}\mathbf{y}_k(\xi)\mathbf{y}_k^\dagger(\xi). \quad (14)$$

Utilizing (5), we can write the cost function $\bar{J}(\xi) = (K-1)J(\xi)$ as

$$\bar{J}(\xi) \approx \sum_{k=1}^{K-1}\sum_{i\in\mathcal{S}_1}(r'_k(i) - e^{j2\pi\xi}\tilde{r}_k(i))(r'_k(i) - e^{j2\pi\xi}\tilde{r}_k(i))^*, \quad (15)$$

where $r'_k(i)$ and $\tilde{r}_k(i)$ are the ith entries of $\mathbf{r}'_k$ and $\tilde{\mathbf{r}}_k$ respectively, and

$$\mathcal{S}_1 = \{0, 1, \ldots, L-1, N, N+1, \ldots, N+L-1\}$$

is a set having a cardinality of $2L$. The CFO can be estimated by

$$\hat{\theta} = \arg\min_{\xi\in(-0.5,0.5]}\bar{J}(\xi).$$

The minimum can be found by taking the derivative of $\bar{J}(\xi)$ with respective to ξ and setting it to zero. Utilizing (15), it can be shown that for $\xi \in (-0.5, 0.5]$, the minimum is unique and an estimate of the CFO is given by

$$\hat{\theta}_{\text{coarse}} = \frac{1}{2\pi}\angle\left(\sum_{k=1}^{K-1}\sum_{i\in\mathcal{S}_1}(\tilde{r}_k(i))^* r'_k(i)\right), \quad (16)$$

where the phase $\angle(\cdot)$ of a complex number is defined in the region $(-\pi, \pi]$. The subscript "coarse" indicates that this gives a coarse estimate of the CFO. In the simulation, we find that the mean square error (MSE) for $\hat{\theta}_{\text{coarse}}$ floors at around 10^{-4} for high signal-to-noise ratio (SNR). To solve this issue, we propose the following refinement on the CFO estimate.

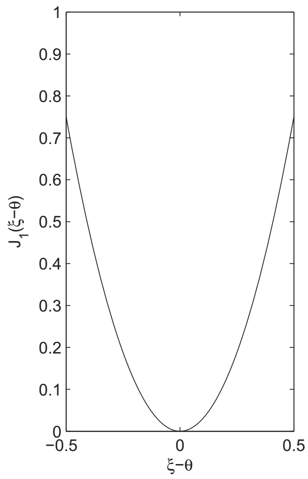

Fig. 1: Two cost functions with different curvatures

B. Fine estimation

Let us consider the two cost functions $J_1(\xi - \theta)$ and $J_2(\xi - \theta)$ in Fig. 1. It is seen that the second order derivative $(\partial^2 J_1)/(\partial \xi^2)|_{\xi=\theta}$ is larger than $(\partial^2 J_2)/(\partial \xi^2)|_{\xi=\theta}$. For the cost function $J_1(\xi - \theta)$, a small estimation error $\xi - \theta = \Delta\theta$ leads to a large increase in $J_1(\xi - \theta)$. On the other hand, a small estimation error leads to a small increase in $J_2(\xi - \theta)$. Therefore, when the cost functions contain a noise term, the estimate based on $J_1(\xi - \theta)$ will be more accurate than the estimate based on $J_2(\xi - \theta)$. For this reason, let us calculate the second order derivatives of the main diagonal of $\mathbf{R}_y(\xi)$. From (11), we can get

$$\left[\frac{\partial^2 \mathbf{R}_y(\xi)}{\partial \xi^2}\right]_{i,i} = \begin{cases} 8\pi^2 \epsilon_1 \sigma_x^2 \sum\limits_{l=i+1}^{L} |h(l)|^2 & \text{if } 0 \le i < L, \\ 8\pi^2 \epsilon_1 \sigma_x^2 \sum\limits_{l=0}^{i-N} |h(l)|^2 & \text{if } N \le i < N+L, \\ 0 & \text{otherwise,} \end{cases} \tag{17}$$

If $|\xi - \theta| < 1/4$, then $\epsilon_1 > 0$. Comparing (17) with (11), we find that those smaller diagonal entries $[\mathbf{R}_y(\xi)]_{i,i}$ have larger second order derivatives. Therefore, instead of including all $2L$ terms of $\mathcal{S}_1$ in the cost function $\bar{J}(\xi)$ as in (15), we include only the m (where $m < 2L$) smallest terms. In order to find these m smallest terms, one can use the coarse estimate $\hat{\theta}_{\text{coarse}}$ to evaluate $[\mathbf{R}_y(\hat{\theta}_{\text{coarse}})]_{i,i}$. In summary, our algorithm is as follows.

1) Compute a coarse estimate $\hat{\theta}_{\text{coarse}}$ using (16).
2) Calculate $[\mathbf{R}_y(\hat{\theta}_{\text{coarse}})]_{i,i}$ in (14) for $i \in \mathcal{S}_1$.
3) Let the new set $\mathcal{S}_2$ contain those indices of the m smallest diagonal values.
4) Obtain the fine estimate as

$$\hat{\theta}_{\text{fine}} = \frac{1}{2\pi} \angle \left(\sum_{k=1}^{K-1} \sum_{i \in \mathcal{S}_2} (\tilde{r}_k(i))^* r'_k(i) \right). \tag{18}$$

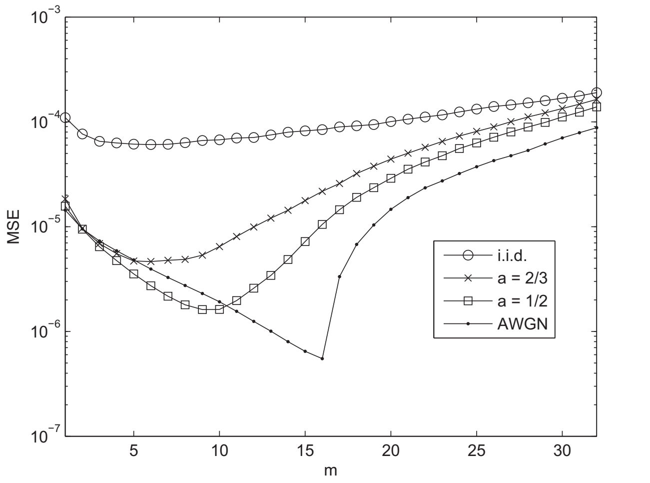

Fig. 2: MSE versus number of the smallest diagonal entries in the cost function

TABLE I: Comparison of the number of CMUL

Blind CFO Estimation	CMUL in terms of N, L, and K	CMUL with the setting in Section IV
Proposed	$6LK - 4L$	896
FA [3]	$K(2N\log_2 N + 3N - 6L - 3)$	8610
Kurtosis [4]	$K(N\log_2 N + 6N)$	7680
MUSIC [6]	$QK((N\log_2 N)/3 + N - P)$	140000

C. On the choice of m

In Fig. 2, we plot MSE versus m for different classes of channel models. In the plot, we choose $L = 16$, $N = 64$, and SNR = 25dB. We plot the MSE for the random channels with exponentially decaying power a^l for $0 \le l \le 16$. “i.i.d.” means that the channel taps are generated as independent and identically distributed (i.i.d.) random variables, or equivalently, $a = 1$. In practice, we do not know the channel information. Hence, from the plot, we see that $m = L/2$ yields a satisfactory performance.

D. Complexity

The computation of the proposed method consists of the generation of $\mathcal{S}_2$ and the two inner products in (16) and (18). Calculating (16) and (18) only need $2L(K-1)$ complex multiplications (CMUL) since $\mathcal{S}_2$ is a subset of $\mathcal{S}_1$. Constructing $\mathcal{S}_2$ requires computing $[\mathbf{R}_y(\hat{\theta}_{\text{coarse}})]_{i,i}$ for $i \in \mathcal{S}_1$ and choosing the m smallest values, so we need $4L(K-1)$ and $2L$ CMUL respectively. Hence, the complexity of the proposed method is $6LK - 4L$.

The comparison of complexity with some existing methods is given in Table I, where Q represents the number of times that the one-dimensional search repeats in MUSIC method [6]. In practice, the CP length L is usually much smaller than the DFT size N. Therefore, the proposed method has a much lower complexity than other methods. In the table, we also explicitly calculate the number of CMUL for the setting given

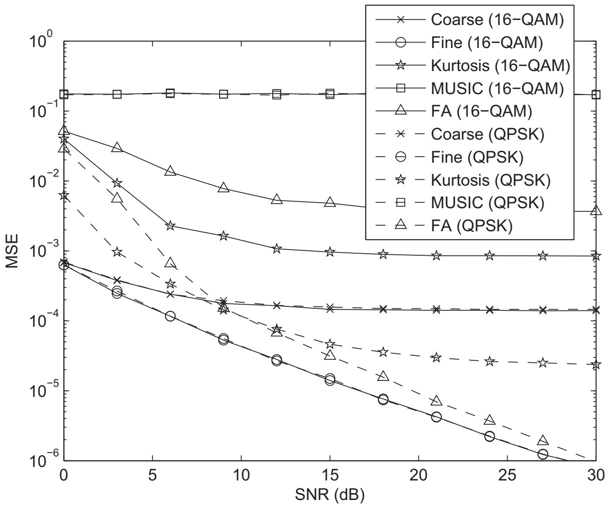

Fig. 3: Comparison of the MSE performance without VCs

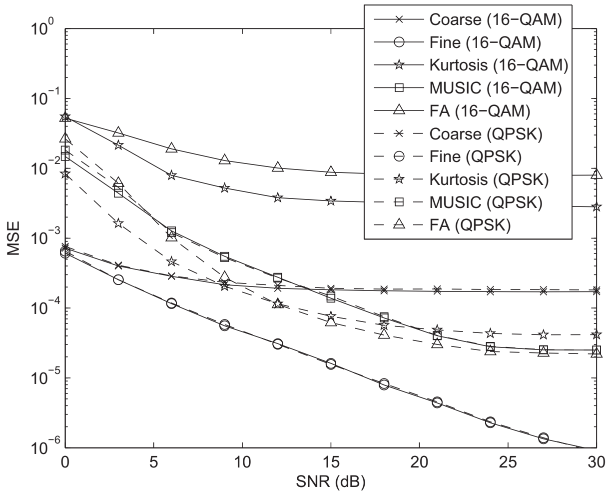

Fig. 4: Comparison of the MSE performance with VCs

in the simulation. It can be seen that the proposed method has very low complexity.

IV. Simulation results

We assume that the channel does not change while CFO estimation is performed. Channel taps are generated as independent random variables with exponentially decaying powers of 2^{-l}. The channel noise is AWGN. Two cases of the transmission symbols are considered (i) 16-QAM; (ii) QPSK.

The OFDM block size is $N = 64$. The length of CP is $L = 16$, and the set $\mathcal{S}_2$ has a cardinality of $m = L/2 = 8$. The number of blocks is $K = 10$.

The MSE is defined as

$$\text{MSE} = \frac{1}{R}\sum_{r=1}^{R} |\hat{\theta}^{(r)} - \theta|^2,$$

where $\hat{\theta}^{(r)}$ represents the estimated CFO in the rth trial. $R = 2000$ denotes the total number of Monte Carlo trials.

First assume that there is no virtual carriers. In Fig. 3, we plot the performances of the "Coarse" estimation in (16), the "Fine" estimation in (18), the "FA" algorithm of [3], the "Kurtosis"-based algorithm of [4], and the "MUSIC"-like algorithm of [6]. It can be seen that the MUSIC-like algorithm does not work since there is no VC. The FA algorithm does not perform well for 16-QAM because it is based on constant modulus modulation. For both cases of 16-QAM and QPSK, the coarse estimation has an error floor at about 10^{-4} at high SNR. The performance of Kurtosis method degrades for the case of 16-QAM. The proposed fine estimation method has the best performance, and its performance does not dependent on the modulation symbols.

Next we consider OFDM systems with virtual carriers.[1] The numbers of data subcarriers and VCs are $P = 52$ and $N - P = 12$ respectively. The indices for VCs are $\{0, \ldots, 5, 32, 59, \ldots, 63\}$. Fig. 4 shows the results. The performance of MUSIC method depends on the search resolution, and with a search resolution of $1/Q$ ($Q = 100$), its MSE floors at 2.5×10^{-5}. Comparing Fig. 3 and 4, we find that the presence of VCs seriously degrades the performance of the FA algorithm. The performance of the Kurtosis method is also affected by the presence of VCs. On the other hand, the presence of VCs has little effect on the proposed method. The proposed method has the best performance in the case of VCs as well.

[1]Though our results are derived for OFDM systems with no VC, the proposed method continues to work well in the presence of VCs.

V. Conclusions

In this paper, we propose a new blind CFO estimation method. The proposed method has very low complexity and its performance is robust to different modulation symbols and the presence of VCs. Simulation results show that the proposed method outperforms the existing methods while its complexity is only a fraction of those methods.

References

[1] T. Pollet, M. Van Bladel, and M. Moeneclaey, "BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise," *IEEE Transactions on Communications*, vol. 43, pp. 191-193, Feb.-Apr. 1995.

[2] M. Morelli and M. Moretti, "Carrier Frequency Offset Estimation for OFDM Direct-Conversion Receivers," *IEEE Transactions on Wireless Communications*, vol. 11, no. 7, pp. 2670-2679, July 2012.

[3] L. Wu, X-D. Zhang, P-S. Li, and Y-T. Su, "A closed-form blind CFO estimator based on frequency analysis for OFDM systems," *IEEE Transactions on Wireless Commun.*, vol. 57, no.6, pp. 1634-1637, June 2009.

[4] Y. Yao and G. B. Giannakis, "Blind Carrier Frequency Offset Estimation in SISO, MIMO, and Multiuser OFDM Systems," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 173-183, Jan. 2005.

[5] J-J. van de Beek, M. Sandell, and P. O. Borjesson, "ML Estimation of Time and Frequency Offset in OFDM Systems," *IEEE Transactions on Signal Processing*, vol. 45, no. 7, pp. 1800-1805, July 1997.

[6] H. Liu and U. Tureli, "A High-Efficiency Carrier Estimator for OFDM Communications," *IEEE Commun. Letters*, vol. 2, pp. 104-106, Apr. 1998.

[7] U. Tureli, H. Liu, and M. D. Zoltowski, "OFDM blind carrier offset estimation: ESPRIT," *IEEE Trans. Commun.*, pp. 1459-1461, Sep. 2000.

[8] F. Gao, Y. Zeng, A. Nallanathan, and T-S. Ng, "Robust Subspace Blind Channel Estimation for Cyclic Prefixed MIMO OFDM Systems: Algorithm, Identifiability and Performance Analysis," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 2, pp. 378-388, Feb. 2008.