Cepstrum based detection and classification of OFDM waveforms

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Abstract—This paper presents cepstral analysis of OFDM signals. Cepstrum can reveal periodicities in a signal and has been widely used in audio and speech processing applications. In this work, the focus is on cepstrum based detection and classification of OFDM signals for cognitive radio applications such as flexible spectrum reuse and coexistence of heterogeneous networks. Two cepstrum based sensing schemes formulated as hypothesis testing are proposed. The distributions of the test statistics are derived under the null hypothesis so that the thresholds for the Neyman-Pearson detectors can be computed analytically. These cepstrum based schemes are compared to the traditional energy detector. First scheme is robust to noise uncertainty which is a clear benefit when compared to the energy detector. On the other hand, the second cepstrum based scheme has performance similar to the energy detection. Later, it is shown that the cepstral analysis can be used to estimate parameters of OFDM waveforms such as number of samples in data and cyclic prefix (CP) parts of an OFDM symbol. These features can be used to distinguish among different OFDM waveforms, which is not possible with energy detection.

Index Terms—Cepstrum analysis, classification, cognitive radios, heterogeneous networks, sensing.

I. INTRODUCTION

Cognitive radios and coexistence of heterogeneous wireless systems have become important research topics lately [1], [2], [3]. Cognitive radios offer a novel solution to increase the spectrum utilization through dynamic spectrum access along with flexible allocation policy. Similarly, the number of heterogeneous networks potentially occupying same frequency bands is going to rise in the future [1]. In such scenarios, acquiring awareness of the spectrum state such as identifying idle spectrum and classifying different users (e.g., primary, secondary, noise, etc.) in the network is of utmost importance.

Orthogonal frequency division multiplexing (OFDM) is a key wideband digital transmission technology for the present as well as future wireless systems [4]. Therefore, it is reasonable to assume that many licenced and secondary spectrum users will transmit OFDM waveforms with different parameters. Thus detection and classification of OFDM waveforms is an important research problem for spectrum monitoring and coexistence of heterogeneous networks. Several algorithms for detecting OFDM based primary users are discussed in [5].

The cepstrum is defined as the inverse discrete Fourier transform (IDFT) of the logarithm magnitude of the discrete Fourier transform (DFT) of a signal [6]. It captures the information about the rate of change in different spectrum bands. Cepstral processing was first used for characterizing the seismic echoes resulting from earthquakes [7]. Since then, it has been used in various applications such as radar and sonar, speech processing, deconvolution of two or more signals [8]. It has been often used in speech processing applications such as pitch estimation for voice coding or voice activity detection [9], [10].

The use of cyclic prefix (CP) in an OFDM symbol induces periodicity, which is a property that can be used for detection as shown in [11], [12]. Since the cepstrum has the ability to reveal periodocities in a signal, it can detect a CP-OFDM signal. This motivates the use of cepstral analysis of OFDM signals in this paper. It is shown that in addition to the DC peak, the mean of OFDM cepstrum has distinct peaks at indices which are integer multiples of the number of samples in the data part of an OFDM symbol. Also variance of the OFDM cepstrum shows triangular peaks at the integer multiples of length of one OFDM symbol. On the other hand, the mean and variance of noise cepstrum is flat, except for the DC index, under the assumption that the noise is uncorrelated.

Based on the above mentioned differences between the cepstrums of OFDM signal and noise, we propose two OFDM waveform detection methods in this paper. The distributions of the two test statistics are established for noise only hypothesis so that the threshold for a Neyman-Pearson detector can be calculated analytically. Next, the performance of these detectors is compared to the classical energy detector [15] both in the absence and presence of noise uncertainty. It is shown that the first detector is robust to noise uncertainty and outperforms the energy detector even in the case of small noise uncertainty while the performance of the second detector is closer to that of the energy detector. Moreover, we propose algorithms to estimate the number of samples in the data and CP parts of an OFDM symbol based on the features in the mean and variance of the OFDM cepstrum. The parameters of payload data length and CP length in an OFDM symbol are generally distinct for different wireless standards and therefore can be used to classify OFDM signals. This is an added advantage over the energy detection scheme which cannot distinguish between differents signals.

The organization of this paper is as follows. In Sec. II, a brief introduction to cepstral analysis is given followed by cepstral analysis of an OFDM signal and derivation of the distributions of the decision statistics. In Sec. III, proposed detection and classification algorithms are presented. In Sec. IV, results are presented to show the performance of the proposed algorithms and comparison with classical energy detector is also carried out. Sec. V concludes the paper.

II. CEPSTRUM ANALYSIS OF OFDM SIGNAL

A. Cepstrum

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Cepstrum is defined to be an IDFT of the logarithmic magnitude spectrum of a signal [6]. There are different cepstrum definitions such as complex cepstrum, real cepstrum, power cepstrum, and phase cepstrum. The most appropriate definition for our purposes is real cepstrum, which is given by



Fig. 1. In an OFDM symbol, N_c last samples of the useful data are prefixed to the data samples. Here N_d is the number of samples in the data part of an OFDM symbol, $N_s = N_c + N_d$ is the number of samples in one OFDM symbol, and B is the number of OFDM symbols.

for $n = 0, 1, ..., N_r - 1$, where log denotes natural logarithm, N_r is the size of DFT/IDFT in the cepstrum process while \mathcal{F} and \mathcal{F}^{-1} denote DFT and IDFT respectively.

B. OFDM transmission

An OFDM signal consists of sum of subcarrier signals that are typically modulated by using phase shift keying (PSK) or quadrature amplitude keying (QAM). If N_d is the number of subcarriers, then N_d symbols are fed to IDFT. The N_d symbols long IDFT output forms the data part of one OFDM symbol. A complete OFDM symbol is created by prefixing the data part with its own last N_c samples as shown in Fig.1. If OFDM signal contains *B* OFDM symbols each of $N_s = N_c + N_d$ samples, then the total number of samples in OFDM signal is $M = B(N_c + N_d)$.

C. Binary hypothesis test

In this paper, the detection of OFDM signal is formulated as binary hypothesis test. Let H_0 denote the null hypothesis that the OFDM based primary (or secondary) user transmission is not present while let H_1 denote the alternate hypothesis that the OFDM based primary (or secondary) user is active.

$$H_0: \quad x(n) = w(n) H_1: \quad x(n) = s(n) + w(n),$$
(2)

where x(n) is the received signal, w(n) additive white Gaussian noise (AWGN) and s(n) is an OFDM signal to be detected. Let σ_s^2 and σ_w^2 denote the signal and noise variances, respectively. Assuming sufficiently large IDFT size for the OFDM signal, and invoking the central limit theorem (CLT), we have $s(n) \sim \mathcal{N}_c(0, \sigma_s^2)$. Here it has been assumed that s(n) and w(n) are independent of each other and $\mathcal{N}_c(.)$ denotes distribution for circularly symmetric complex Gaussian random variable. Therefore, the distribution of x(n) under the two hypotheses is given by

$$H_0 : x(n) \sim \mathcal{N}_c(0, \sigma_w^2) H_1 : x(n) \sim \mathcal{N}_c(0, \sigma_s^2 + \sigma_w^2).$$
(3)

D. Cepstrum of OFDM signal

Figs. 2 and 3 show mean and variance of the received signal cepstrum under the null and alternate hypotheses averaged over 10000 realizations and simulation parameters of $N_d = 64$, $N_c = 16$, B = 26, $N_r = 2048$, SNR = 10 dB. Fig. 2 shows that the mean of the cepstrum for AWGN has only one peak appearing at the zeroth index (or the DC index) while other values are zero. On the other hand, the mean of the real part of the OFDM cepstrum shows distinct peaks at integer multiples of N_d in addition to the DC peak. This property can be exploited in designing detection and classification methods for OFDM signals. The additional peaks do not appear in the imaginary part of the cepstrum which justifies the use of real cepstrum only.

It can be seen from Fig. 3 that the real parts of the coefficients have constant variance under H_0 , which is not the case under H_1 . When OFDM signal is present the variances start to rise at the indices



Fig. 2. A zoom of mean of real part of ceptrum of signals under H_0 and H_1 averaged over 10000 realizations and simulation parameters of $N_d = 64$, $N_c = 16$, B = 26, $N_r = 2048$, SNR= 10dB. The AWGN ceptrum has only one peak at the DC peak. On the other hand, when OFDM signal is also present, there are additional peaks at the indices of N_d and $2N_d$.



Fig. 3. Variances of cepstral coefficients under both hypotheses H_0 and H_1 averaged over 10000 realizations and simulation parameters of $N_d = 64, N_c = 16, B = 26, N_r = 2048$, SNR= 10dB. While variance plot of AWGN cepstrum is flat, presence of OFDM causes cepstrum to have pyramidal shapes peaking at the integer multiples of N_s .

of $lN_s - N_d$ (l = 1, 2, 3, ...), peaking at the indices of lN_s . Also variances at indices smaller than N_d are significantly lower than under H_0 . Similar behavior is observed also in the imaginary parts of the coefficients. It can be suggested that these effects can be used either to estimate cyclic prefix length or to detect the presence of OFDM transmission.

E. Distributions of cepstral coefficients under H_0

This section focuses on deriving the distributions of cepstral coefficients c(n). In this paper, we derive the distribution for c(n) under H_0 only as finding closed form expression for the distributions under H_1 is difficult.

The x(n) has Gaussian distribution under H_0 given by (3). After taking DFT, which is a unitary transform, the distribution remains the same and therefore $X(k) \sim \mathcal{N}_c(0, \sigma_w^2)$. In [13], $Z(k) = \log |X(k)|$ is shown to follow circular Log-Rayleigh distribution, i.e., $Z(k) \sim LogRay(\sigma_w^2/2)$. The probability distribution function for a circular Log-Rayleigh random variable Z is given in [13] by

$$p_Z(z) = \frac{(e^z)^2}{\beta^2} \exp\left(-\frac{(e^z)^2}{2\beta^2}\right),$$
 (4)

where $\beta^2 = \sigma_w^2/2$ is the localization parameter. The circular Log-Rayleigh distribution is characterized by a property that all its central moments of order higher than one are independent of localization parameter. The mean and variance of Z are given by

$$\mu_z = \log \beta + \frac{\log 2}{2} - \frac{\gamma}{2}, \text{ and } \sigma_z^2 = \frac{\pi^2}{24},$$
 (5)

where γ is the Euler's constant ($\gamma \approx 0.577216$). Substituting Z(k) =



Fig. 4. Distribution of $\mathcal{R}\{c(N_d)\}$ under hypothesis \mathcal{H}_0 . The theoretical and simulated distributions are on par.

 $\log |X(k)|$, we can rewrite (1) as

$$c(n) = \frac{1}{\sqrt{N_r}} \sum_{k=0}^{N_r - 1} Z(k) e^{\frac{j2\pi kn}{N_r}}.$$
 (6)

Since c(n) is sum of rotated i.i.d. Log-Rayleigh random variables $Z(k)e^{\frac{j2\pi kn}{N_r}}$, assuming N_r to be sufficiently large we can use CLT to approximate the distribution of c(n) under the null hypothesis to be Gaussian. The mean of c(n) can be derived as follows

$$\mathbf{E}[c(n)] = \frac{1}{\sqrt{N_r}} \sum_{k=0}^{N_r - 1} \mathbf{E}[Z(k)] e^{\frac{j2\pi kn}{N_r}} = \frac{1}{\sqrt{N_r}} \mu_z \sum_{k=0}^{N_r - 1} e^{\frac{j2\pi kn}{N_r}}.$$
(7)

Noting that $\sum_{k=0}^{N_r-1} e^{\frac{j\pi kn}{N_r}} = N_r$ for n = 0 and 0 otherwise, we get

$$E[c(0)] = \mu_z \sqrt{N_r} E[c(n)] = 0, \quad n \neq 0.$$
(8)

Since DFT is a unitary transform, the variances of all c(n) coefficients come straight from the Log-Rayleigh distribution

$$\operatorname{Var}[c(n)] = \frac{1}{N_r} \sum_{k=0}^{N_r - 1} \operatorname{Var}[Z(k)] = \sigma_z^2 = \frac{\pi^2}{24}$$
(9)

So, we get the following distributions for c(n) under \mathcal{H}_0

$$c(n) \sim \mathcal{N}(\mu_z \sqrt{N_r}, \pi^2/24), \quad n = 0$$

$$c(n) \sim \mathcal{N}(0, \pi^2/24), \qquad n \neq 0.$$
(10)

Fig. 4 shows the theoretical and simulated distribution for $\mathcal{R}\{c(N_d)\}$. Note that $\mathcal{R}\{c(N_d)\} \sim \mathcal{N}(0, \pi^2/48)$. It can be seen that the theoretical and simulated distributions are on par.

III. DETECTION AND CLASSIFICATION OF OFDM SIGNALS

In this section, we propose two cepstrum based detection algorithms. The decision is made by comparing a scalar test statistic T to a threshold value η . If the test statistic is greater than η , H_1 is declared. Otherwise H_0 is declared. The threshold η depends on the detection strategy and in our case we consider Neyman-Pearson criterion which maximizes probability of detection with a constraint on the maximum false alarm probability P_f . Note that we only need establish the distribution of the test statistic under H_0 to design a Neyman-Pearson detector. Later in this section, we present algorithms to estimate N_d and N_c parameters of the OFDM signal from its cepstral analysis.

A. Detection based on the cepstral coefficient $c(N_d)$

In Sec. II-D, it was seen that the mean of the real part of OFDM cepstrum has peaks at integer multiples of N_d in addition to the DC peak while those peaks are not present in AWGN cepstrum. The first cepstrum based sensing scheme is proposed on this distinguishing factor. Note that although there are multiple peaks, the magnitude of

the peak values decrease rapidly after the first peak at N_d . As the peak values are unknown, weighted combining of these multiple peaks for detection is difficult. Also using equal gain combining (EGC) may deteoriate the performance. Therefore we suggest to use the following test statistic based on only the first peak at N_d :

$$T_1 = \mathcal{R}\{c(N_d)\}.$$
(11)

Now, from (10), $T_1 \sim \mathcal{N}(0, \pi^2/48)$. The false alarm probability for a test statistic with Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$ is given by

$$P_f = \frac{1}{2} \operatorname{erfc}\left(\frac{\eta - \mu}{\sqrt{2}\sigma}\right). \tag{12}$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function. Using (12), the threshold for Neyman-Pearson detector using test statistic T_1 is given by

$$\eta_1 = \sqrt{\frac{\pi^2}{24}} \operatorname{erfc}^{-1}(2P_f).$$
 (13)

It can be seen that the threshold value is independent of the noise power. Therefore the proposed detector is robust to noise power uncertainty.

B. Detection based on the cepstral coefficient c(0)

It was seen in Sec.II-D that the mean of the cepstral coefficient c(0) increases in the presence of OFDM signal as compared to AWGN alone. Using this property, a detection scheme with the following test statistic is proposed

$$T_2 = c(0).$$
 (14)

From the distribution of (10), $T_2 \sim \mathcal{N}(\mu_z \sqrt{N_r}, \pi^2/24)$. Using (12), the threshold for a Neyman-Pearson detector using test statistic T_2 is given by

$$\eta_2 = \sqrt{\frac{\pi^2}{12}} \operatorname{erfc}^{-1}(2P_f) + \sqrt{N_r} (\log\beta + \frac{\log 2}{2} - \frac{\gamma}{2}).$$
(15)

In this scheme, the localization parameter $\beta^2 = \sigma_w^2/2$ and thus the threshold depends on the noise power σ_w^2 . Hence, the algorithm is not robust to the noise power uncertainty, which is a drawback when compared to the scheme based on $c(N_d)$.

C. Estimation of the data length in an OFDM symbol

It was seen in Fig. 2 that the mean of the real part of the OFDM cepstrum has a dominant peak at N_d in addition to the DC peak. This feature can be used to extract payload data length parameter of a OFDM symbol from the observed signal by searching the index of the maximum cepstral coefficient value excluding the DC peak. Therefore, the estimate of N_d is given by

$$\hat{N}_d = \arg\max_n \mathcal{R}\{c(n)\}, \quad n = 1, \dots, \left\lfloor \frac{N_r}{2} \right\rfloor.$$
 (16)

Note that being the DFT of a real signal, the cepstrum is symmetrical and hence only the first half of the cepstral coefficients need to be used in detection and classification purposes.

The error in the estimate will depend on the set of allowed values. Generally there are only few possible values of N_d and N_c for each OFDM signal. For example, DVB-T has option of 2048 and 8192 values for N_d while N_c can take values from 1/4, 1/8, 1/16 and 1/32 of N_d [14]. The accuracy of the estimate can be improved if the estimate \hat{N}_d is allowed to take values from a finite set of possible values. Without loss of generality, we consider a set containing powers of two because generally the number of subcarriers in an OFDM symbol is a power of 2 since it facilitates efficient



Fig. 5. Probability of detection vs. SNR for the two proposed cepstrum based sensing schemes in addition to energy detection. Neyman-Pearson detection strategy is used for all of the detectors. All schemes maintain the desired false alarm rate. Scheme based on the cepstral coefficient $c(N_d)$ is robust to noise uncertainty and performs better than the energy detector even in the presence of noise uncertainty of only 1 dB. The performance of the proposed scheme based on the cepstral coefficient c(0) is close to that of energy detection.

FFT/IFFT implementation. Limiting the set of values allowed for N_d , the estimate of N_d is given by

$$\hat{N}_d = \arg\max_{\mathbf{n}} \ \mathcal{R}\{c(n)\}, \quad n = 2^m, m \in (1, 2, \dots, \log_2 \left\lfloor \frac{N_r}{2} \right\rfloor).$$
(17)

D. Estimation of the CP length in an OFDM symbol

As seen in Fig. 3, the variances of the different cepstral coefficients follow a repetitive pattern when an OFDM signal is present. The period of pyramid-like peaks is N_s samples, which can be estimated by finding the highest correlation for different lags τ by

$$\hat{N}_{s} = \arg \max_{\tau \in \mathcal{A}} \ \frac{1}{M_{v}} \sum_{n=N_{d}}^{N_{d}+M_{v}} V_{c}(n) V_{c}(n+\tau),$$
(18)

where $V_c(n)$ is the estimated variance of $\mathcal{R}\{c(n)\}$, $M_v < N_r/2 - N_d$ and \mathcal{A} is set of all integers between $N_d + 2$ and $\frac{3N_d}{2}$ under a reasonable assumption that N_c can take integer values between 2 and $\frac{N_d}{2}$, and N_d is an even number. Now the estimate of number of samples in cyclic prefix $\hat{N}_c = \hat{N}_s - N_d$.

Similar to the case of N_d it is also possible to limit the possible values for \hat{N}_c to be integer powers of 2 less than $N_d/2$. For such case, $\mathcal{A} = \{\frac{33N_d}{32}, \frac{17N_d}{16}, \frac{9N_d}{8}, \frac{5N_d}{4}, \frac{3N_d}{2}\}$.

IV. RESULTS

All simulations are done in Matlab and following parameters are used: $P_f = 0.05$, $N_d = 64$, $N_c = 16$, B = 26, $N_r = 2048$ and $\sigma_w^2 = 1$. In all simulations, the results are averaged over 10000 realizations.

A. Detection of OFDM signal

Fig.5 shows probability of detection vs SNR (dB) curves for the proposed cepstrum based schemes and the traditional energy detector when noise power is assumed to be known. Neyman-Pearson based detection strategy is used for all the detectors. In practice, noise level is always uncertain and needs to be estimated [16]. Hence, we have also included detection results with noise power uncertainty of 1 dB. It can be seen that the detection scheme based on the cepstral coefficient $c(N_d)$ performs worse than energy detection when noise power is perfectly known. However, even when a small uncertainty of 1 dB is present in the noise power, the performance of energy detection degrades drastically. On the other hand, the scheme based on $c(N_d)$ is insensitive to noise uncertainty and is very valuable for practical scenarios as it provides a good trade-off between detection performance and robustness. Although the



Fig. 6. Mean and variance of the estimate \hat{N}_d . The mean converges to the correct value 64 and variance converges to zero as the SNR increases.



Fig. 7. Mean and variance of the \hat{N}_c value. Mean converges to the correct value 16 and variance reduces as the SNR increases. When the set of possible N_c values is finite containing a set of known possible values, the variance of N_c converges to zero.

performance of the detection scheme using c(0) is close to that of the energy detector, the performance degrades similar to energy detector in the presence of noise uncertainty.

B. Estimation of N_d and N_c

Figs. 6 and 7 show the mean and the variance of the estimates \hat{N}_d and \hat{N}_c , respectively, with different SNR values. It can be seen that both estimates converge to their desired values and their variances decrease as the SNR increases.

Note that in the low SNR regime, the estimates of integer valued N_d and N_c behave like uniform random variables and hence the mean and variance depend on the set of allowed values. For example, the mean and variance for $\hat{N_c}$ from simulation over the set $\{2, 4, 8, 16, 32\}$ are 12.4 and 119 which are same as that for a uniform discrete random variable over the same set.

V. CONCLUSION

In this paper we show that the cepstral features of OFDM signals can be used to detect and classify different OFDM waveforms. Two cepstrum based test statistics are proposed and their distributions under the null hypothesis have been derived. The proposed detection scheme based on the cepstral coefficient $c(N_d)$ is robust to the noise uncertainty and performs better than the energy detection even in the presence of small noise uncertainty (1 dB). Although the performance of the second scheme based on c(0) is very close to that of the energy detector, it is not robust to the noise uncertainty. It has also been shown that OFDM symbol's data size N_d can be estimated based on the location of peak in the mean of cepstrum. Similarly the number of samples in cyclic prefix N_c can be estimated based on the peak in the cepstrum variance. For both the estimators, the mean of the estimates converge to the true value and variance of the estimates decrease as the SNR increases. These properties can be used in automatic classification and recognition of OFDM signals, for example.

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