SHORT PREAMBLE-BASED ESTIMATION OF HIGHLY FREQUENCY SELECTIVE CHANNELS IN FBMC/OQAM

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ABSTRACT

Channel estimation in multicarrier systems employing filter banks with offset quadrature amplitude modulation (FBMC/ OQAM) is known to face challenges related to the interference effect intrinsic to these systems. Most of the methods reported thus far rely on the assumption of (almost) flat subchannels - implying a low frequency selective channel - to more easily tackle this problem. Others, more recent ones, relax this assumption but they either depend on a model for the channel or they concentrate on per-subchannel estimation and often require long training sequences. This paper revisits this problem by proposing a method that makes no assumption on the channel frequency selectivity, while using a very short preamble for training. The latter is optimized with respect to the channel estimation mean squared error. The superiority of the proposed method over classical ones is demonstrated via simulation results for both mildly and highly frequency selective channels.

Index Terms— Channel estimation, filter bank multicarrier, intrinsic interference, offset quadrature amplitude modulation, orthogonal frequency division multiplexing, preamble.

1. INTRODUCTION

Filter bank-based multicarrier (FBMC) systems based on offset quadrature amplitude modulation (known as FBMC/OQAM or OFDM/OQAM) have recently attracted increased interest (in applications including DVB-T [2], cognitive [3] and professional mobile radio [28], and powerline communications [1]) due to their enhanced flexibility, higher spectral efficiency, and better spectral containment compared to conventional OFDM [10]. Notably, there is no need to insert a guard interval such as the cyclic prefix (CP) common in OFDM.

This work was supported by an FP7 grant, project EMPhAtiC (http://www.ict-emphatic.eu/). Part of this work was done while the author was in the Centre Tecnològic de Telecomunicacions de Catalunya (CTTC), Spain, on leave from the University of Piraeus, Greece.

FBMC/OQAM suffers, however, from an intrinsic intercarrier/inter-symbol interference that complicates signal processing tasks such as channel estimation [14]. Most of the methods reported thus far in the literature rely on the assumption of (almost) flat subchannels – valid for channels with sufficiently low frequency selectivity – to more easily tackle this problem, with the aim of addressing it in a way similar to OFDM; see [21] and references therein. However, this assumption may be often quite inaccurate, due to the high frequency selectivity of the channel [30, 25, 5] and/or the small number of subcarriers employed to cope with frequency dispersion in fast fading environments [11]. In such cases, and due to the residual interference generated by the inaccuracies in the system model adopted, severe error floors are exhibited at medium to high signal-to-noise ratio (SNR) values, which cancel the advantage of this modulation over CP-OFDM at these SNR regimes. Other, more recent methods relax the subchannel flatness assumption but they either rely on a model for the channel [12] or they only provide estimates of the subchannel responses (e.g., [5, 4]) often requiring quite long preambles for their training [30, 26].

The goal of this paper is to revisit this problem through an alternative formulation that focuses on the estimation of the channel impulse response (CIR) itself and makes no assumption on the degree of frequency selectivity of the subchannels. The possible gains in estimation performance offered by such an approach are investigated through the design of optimal (in the mean squared error (MSE) sense) preambles, consisting of only *one* pilot FBMC symbol. Simulation results are presented, for both moderately and highly frequency selective channels, that demonstrate the significant improvements in performance and robustness to channel dispersion offered by the proposed approach over both CP-OFDM and the optimal flat subchannel-based FBMC/OQAM method. Most notably, no error floors appear anymore over a quite wide range of SNR values.

The rest of the paper is organized as follows. An alternative formulation of the FBMC/OQAM system in the presence

of channel and noise is developed in Section 2. Section 3 then goes ahead with deriving the CIR estimator and paving the way to preamble design. The latter is addressed in Section 4, for the more interesting case of *full* preambles, i.e. with all subcarriers carrying pilots [21]. Simulation results are reported and commented in Section 5.

2. SYSTEM MODEL

The (baseband) output of an FBMC/OQAM synthesis filter bank (SFB) can be written as [29]

$$s(l) = \sum_{m=0}^{M-1} \sum_{n} a_{m,n} g_{m,n}(l), \tag{1}$$

where (m,n) refers to the mth subcarrier and the nth FBMC symbol, $a_{m,n}$ are real OQAM symbols, M is the (even) number of subcarriers, and

$$g_{m,n}(l) = g\left(l - n\frac{M}{2}\right)e^{j\frac{2\pi}{M}m\left(l - \frac{L_g - 1}{2}\right)}e^{j\varphi_{m,n}},$$

with g being the employed prototype filter impulse response (assumed of unit energy) with length L_g , and $\varphi_{m,n}=(m+n)\frac{\pi}{2}+mn\pi$. Moreover, usually $L_g=KM$, with K being the overlapping factor. The corresponding output of a channel with impulse response h of length L_h will be

$$y(l) = \sum_{k=0}^{L_h-1} h(k)s(l-k) + w(l), \tag{2}$$
 with $w(l)$ assumed to be zero mean Gaussian noise with vari-

with w(l) assumed to be zero mean Gaussian noise with variance σ^2 . The analysis filter bank (AFB) output at the (p,q) frequency-time (FT) point is given at the top of the next page, with $\eta_{p,q}$ denoting the corresponding noise component. The latter equation describes the AFB output samples in terms of the channel impulse response, that is, in a *time domain* formulation. Its value is found in the fact that it is *exact* regardless of the relative channel delay spread (only assumed not to exceed M). Recall that the usual frequency-domain input/output description [23]

$$y_{p,q} = H_{p,q} a_{p,q} + \text{interference} + \eta_{p,q}$$

is only valid (to an approximation) for short enough (relative to the extent of g) channels. Eq. (3) is thus a good candidate in providing a way to address channel estimation for the general case, namely when the subchannels are not well approximated by the frequency flat model.

3. CHANNEL ESTIMATION

Consider the preamble-based channel estimation problem. Assume, moreover, that the preamble is constructed in the usual manner [21], namely consisting of 2 FBMC symbols, with the second consisting of all zeros.¹ This is to protect the

pilots from being interfered by the unknown data (or control) samples of the current frame. Then, $a_{m,n}$ is nonzero only for n=0. Also, only the case of q=0 is of interest. Rewriting eq. (3) with these assumptions yields

$$y_{p,0}=\sum_{k=0}^{L_h-1}\Gamma_{p,k}h(k)+\eta_{p,0} \tag{4}$$
 where $\Gamma_{p,k}$, defined at the top of the next page, can be seen to

where $\Gamma_{p,k}$, defined at the top of the next page, can be seen to be the response of the transmultiplexer to this particular input for a channel equal to a k-samples delay, i.e., $h(l) = \delta(l-k)$, and can be easily computed. The AFB output sample is the sum of those responses, multiplied with the channel gains. In matrix-vector notation:²

 $y = \Gamma h + \eta$, (6) with $y = \begin{bmatrix} y_{0,0} & y_{1,0} & \cdots & y_{M-1,0} \end{bmatrix}^T$ and similarly for η , while $h = \begin{bmatrix} h(0) & h(1) & \cdots & h(L_h - 1) \end{bmatrix}^T$ and $[\Gamma]_{p+1,k+1} = \Gamma_{p,k}$. The matrix Γ is of dimensions $M \times L_h$, i.e., tall, and hence the equation above can be solved for the channel impulse response h using, for example, least squares (LS).

It must be emphasized here that the noise η is known to be zero mean Gaussian with the same variance as w, however it is not uncorrelated among subcarriers [21]. Its $M \times M$ covariance matrix is known to be given by

$$\boldsymbol{C}_{\boldsymbol{\eta}} = \sigma^{2} \begin{bmatrix} 1 & j\beta & 0 & \cdots & 0 & j\beta \\ -j\beta & 1 & j\beta & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ -j\beta & 0 & 0 & \cdots & -j\beta & 1 \end{bmatrix} \equiv \sigma^{2}\boldsymbol{B},$$
(7)

with $\pm j\beta$ being the correlation of $g_{m,1}$ with $g_{m\pm 1,1}$, a priori computable based on the knowledge of g [21]. C_{η} is (almost) tridiagonal and circulant (and hence diagonalizable via DFT [13]). The invertibility of this matrix is verified in [18].

In view of the color of the noise component in (6), Gauss-Markov estimation is better suited here than LS, resulting in the following estimate for h [20]

$$\hat{\boldsymbol{h}} = (\boldsymbol{\Gamma}^{H} \boldsymbol{C}_{\boldsymbol{\eta}}^{-1} \boldsymbol{\Gamma})^{-1} \boldsymbol{\Gamma}^{H} \boldsymbol{C}_{\boldsymbol{\eta}}^{-1} \boldsymbol{y}$$
 (8)

Because of the Gaussianity of the noise, this can be seen to be also the maximum likelihood (ML) estimate [20].

4. OPTIMAL PREAMBLE DESIGN

Rewrite (5) as $\Gamma_{p,k} = \mathcal{G}_{p,k}^H a$, with the obvious definition for the $M \times 1$ vectors $\mathcal{G}_{p,k}$ and a. One can then express the matrix Γ in the form:

$$\Gamma = \mathcal{G}A. \tag{9}$$

$$\mathcal{G} = \begin{bmatrix} \mathcal{G}_{0,0}^{H} & \mathcal{G}_{0,1}^{H} & \cdots & \mathcal{G}_{0,L_{h}-1}^{H} \\ \mathcal{G}_{1,0}^{H} & \mathcal{G}_{1,1}^{H} & \cdots & \mathcal{G}_{1,L_{h}-1}^{H} \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{G}_{M-1,0}^{H} & \mathcal{G}_{M-1,1}^{H} & \cdots & \mathcal{G}_{M-1,L_{h}-1}^{H} \end{bmatrix}$$

¹Note that the null FBMC symbol usually placed before the pilots one is in general unnecessary, in view of the interframe time gaps commonly used in wireless transmissions. Hence, the preamble as a whole may last one OFDM symbol.

²Such a formulation was also independently derived in [22], however not addressing the training design problem.

$$y_{p,q} = \sum_{l} y(l) g_{p,q}^{*}(l) = \sum_{k=0}^{L_{h}-1} h(k) \sum_{m=0}^{M-1} e^{-j\frac{2\pi}{M}mk} \sum_{n} a_{m,n} j^{m+n-p-q} (-1)^{mn-pq} \times \sum_{l} g\left(l-k-n\frac{M}{2}\right) g\left(l-q\frac{M}{2}\right) e^{j\frac{2\pi}{M}(m-p)\left(l-\frac{L_{g}-1}{2}\right)} + \eta_{p,q}$$
(3)

$$\Gamma_{p,k} = \sum_{m=0}^{M-1} e^{-j\frac{2\pi}{M}mk} a_{m,0} j^{m-p} e^{-j\frac{2\pi}{M}(m-p)\left(\frac{L_g-1}{2}\right)} \sum_{l=k}^{L_g-1} g(l-k)g(l) e^{j\frac{2\pi}{M}(m-p)l}$$
(5)

and

$$A = I_{L_h} \otimes a$$
,

with \otimes denoting the (left) Kronecker product. The matrix \mathcal{G} has a special structure, to be elaborated in the sequel.

The MSE-optimal design of the preamble $a_{m,0}$, m = 0, 1, ..., M-1, can then be stated as follows [20]:

$$\min_{\boldsymbol{a}} \operatorname{tr} \left\{ (\boldsymbol{\Gamma}^{H} \boldsymbol{C}_{\boldsymbol{\eta}}^{-1} \boldsymbol{\Gamma})^{-1} \right\} \tag{10}$$

subject to a constraint on the transmitted energy. The latter could refer to the energy input to the SFB (e.g., $\sum_{m=0}^{M-1} |a_{m,0}|^2$) or, more realistically, to the energy of the modulated preamble at the SFB output as in [16, 18]. For the more interesting case of a *full* preamble (i.e., with all subcarriers carrying pilots), the latter is not trivially related to the former and is known [17] to equal $a^H B a$. The vector a will first be allowed to be complex-valued, as the optimal preamble does not have to be OQAM modulated (see, e.g., the IAM-I and (E)IAM-C preambles in [21]). The real-valued case will be considered later on.

Write

$$\mathcal{G} = \begin{bmatrix} \mathcal{G}_0 & \mathcal{G}_1 & \cdots & \mathcal{G}_{L_h-1} \end{bmatrix}, \qquad (11)$$
 where, for $k = 0, 1, \dots, L_h - 1$, $\mathcal{G}_k = \begin{bmatrix} \mathcal{G}_{0,k} & \mathcal{G}_{1,k} & \cdots \\ \mathcal{G}_{M-1,k} \end{bmatrix}^H$ is an $M \times M$ matrix. Regarding the optimality criterion above, it is known that an optimal a will diagonalize the matrix $(\mathbf{\Gamma}^H \mathbf{C}_{\eta}^{-1} \mathbf{\Gamma})^{-1}$ while satisfying the energy constraint, say $a^H \mathbf{B} a \leq \mathcal{E}$. Thus, the following system of $\frac{L_h(L_h-1)}{2}$ (quadratic) equations in a results:

 $a^{H} \mathcal{G}_{k}^{H} B^{-1} \mathcal{G}_{l} a = 0, \ k = 0, 1, \dots, L_{h} - 2, \ l > k$ (12)

In view of the fact that a given subcarrier only overlaps with the immediately adjacent ones, it is not unexpected that the blocks of \mathcal{G} are tridiagonal. Moreover, it is not hard to show (details are omitted here) that

$$G_k = W^k G_k, \quad k = 0, 1, 2, \dots, L_h - 1$$
 (13)

where $W = \operatorname{diag}(1, e^{-j\frac{2\pi}{M}}, e^{-j2\frac{2\pi}{M}}, \dots, e^{-j(M-1)\frac{2\pi}{M}})$ and G_k is a Hermitian (almost) tridiagonal circulant matrix. In fact, one can readily verify that $\mathcal{G}_0 = B$. Recall that $\mathcal{G}_k a$ is the response of the entire system when the channel is only a delay of k samples. Hence, $\mathcal{G}_0 a$ is the vector of *pseudo-pilots* [23, 21], already known to equal Ba.

The circulant structure of the G_k matrices can be exploited to facilitate the task of deducing the solution for a. Indeed, G_k is diagonalized with the DFT matrix [13], that is,

$$G_k = F \Lambda_k F^H, \tag{14}$$

where F is the *unitary M*-point DFT matrix and Λ_k is diagonal with the (real) eigenvalues of G_k on its main diagonal.³ Making use of the readily verified identity

$$\boldsymbol{F}^{H}\boldsymbol{W}^{k} = \boldsymbol{Z}^{k}\boldsymbol{F}^{H} \tag{15}$$

with

$$oldsymbol{Z} = \left[egin{array}{ccc} oldsymbol{0}_{1 imes (M-1)} & 1 \ oldsymbol{I}_{M-1} & oldsymbol{0}_{(M-1) imes 1} \end{array}
ight],$$

the conditions (12) then translate to the requirements

$$\tilde{\boldsymbol{a}}^{H}\boldsymbol{\Lambda}_{k}\boldsymbol{Z}^{-k}\boldsymbol{\Lambda}_{0}^{-1}\boldsymbol{Z}^{l}\boldsymbol{\Lambda}_{l}\tilde{\boldsymbol{a}}=0, \text{ for } l>k,$$
 (16)

where $\tilde{a} = F^H a$.⁴ It can be verified that the matrix in (16) has the form

$$\begin{bmatrix} \mathbf{0}_{(M-l)\times(l-k)} & \mathbf{L}_1 & \mathbf{0}_{(M-l)\times k} \\ \mathbf{0}_{k\times(l-k)} & \mathbf{0}_{k\times(M-l)} & \mathbf{L}_2 \\ \mathbf{L}_3 & \mathbf{0}_{(l-k)\times(M-l)} & \mathbf{0}_{(l-k)\times k} \end{bmatrix}$$
(17)

with the L matrices being diagonal of order M-l, k, and l-k, respectively. Invoking results from [13, 31], one can show that the sequence of the first $\frac{M}{2}+1-k$ (or last $k-1-\frac{M}{2}$, whichever is nonnegative) diagonal entries of Λ_k and that of the remaining ones are each evenly symmetric. This property, along with (16), (17), and the equivalent energy constraint $\tilde{\boldsymbol{a}}^H \Lambda_0 \tilde{\boldsymbol{a}} \leq \mathcal{E}$ suggest the following choice for the optimal $\tilde{\boldsymbol{a}}$:

$$\tilde{a} = \sqrt{\mathcal{E}/\lambda_{0,i}} \, e_i, \tag{18}$$

where e_i denotes the ith pinning vector, namely a vector of all zeros except for a unity at the ith position, and $\lambda_{0,i} = [\mathbf{\Lambda}_0]_{i,i}.$ $i \in \{1,2,\ldots,M\}$ is to be determined so that (10) is minimized. Defining from (16) the diagonal matrix $\mathbf{\Delta}_k = \mathbf{\Lambda}_k \mathbf{Z}^{-k} \mathbf{\Lambda}_0^{-1} \mathbf{Z}^k \mathbf{\Lambda}_k = \mathbf{\Lambda}_k^2 \left(\mathbf{Z}^{-k} \mathbf{\Lambda}_0 \mathbf{Z}^k\right)^{-1}$, the diagonal entries of the matrix in (10) are proportional to $\sum_{k=0}^{L_h-1} \frac{1}{\delta_k}$, with the quantities $\delta_k = \tilde{a}^H \mathbf{\Delta}_k \tilde{a}$ being clearly nonnegative and $\delta_0 = \mathcal{E}$ for \tilde{a} as in (18). The optimal i can then be determined

³For a closed form expression of the eigenvalues of $G_0 = B$, see [18].

⁴Note that \tilde{a} is the output of the IDFT block in the polyphase realization of the synthesis filter bank, and hence the input to the polyphase network [29].

as $i_{\mathrm{opt}} = \arg\min_i \lambda_{0,i} \sum_{k=1}^{L_h-1} \frac{\lambda_{0,((k+i))_M}}{\lambda_{k,i}^2}$, where $((\cdot))_M$ denotes that the argument is circularly shifted back to the set $\{1,2,\ldots,M\}$ if it exceeds M. This (offline) search can be facilitated if the symmetries of the Λ matrices described above are utilized. Then (18) results in the following choice for the SFB input:

 $\boldsymbol{a} = \sqrt{\frac{\mathcal{E}}{\lambda_{0,i_{\text{opt}}}}} \boldsymbol{f}_{i_{\text{opt}}}, \tag{19}$

with $f_{i_{\mathrm{opt}}}$ being the corresponding column of the matrix F, which is complex-valued in general.

Constraining a to be real-valued implies a conjugate symmetry for \tilde{a} , which in the light of (18) permits for i_{opt} only one of the values 1 and $\frac{M}{2}+1$. (Note also that $\lambda_{0,1}=\lambda_{0,\frac{M}{2}+1}$.) These correspond to all equal $a_{m,0}$'s or equal with alternating signs, respectively.⁵ The former choice turns out to be the best.

5. SIMULATION RESULTS

Training with preambles designed as above leads to the results shown in Fig. 1. Filter banks designed as in [6] were employed. Two examples corresponding to channels with relatively moderate and high frequency selectivity are provided. The frequency domain normalized MSE (NMSE) is plotted, that is $E(\|\boldsymbol{H} - \hat{\boldsymbol{H}}\|^2 / \|\boldsymbol{H}\|^2)$, with \boldsymbol{H} denoting the M-point channel frequency response (CFR). Real pilots are employed for the proposed estimator and, in addition to the optimal ones derived above, pseudorandom (equipowered) pilots are also tested for the sake of comparison. Due to its optimality among the preambles of the given structure for channels of low delay spread, the IAM-C method is employed as a benchmark representative of the methods assuming flat subchannels [8, 27, 21]. CP-OFDM is also included in the comparison, using the shortest allowable CP, namely equal to the channel order. For CP-OFDM, the full preamble is optimally chosen as one of the columns of the DFT matrix, as suggested by [16, 15]. For IAM-C, the preamble is as in [18]. For a fair comparison, the SFB output training signals for the FBMC/OOAM methods were scaled so as to have the same power with that of CP-OFDM. For the sake of the completeness of the comparison, and to demonstrate the effect of the channel length knowledge, the frequency-domain methods (i.e., IAM-C and CP-OFDM) are also enriched with a DFT-interpolation step. This consists of filtering the CFR estimates through taking the inverse DFT and truncating it to the assumed known length L_h (see [7, 24, 19]). To simplify these steps, sampled-spaced channels were considered [9]. These

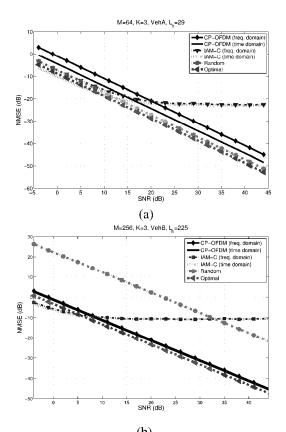


Fig. 1. MSE performance of the methods under study for channels of (a) medium $(L_h \approx M/2)$ and (b) high $(L_h \approx M)$ frequency selectivity.

variations are named "time domain," to distinguish them from the "frequency domain" estimation methods.

The method developed above generally outperforms IAM-C in both examples and as expected does not exhibit error floors, typical for IAM at medium to high SNR values. The gain from the use of the proposed preamble design over a pseudorandom one is made evident. The latter is useless with long channels (cf. Fig. 1(b)) as it then induces ill conditioning for the matrix of (8). However, the method performs slightly worse than IAM-C at low SNRs. This may be due to the inadequateness of a single FBMC preamble symbol in these scenarios. Extending the method to employ longer preambles is a subject of on-going research. On the other hand, IAM-C has already shown advantage at low SNRs over methods relying on less inaccurate assumptions than itself [18]. This is due to the strong noise attenuation achieved by its maximized "pseudo-pilots" in noise-(not interference-) limited regimes. Concluding, it must be emphasized that the good performance attained by the proposed method is achieved with only one pilot FBMC symbol, in contrast to other methods (e.g., [30, 27, 5, 26]) requiring long training sequences that imply considerable bandwidth loss, complexity increase, and lack of robustness to channel time variations.

⁵That the real pilots must all be of the same energy can also be seen by applying in (16) the symmetry of $\tilde{\boldsymbol{a}}$ together with the property of each of the $\boldsymbol{\Lambda}_k$ matrices that the sums $\lambda_{k,r} + \lambda_{k,r+\frac{M}{2}}$ are equal for $r=1,2,\ldots,\frac{M}{2}$. This finally leads to the conditions $\tilde{\boldsymbol{a}}^H \boldsymbol{Z}^{-k} \tilde{\boldsymbol{a}} = 0$, for $k=1,2,\ldots,L_h-1$, which can (by (15)) in turn be written as $\sum_{m=0}^{M-1} a_{m,0}^2 e^{j\frac{2\pi}{M}mk} = 0$ for all k's of interest.

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