

COMPRESSED SPECTRUM SENSING IN COGNITIVE RADAR SYSTEMS

Pietro Stinco¹, Maria Greco¹, Fulvio Gini¹ and Mario La Manna²

¹Department of “Ingegneria dell’Informazione”, University of Pisa, Via G. Caruso 16, Pisa, 56122, Italy

²SELEX-ES, Via Tiburtina Km 12, Rome, Italy

ABSTRACT

Compressed Sensing is a new signal processing methodology that allows to reconstruct sparse signals using a relatively small number of samples in the form of random projections. These samples are collected at a much lower rate than Nyquist rate. This paper focuses on the application of Compressed Sensing in Cognitive Radar systems that use wide operating frequency bandwidths for spectrum sensing and sharing. Compressed sensing can provide a significant reduction in acquisition time reducing the cost for high resolution analog-to-digital converters with large dynamic range, and high speed signal processors.

Index Terms— Compressed Sensing, Spectrum Sensing, Cognitive Radar, Analog-to-Digital Converter.

1. INTRODUCTION

In recent years, interest is growing in the design of cognitive radar systems. Much of this interest comes from the fact that new applications in the communication field are pushing towards the redefinition of international regulations to allocate these applications in the bands previously used by radar systems. As a consequence, the availability of frequency spectrum for radar sensors is going to be severely reduced. On the other hand, owing to the introduction of new requirements (e.g. classification, discrimination, identification), future radar sensors should be able to work on a wider range of frequencies than in the past. In order to cope with the above two opposite issues, future radars should be able to coexist with other radio frequency systems and therefore they require the ability to recognize and react to the behavior of other users radiating in the same operational environment. This, in turn, leads to the need of new methodologies and techniques, based upon cognition as enabling technology.

The cognitive methodology to reduce mutual interference between the radar and the other radiating elements is based on two main concepts: Spectrum Sensing and Spectrum Sharing. Spectrum Sensing has the goal to recognize the frequencies used by other systems using the same spectrum in real time, while Spectrum Sharing has the goal to limit interference from the radar to other services and vice versa. In [1]-[2] we analyzed the problem of spectrum allocation and sharing, designing a cognitive radar that provides good radar performance in the presence of

interference from other users and low impact on the performance of the other users by the presence of the radar. From the concluding remarks of our previous works [1]-[2], it is apparent that the presence of an observation block for spectrum sensing and analysis is paramount and its design is decisive to exploit the full potentiality of the system. In other words, Spectrum Sensing is the most important function that makes a radar to be cognitive. Through this function, a cognitive radar can obtain necessary observations about the radio frequency channel, such as the presence of other users and the appearance of spectrum opportunities, that is spectrum holes where it is possible to transmit without interfering with other users of the channel. After using this information a cognitive radar is able to adapt its transmitting and receiving parameters, such as the transmission power and the operating frequency, in order to achieve efficient spectrum utilization. In this work we focus on the problem of Spectrum Sensing, in particular we study how the emerging technology of Compressed Sensing (CS) can solve the crucial problems in the implementation of a responsive spectrum sensing system which is able to quickly react to the changes of the operating frequency channel. As a matter of fact, to have high spectrum efficiency and high sensing accuracy, a cognitive radar has to perform real-time wideband monitoring of the licensed spectrum, using a dual-radio architecture [6]-[8] where one chain is dedicated to radar operations while the other chain is dedicated for spectrum monitoring. The drawback of such approach is the hardware cost, as the related systems requires high sampling rate and high resolution analog-to-digital converters (ADCs) with large dynamic range, plus the use of high speed signal processors. Moreover, when the required time used to estimate the spectrum occupancy is very short and the monitored frequency band is very large, the current generation ADCs are even unable to collect the required samples at the Nyquist-rate. A signal processing technique that can solve this problem has to be based on the use of Compressed Sensing. Recent results on CS [9]-[12] state that it is possible to reconstruct a sparse signal from a random projections of the sensor data. The number of random projections can be very small, in proportion to the number of the channels occupied by the other users. Under the hypothesis that the frequency spectrum of the other users is sparse, CS can be profitably used to solve the hardware constraints by reducing the sampling rate, decreasing the

This work has been partially funded by the SELEX ES “Signal Processing Technologies for Radio Frequency Systems” - SPRINT Project.

computational complexity thus limiting the coherent integration time.

2. PROBLEM FORMULATION

In this work we consider a cognitive radar that shares the same frequency band with a set of communication systems. The frequency band used by the communication systems is subdivided into N frequency channels used for frequency division multiple access. The cognitive radar uses a spectrum sensing module that scans and senses the frequency channels to evaluate the spectrum occupancy of the communication systems. As described in [1]-[2], the cognitive radar transmits a wideband pulse, suitably modified by notching the frequencies occupied by the other systems. As stated before, spectrum sensing serves to minimize the interference between the radar and the other radiating elements. For each frequency channel occupied by communication systems, the traffic is modeled with two random processes. The arrival traffic is modeled as a Poisson process with rate λ , while the spectrum access duration is exponentially distributed with mean time $1/\mu$, so the departure of the user traffic is another Poisson process with rate μ [3]-[5]. In each frequency channel, if the primary user is transmitting, the received signal is given by an oscillation at that frequency whose amplitude is a Gaussian random variable with zero mean and standard deviation σ_s , if the channel is free, the received signal is zero. The multiband received signal is given by the combination of the signal in each frequency channel plus Additive White Gaussian Noise with zero mean and standard deviation σ_w . Indicating with Δt the time slot used by the cognitive radar to estimate the frequency occupancy of the operating channel and with $\mathbf{f}$ the multiband time signal received every Δt seconds, the frequency spectrum of the received signal in absence of noise is given by

$$\mathbf{x} = \Psi^T \mathbf{f}, \quad (1)$$

where Ψ is the Discrete Cosine Transform (DCT) matrix. Note that, in this work, without any lack of generality, we can consider real signals, instead of the complete complex signals. As a matter of fact, to monitor the spectrum occupancy of the primary users and to further reduce the cost of the receiver, it is not necessary to process the In-Phase and Quadrature components of the received signal, but only one of this two components. Figure 1 shows the channel occupancy evolution $\mathbf{x}^2$ during an observation time T composed of ten time slots Δt . The channel is composed of $N=256$ frequency bands and the Signal to Noise Ratio (SNR), defined as $\text{SNR} = \sigma_s^2 / \sigma_w^2$, is 20dB.

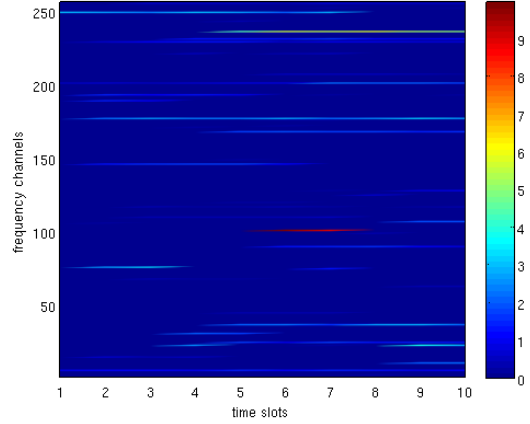


Figure 1 – Channel occupancy evolution in T seconds composed of 10 time slots Δt , $N=256$ frequency channels. Additive Gaussian noise, SNR=20dB.

To evaluate channel occupancy evolution it is necessary to perform the DCT of the received time samples every Δt seconds. When the frequency band to be monitored tends to be very wide and/or the time slot Δt tends to be very short, it should be very difficult to collect the N time samples at the Nyquist-rate. As apparent from Figure 1, since the frequency spectrum $\mathbf{x}$ is sparse, CS may be used to alleviate this hardware constraint. In fact, CS states [10]-[12] that the sparse vector $\mathbf{x}$ can be recovered from a small number $K \ll N$ of random projections of the time signal $\mathbf{f}$.

3. COMPRESSED SPECTRUM SENSING

In this section, after a brief overview on the principles of CS, we analyze how it is possible to adopt this signal processing methodology for Spectrum Sensing, i.e. Compressed Spectrum Sensing (CSS). For more details on CS, we refer the reader to [10]-[12] and references therein.

CS is a signal processing methodology for signal recovery from highly incomplete information.

The central results state that a sparse vector¹ $\mathbf{x} \in \mathbb{R}^N$ can be recovered from a small number of linear measurements $\mathbf{y} = \mathbf{A}\mathbf{x} \in \mathbb{R}^K$, $K \ll N$ (or $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$ when there is measurement noise) by solving a convex program [10]-[12]. To make this possible, CS relies on two principles: sparsity, which pertains to the signals of interest, and incoherence, which pertains to the sensing modality. Let be $\mathbf{f} \in \mathbb{R}^N$ a real signal and let be $\Psi = [\psi_1, \dots, \psi_N]$ an orthonormal basis (such as the DCT), then the representation of $\mathbf{f}$ on the basis Ψ is given by $\mathbf{f} = \Psi\mathbf{x}$, where $\mathbf{x}$ is the sparse coefficient vector. Given a set of vectors $[\phi_1, \dots, \phi_K]$ and denoting with Φ the $K \times N$ sensing matrix with these vectors as rows, the measures are

¹ A signal is said to be s -sparse if it has at most s nonzero entries.

collected by means of linear functionals $\mathbf{y}=\Phi\mathbf{f}=\Phi\Psi\mathbf{x}\in\mathbb{R}^K$ [10]-[11]. The interest is in undersampled situations in which the number K of available measurements is much smaller than the dimension N of the signal $\mathbf{f}$. The process of recovering the K dimensional vector $\mathbf{x}=\Psi^T\mathbf{f}$ from the N dimensional measurement vector $\mathbf{y}=\Phi\mathbf{f}$ is, in general, ill-posed when $K<N$. However, if $\mathbf{x}$ is s -sparse, then the problem can be solved provided $K\geq s$. A necessary and sufficient condition for this problem is that, for some small $\delta>0$, the matrix $\mathbf{A}=\Phi\Psi$ satisfies the Restricted Isometry Property (RIP) [13]

$$(1-\delta)\|\mathbf{x}\|_2 \leq \|\mathbf{Ax}\|_2 \leq (1+\delta)\|\mathbf{x}\|_2. \quad (2)$$

The RIP implies that matrix $\mathbf{A}$ must preserve the length of s -sparse vectors. A related condition to RIP is referred as incoherence. The coherence between the measurement matrix Φ and the representation matrix Ψ measures the largest correlation between any two columns of these matrix and is defined as

$$\mu(\Phi, \Psi) = \sqrt{N} \max_{1 \leq k, j \leq N} |\langle \phi_k, \psi_j \rangle|. \quad (3)$$

Moreover, it can be shown [10]-[12] that $\mu(\Phi, \Psi) \in [1, N^{1/2}]$.

The design of a measurement matrix Φ such that $\mathbf{A}=\Phi\Psi$ has the RIP requires that all possible combination of s non zero entries on the vector $\mathbf{x}$ of length N have to satisfy (2). However, both the RIP and incoherence can be achieved with high probability simply by designing Φ as a random matrix [10]. Now, it is natural to attempt to recover $\mathbf{x}$ by solving the following optimization problem

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \|\mathbf{x}\|_0, \text{ subject to } \Phi\Psi\mathbf{x}=\mathbf{y}, \quad (4)$$

The main problem of this approach is that the ℓ_0 -norm is a discrete and non convex function and hence it is potentially very difficult to solve the optimization problem in (4). A way to reformulate this problem into something more tractable is to replace the ℓ_0 -norm with its convex approximation, i.e. the ℓ_1 -norm

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \|\mathbf{x}\|_1, \text{ subject to } \Phi\Psi\mathbf{x}=\mathbf{y}. \quad (5)$$

In literature, this minimization is referred as the Basis Pursuit method which, for real valued signals, can be recast as a linear programming problem. The Basis Pursuit method is guaranteed to find a reconstruction of a s -sparse signal if there is not measurement noise. However, in the presence of measurement noise, its influence on the signal reconstruction can be minimized by applying the Basis Pursuit De-Noising method which find a solution of the following problem [14]

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathbb{R}^N} \|\mathbf{x}\|_1, \text{ subject to } \|\mathbf{y} - \Phi\Psi\mathbf{x}\|_2 \leq \sigma, \quad (6)$$

where the positive parameter σ is an estimate of the noise level in the data. The case of $\sigma=0$ corresponds to the basis pursuit problem. Also the Basis Pursuit De-Noising method can be solved by algorithms that are used in linear

programming. As discussed, when the frequency spectrum of the user radiating in the same channel as the cognitive radar is a sparse signal, it is possible to adopt CS for Spectrum Sensing, i.e. Compressed Spectrum Sensing (CSS).

For the problem at hand, the representation matrix Ψ is the DCT, whose elements are

$$[\Psi]_{i,j} = \zeta_i \cos\left(\frac{\pi(i-1)(2j-1)}{2N}\right), i,j=1,\dots,N, \quad (7)$$

$$\zeta_i = \begin{cases} 1/\sqrt{N} & i=1 \\ \sqrt{2/N} & 2 \leq i \leq N \end{cases}. \quad (8)$$

In this work we consider two kind of measurement matrices Φ , the first one is the Gaussian matrix which is formed by sampling i.i.d. entries from the normal distribution with zero mean and variance $1/K$, i.e.

$$[\Phi]_{i,j} \sim N(0, 1/K), i=1,\dots,K \text{ and } j=1,\dots,N. \quad (9)$$

The second measurement matrix is the Spiky matrix given by random selecting K rows of the $N \times N$ identity matrix. The latter case is the more interesting because, from the definition of this matrix, the measurement vector $\mathbf{y}$ is obtained by selecting K samples of $\mathbf{f}$ at random. The use of CS allows to use an ADC with a rate of $K/\Delta t$ instead of an ADC with rate $N/\Delta t$. For the physical implementation of the CS filters we refer the reader to [15]-[17]. Figure 2 shows the channel occupancy evolution in Figure 1 recovered using the Gaussian measurement matrix while Figure 3 shows the result obtained using the Spiky measurement matrix. In both the cases $K=N/2$ and $\text{SNR}=20\text{dB}$.

4. SIMULATION RESULTS

This section deals with the performance evaluation of the CSS technique defined in the previous section. Figure 4 shows the Root Mean Square Error (RMSE) after reconstructing the channel occupancy signal. The RMSE measures the error in reconstructing $\mathbf{x}$ using CSS w.r.t. the reference signal estimated with all the N samples, that is

$$\text{RMSE} = \sqrt{\sum_{h=1}^H \sum_{m=1}^M |\hat{\mathbf{x}}_m^{(h)} - \mathbf{x}_m^{(h)}|^2}. \quad (10)$$

where m and h are the time slot and the Monte Carlo run indexes, respectively. The results are shown as a function of K (percentage of N) for both the measurements matrices and for different values of the SNR. The performance results obtained using the two matrices are about the same. It is also apparent that, in absence of noise, it is possible to reconstruct the signal of interest using a very low number of samples (30% of N). However, as much as the noise power increases, the higher number of samples we need to minimize the influence of the noise on the signal reconstruction. Anyway when the SNR tends to be high, the signal can be almost perfectly reconstructed using few samples (40% of N).

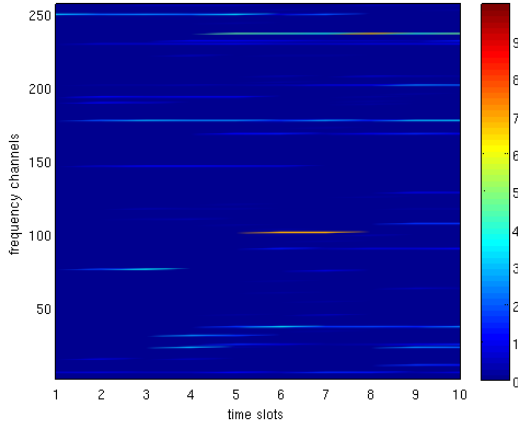


Figure 2 - Channel occupancy evolution recovered using the Gaussian measurement matrix, $K=N/2$.

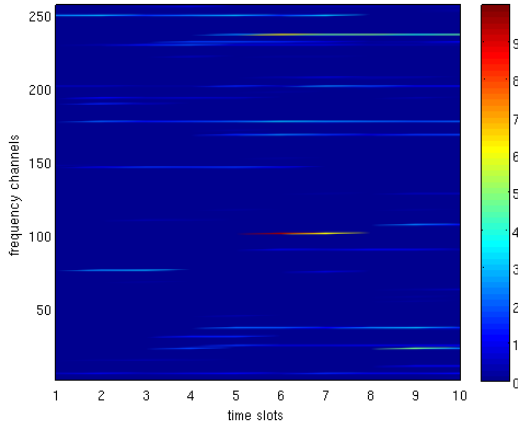


Figure 3 - Channel occupancy evolution recovered using the Spiky measurement matrix, $K=N/2$.

From our analysis, and also from Figures 1-3, the RMSE in reconstructing the signal is strictly related to the fact that, when the channel is busy, we need a high number of samples to reconstruct the whole spectrum with high precision. However, in this case, even if we use a low number of samples, a busy channel is recovered as busy, as well. As a matter of fact, when performing the cognitive spectrum sensing function, we are not interested in the operation of reconstructing the whole spectrum with high accuracy, but more in the operation of deciding which are the busy channels. With regard to this latter operation, we apply the classical energy detector technique [18], which compares $\mathbf{x}^2$ with a threshold ξ to evaluate if the channel is busy/free and we evaluate the percentage of error in the decision on the channel occupancy (i.e. if a free channel is declared as busy and vice versa), the results are shown in Figure 5 when ξ is fixed for a probability of detection of 0.8. The results show that, when the SNR is sufficiently high, the percentage error is reasonably low, which means that the

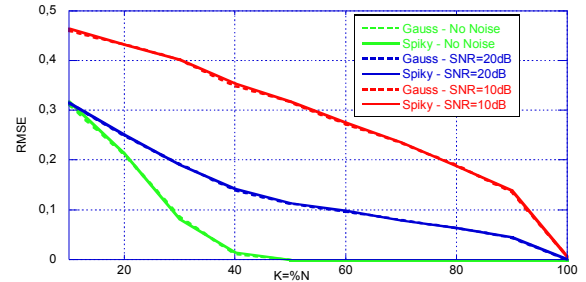


Figure 4 – RMSE for channel occupancy reconstruction as a function of K (percentage of N) for different SNR values.

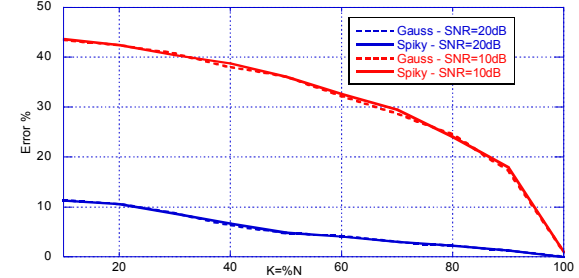


Figure 5 – Percentage error on the decision of channel occupancy as a function of K (percentage of N) for different SNR values.

busy/free decision can still be carried out on the signal reconstructed with few samples ($<30\%$ of N), even if the signal is not accurately reconstructed.

5. CONCLUSIONS

In this work we describe the application of Compressed Sensing in Cognitive Radar systems that use a wide operating frequency bandwidth. In particular, we apply the Compressed Sensing methodology to the Spectrum Sensing function of a Cognitive Radar, specifically conceived for Spectrum Sharing applications. In agreement with the general assumptions for the application of Compressed Sensing, we first assume that the frequency spectrum under consideration is sparse, i.e. it features few non-zero entries. In this case, we demonstrate that Compressed Sensing can provide a significant reduction in acquisition time reducing the cost for high resolution analog-to-digital converters with large dynamic range, and high speed signal processors. In the specific application, where the goal is not reconstructing the whole spectrum with high accuracy, but more deciding which are the busy channels in the considered band, the results show that, when the SNR is sufficiently high, the percentage error on the busy/free decision can be carried out by using only $<30\%$ of the total samples of the original signal. Further work is the design of the whole chain of the Spectrum Sharing application, by putting together the Spectrum Sensing function and the Cognitive Signal Processing chain and the evaluation of its performance.

6. REFERENCES

- [1] M. La Manna, P. Stinco, M. Greco, F. Gini, "Design of a Cognitive Radar for Operation in Spectrally Dense Environments", *IEEE Radar Conference*, 29 April-3 May, 2013 Ottawa, Ontario, Canada.
- [2] M. La Manna, P. Stinco, M. Greco, F. Gini, "Cognitive Techniques for a Wideband Phased Array Radar", *2013 IEEE International Symposium on Phased Array Systems & Technology*, 15-18 October 2013 Waltham, Massachusetts USA.
- [3] J. Bater; H. Tan; K.N. Brown; L. Doyle, "Modelling Interference Temperature Constraints for Spectrum Access in Cognitive Radio Networks," *Communications, 2007. ICC '07. IEEE International Conference on*, vol., no., pp.6493,6498, 24-28 June 2007
- [4] M. Sharma; A. Sahoo; K.D. Nayak, "Channel modeling based on interference temperature in underlay cognitive wireless networks," *Wireless Communication Systems. 2008. ISWCS '08. IEEE International Symposium on*, vol., no., pp.224,228, 21-24 Oct. 2008.
- [5] S.D. Barnes; B.T. Maharaj, "Performance of a Hidden Markov channel occupancy model for cognitive radio," *AFRICON, 2011*, vol., no., pp.1,6, 13-15 Sept. 2011.
- [6] Y. Hur, J. Park, W. Woo, K. Lim, C. Lee, H. Kim, and J. Laskar, "A wideband analog multi-resolution spectrum sensing (MRSS) technique for cognitive radio (CR) systems," in *Proc. IEEE Int. Symp. Circuits and Systems*, Island of Kos, Greece, May 2006, pp. 4090–4093.
- [7] Y. Yuan, P. Bahl, R. Chandra, P. A. Chou, J. I. Ferrell, T. Moscibroda, S. Narlanka, and Y. Wu, "KNOWS: Cognitive radio networks over white spaces," in *Proc. IEEE Int. Symposium on New Frontiers in Dynamic Spectrum Access Networks*, Dublin, Ireland, Apr. 2007, pp. 416–427.
- [8] M. Piezzo, A. De Maio, A. Aubry, A. Farina, "Cognitive radar waveform design for spectral coexistence," *Radar Conference (RADAR), 2013 IEEE*, April 29 2013-May 3 2013
- [9] T. Zhi, G.B. Giannakis, "Compressed Sensing for Wideband Cognitive Radios," *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*, 15-20 April 2007
- [10] E.J. Candes, M.B. Wakin; "An Introduction To Compressive Sampling," *Signal Processing Magazine, IEEE*, vol. 25, no. 2, pp. 21-30, March 2008.
- [11] D.L. Donoho; "Compressed sensing," *Information Theory, IEEE Transactions on*, vol. 52, no. 4, pp. 1289-1306, April 2006.
- [12] D.L. Donoho, X. Huo; "Uncertainty principles and ideal atomic decomposition," *Information Theory, IEEE Transactions on*, vol. 47, no. 7, pp. 2845-2862, Nov 2001.
- [13] R.G. Baraniuk, "Compressive Sensing [Lecture Notes]," *Signal Processing Magazine, IEEE*, vol.24, no.4, pp.118,121, July 2007.
- [14] S.S. Chen, D.L. Donoho, M.A. Saunders; "Atomic decomposition by basis pursuit", *SIAM Journal on Scientific Computing*, 20 (1998), pp. 33-61.
- [15] M. A. Davenport, P. T. Boufounos, M. B. Wakin, R. G. Baraniuk, "Signal Processing With Compressive Measurements," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 445-460, April 2010.
- [16] J. A. Tropp, J. N. Laska, M. F. Duarte, J. K. Romberg, R. G. Baraniuk, "Beyond Nyquist: Efficient Sampling of Sparse Bandlimited Signals," *IEEE Transactions on Information Theory*, vol. 56, no. 1, pp. 520-544, Jan. 2010.
- [17] Y. C. Eldar, G. Kutyniok, *Compressed Sensing*, Cambridge University Press, June 2012.
- [18] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access: signal processing, networking, and regulatory policy", *IEEE Signal Processing Mag.*, vol. 4, no. 3, pp. 79-89, May 2007.