A POISSON SERIES APPROACH TO BAYESIAN MONTE CARLO INFERENCE FOR SKEWED ALPHA-STABLE DISTRIBUTIONS

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ABSTRACT

In this paper we study parameter estimation for α-stable distribution parameters. The proposed approach uses a Poisson series representation (PSR) for skewed α-stable random variables, which provides a conditionally Gaussian framework. Therefore, a straightforward implementation of Bayesian parameter estimation using Markov chain Monte Carlo (MCMC) methods is feasible. To extend the series representation to practical application, we provide a novel approximation of the series residual terms, which exactly characterises the mean and variance of the approximation and maintains its structure. Simulations illustrate the proposed framework applied to skewed α-stable data, estimating the distribution parameter values.

Index Terms— Poisson series representation, conditionally Gaussian, residual approximation, α-stable distribution parameter estimation, Markov chain Monte Carlo

1. INTRODUCTION

Empirical evidence of many real-world processes, which exhibit jumps and asymmetric behaviour, does not support the assumption of an underlying Gaussian distribution. For this reason α-stable distributions have gained significant impact and became present in a wide range of application areas, including radar processing, telecommunications, acoustics and econometrics [1, 2]. Most presented works concentrate on a symmetric α-stable law and are not flexible enough to deal with asymmetric behaviour. In the presence of symmetric stable noise, Godsill and Kuruoğlu [3, 4] introduced Monte Carlo Expectation-Maximisation (MCEM) and MCMC methods, which are based on the Scale Mixtures of Normals (SMiN) representation of stable distributions. A method for inference in models with symmetric Paretian disturbances was proposed by Tsionas [5]. Kuruoğlu [6] addressed positive α-stable probability distributions, providing an analytical approximation based on a decomposition into a product of a Pearson and another positive stable random variable. Bayesian inference for stable distribution parameters by exploiting a particular representation involving a bivariate density function was introduced by Buckle [7].

Our aim is to simplify inference in α-stable models by making use of powerful auxiliary variable representations of α-stable random variables [8, Chapter 1.4, page 28]. In particular, we seek conditionally Gaussian representations allowing for Bayesian parameter estimation using MCMC. In the symmetric stable case previous works by, e.g., [3], [4] and [5] demonstrate such a framework, using SMiN. This is in contrast with the MCMC approach of [7], in which an exact auxiliary variable approach is proposed, but computations are difficult because no conditionally Gaussian structure arises.

The original contributions of this paper include a novel residual method allowing for an exact characterisation of the mean and variance of the residual approximation (RA), which are then very well approximated by a Gaussian with moments matched to the residuals. Additionally, the structure of the conditionally Gaussian framework is maintained in contrast to our previous approaches [9, 10]. Moreover, we introduce the use of the approximated PSR to perform Bayesian MC inference for α-stable distribution parameters, which cannot be found in the literature to date.

The paper is organized as follows. In Section 2, we introduce α-stable distributions and state the definition and the PSR of an α-stable random variable. In Section 3, we present our residual approximation approach. In Section 4, we discuss inference for α-stable distribution parameters via MCMC. In Section 5, we present results of our work, and in Section 6, we conclude the paper.

2. α-STABLE LAW AND SERIES REPRESENTATION

2.1. α-Stable Distribution

The α-stable family of distributions $S_\alpha(\sigma, \beta, \mu)$ is identified by means of the characteristic function [8]:

$$\mathbb{E}[\exp(itX)] \tag{1}$$
$$= \begin{cases} \exp(-\sigma^\alpha |t|^\alpha [1 - i\beta \operatorname{sign}(t) \tan(\frac{\alpha\pi}{2})] + i\mu t), & \alpha \neq 1 \\ \exp(-\sigma |t| [1 - i\beta \frac{2}{\pi} \operatorname{sign}(t) \ln |t|] + i\mu t), & \alpha = 1, \end{cases}$$

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while closed-form density functions do not exist in general. The four parameters are given by $\alpha \in (0, 2]$, which measures the tail thickness; $\beta \in [-1, 1]$ termed the skewness parameter; $\sigma > 0$ and $\mu \in \mathbb{R}$ denote the scale and location parameter, respectively.

2.2. Poisson Sum Representation for Random Variables

The general series representation for random variables (r.v.) as given in [8, page 28, Theorem 1.4.5] states that

$$\sum_{i=1}^{\infty} \left(\Gamma_i^{-1/\alpha} W_i - \mathbb{E} W_1 k_i^{(\alpha)} \right), \tag{2}$$

$$k_i^{(\alpha)} = \begin{cases} 0, & 0 < \alpha < 1 \\ \frac{\alpha}{\alpha-1}(i^{\frac{\alpha-1}{\alpha}} - (i-1)^{\frac{\alpha-1}{\alpha}}), & 1 < \alpha < 2 \end{cases} \tag{3}$$

converges almost surely to a $S_\alpha(\sigma, \beta, 0)$ r.v. with

$$\sigma^\alpha = \frac{\mathbb{E}[|W_1|^\alpha]}{C_\alpha}, \quad \beta = \frac{\mathbb{E}[|W_1|^\alpha \text{sign} W_1]}{\mathbb{E}[|W_1|^\alpha]}, \tag{4}$$

where $C_\alpha = \frac{1-\alpha}{\Gamma(2-\alpha)\cos(\pi\alpha/2)}$; Γ_i are arrival times of a unit rate Poisson process; $\{W_1, W_2, ...\}$ are some independent and identically distributed (i.i.d.) random variables with finite absolute α^{th} moment, $0 < \alpha < 2, \alpha \neq 1$. The $\alpha = 1$ special case is omitted here due to space constraints. Equation (2) gives us the possibility of choosing the W_i as i.i.d. normal distributed, $W_i \sim \mathcal{N}(\mu_W, \sigma_W^2)$, whereby β and σ^α as in (4) can be obtained by matching μ_W and σ_W values numerically. This leads us to a conditionally Gaussian form for the $S_\alpha(\sigma, \beta, 0)$ distributed random variable X:

$$\begin{aligned} X|\{\Gamma_i\}_{i=1}^{\infty} &\sim \mathcal{N}(\mu_X, \sigma_X^2) \qquad (5) \\ &:= \mathcal{N}\left(\sum_{i=1}^{\infty} \mu_W (\Gamma_i^{-1/\alpha} - k_i^{(\alpha)}), \sigma_W^2 \sum_{i=1}^{\infty} \Gamma_i^{-2/\alpha} \right). \end{aligned}$$

3. RESIDUAL APPROXIMATION

Our novel approach to approximating the residual series terms is designed to keep the structure of the PSR, where μ_W appears as a factor in μ_X, and σ_W^2 appears as a factor in σ_X^2, see equation (5). Since we approximate the residuals of the mean and variance of the conditionally Gaussian framework, the RA is referred to as the Gaussian approximation of moments approach (GAMA). $\{\Gamma_i\}_{i \geq 1}$ is defined as a unit rate Poisson process satisfying the properties, on any interval $[c, d]$,

$$|\{\Gamma_i : \Gamma_i \in [c, d]\}| \sim \text{Poisson}(d - c) \quad \text{for} \quad d > c \tag{6}$$

and given the number of Γ_i in $[c, d]$, each Γ_i is uniformly and independenlyt distributed on $[c, d]$,

$$\Gamma_i \overset{\text{i.i.d.}}{\sim} \text{U}([c, d]). \tag{7}$$

With d going to infinity we will account for all residual terms in the PSR from c to ∞.

We approximate the residual terms (R_1, R_2) in the summations of the mean and variance of the conditional framework,

$$X|\{\Gamma_i\}_{i=1}^{M} \overset{\text{approx.}}{\sim} \mathcal{N}\left(\mu_W m, \sigma_W^2 s\right), \tag{8}$$

where

$$m := \sum_{i=1}^{M} \Gamma_i^{-1/\alpha} + R_1, \quad s := \sum_{i=1}^{M} \Gamma_i^{-2/\alpha} + R_2, \tag{9}$$

by a bivariate Gaussian distribution, $\mathcal{N}(\mu_R, \Sigma_R)$, which takes account of the correlation between R_1 and R_2. Note that the number of summation terms M is a random variable itself, defined as $M = |\{i : \Gamma_i < c\}|$. The residuals R_1 and R_2 are expressed as the limits of

$$R_1^{(d)} := \sum_{i:\Gamma_i \in [c,d]} \Gamma_i^{-1/\alpha} - \sum_{n:\Gamma_n \in [0,d]} \tilde{k}_n^{(\alpha)}, \tag{10}$$

$$R_2^{(d)} := \sum_{i:\Gamma_i \in [c,d]} \Gamma_i^{-2/\alpha} \tag{11}$$

as $d \to \infty$. In a next step the number of terms in the sums is approximated by the expectation $\mathbb{E}[|\{\Gamma_i : \Gamma_i \in [c, d]\}|] = d - c$ and $\mathbb{E}[|\{\Gamma_i : \Gamma_i \in [0, d]\}|] = d$ to compute

$$\mu_R := \lim_{d \to \infty} \begin{bmatrix} \mathbb{E}[R_1^{(d)}] \\ \mathbb{E}[R_2^{(d)}] \end{bmatrix} = \begin{bmatrix} \frac{\alpha}{1-\alpha} c^{\frac{\alpha-1}{\alpha}} \\ \frac{\alpha}{2-\alpha} c^{\frac{\alpha-2}{\alpha}} \end{bmatrix} \tag{12}$$

and

$$\begin{aligned} \Sigma_R &:= \lim_{d \to \infty} \begin{bmatrix} \text{Var}[R_1^{(d)}] & \text{Cov}[R_1^{(d)}, R_2^{(d)}] \\ \text{Cov}[R_2^{(d)}, R_1^{(d)}] & \text{Var}[R_2^{(d)}] \end{bmatrix} \\ &= \begin{bmatrix} \frac{\alpha}{2-\alpha} c^{\frac{\alpha-2}{\alpha}} & \frac{\alpha}{3-\alpha} c^{\frac{\alpha-3}{\alpha}} \\ \frac{\alpha}{3-\alpha} c^{\frac{\alpha-3}{\alpha}} & \frac{\alpha}{4-\alpha} c^{\frac{\alpha-4}{\alpha}} \end{bmatrix}. \qquad (13) \end{aligned}$$

4. INFERENCE FOR α-STABLE DISTRIBUTIONS VIA MCMC

The parameters of interest are α, σ and β. Given the samples $\boldsymbol{X} := \{X_n\}_{n=1}^{N}$, we aim for the posterior of the latent variable set $\boldsymbol{\Gamma} := \{\{\Gamma_{i,n}\}_{i=1}^{M}\}_{n=1}^{N}$, the variables μ_W and σ_W, as well as the residual approximations $\boldsymbol{R} := \{\boldsymbol{R}_n = (R_{1,n}, R_{2,n})\}_{n=1}^{N}$.

4.1. Marginal and conditional distributions

Considering that our conditionally Gaussian framework including the GAMA for the RA exhibits a certain structure, we explore possible simplifications and improvements with regard to the following inference section first.

Suppose there are N i.i.d. samples

$$X_n \sim S_\alpha(\sigma, \beta, 0) \quad \text{for } n = 1, \ldots, N.$$

We make use of the conditionally Gaussian framework for the α-stable vector $\boldsymbol{X}$ and write

$$\begin{aligned}
&p(\boldsymbol{X}|\mu_W,\sigma_W,\boldsymbol{\Gamma},\alpha)\\
&=\prod_{n=1}^{N}\mathcal{N}\left(X_n\middle|\sum_{i=1}^{\infty}\left(\mu_W\Gamma_{i,n}^{-1/\alpha}-k_i^{(\alpha)}\right),\sigma_W^2\sum_{i=1}^{\infty}\Gamma_{i,n}^{-2/\alpha}\right)\\
&=((2\pi\sigma_W^2)^{N/2}\prod_{n=1}^{N}\sqrt{s_n})^{(-1)} \qquad (14)\\
&\quad\times\exp\left\{-\left(A(\mu_W-B/A)^2+C-B^2/A\right)/(2\sigma_W^2)\right\},
\end{aligned}$$

where

$$A=\sum_{n=1}^{N}\frac{{m_n}^2}{s_n},\;\; B=\sum_{n=1}^{N}\frac{X_n m_n}{s_n},\;\; C=\sum_{n=1}^{N}\frac{{X_n}^2}{s_n}. \qquad (15)$$

Applying the GAMA, we approximate m_n, s_n as in (9). In the following, we aim for a straightforward Gibbs sampler [11] for μ_W and σ_W^2. Their joint conditional distribution can be rewritten as a composition

$$\begin{aligned}
&p(\mu_W,\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha)\\
&\quad=p(\mu_W|\boldsymbol{X},\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)p(\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha). \qquad (16)
\end{aligned}$$

Taking (15) and a uniform prior on μ_W and σ_W, we obtain the Gaussian full conditional distribution

$$\begin{aligned}
&p(\mu_W|\boldsymbol{X},\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)\\
&\quad\propto p(\boldsymbol{X}|\mu_W,\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)p(\mu_W|\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)\\
&\quad=\mathcal{N}\left(B/A,\sigma_W^2/A\right). \qquad (17)
\end{aligned}$$

Next, we derive the conditional distribution for σ_W^2. With the proportionality

$$p(\mu_W,\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha)\propto p(\boldsymbol{X}|\mu_W,\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)$$

we obtain

$$\begin{aligned}
&p(\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha)\\
&\quad=p(\mu_W,\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha)p(\mu_W|\boldsymbol{X},\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)^{-1}\\
&\quad\propto\mathcal{IG}\left(\frac{N-3}{2},\frac{1}{2}\left(C-\frac{B^2}{A}\right)\right), \qquad (18)
\end{aligned}$$

where $\mathcal{IG}$ denotes an inverse gamma distribution.

Finally, marginalising μ_W and σ_W^2 gives

$$\begin{aligned}
p(\boldsymbol{X}|\boldsymbol{\Gamma},\boldsymbol{R},\alpha)&\propto\frac{p(\boldsymbol{X}|\mu_W,\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)}{p(\mu_W|\boldsymbol{X},\sigma_W,\boldsymbol{\Gamma},\boldsymbol{R},\alpha)p(\sigma_W^2|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R},\alpha)}\\
&\propto\frac{\Gamma((N-3)/2)}{((C-B^2/A)/2)^{(N-3)/2}(2\pi)^{(N-1)/2}\sqrt{A}\prod_{n=1}^{N}\sqrt{s_n}},
\end{aligned} \qquad (19)$$

where $\Gamma(.)$ denotes the gamma function. Note that the above marginal distribution gives a relation between $\boldsymbol{\Gamma},\boldsymbol{R},\alpha$ and the given set of data $\boldsymbol{X}$, which can be used for making joint conditional samples from μ_W and σ_W^2.

4.2. MCMC implementation

The parameters μ_W and σ_W can be sampled straight away according to the available joint conditional distribution, which can be written as the product of a Gaussian and an inverse gamma distribution as derived in (17) and (18).

To sample the stability parameter α, we choose the marginalised conditional distribution with a uniform prior on α to obtain the proportionality

$$p(\alpha|\boldsymbol{X},\boldsymbol{\Gamma},\boldsymbol{R})\propto p(\boldsymbol{X}|\boldsymbol{\Gamma},\boldsymbol{R},\alpha), \qquad (20)$$

where $p(\boldsymbol{X}|\boldsymbol{\Gamma},\boldsymbol{R},\alpha)$ as in (19). Then, the acceptance probability for the Metropolis-Hastings (M.-H.) sampler [12] is computed as

$$\rho(\alpha,\alpha')=\min\left(1,\frac{p(\boldsymbol{X}|\boldsymbol{\Gamma},\boldsymbol{R},\alpha')q(\alpha|\alpha')}{p(\boldsymbol{X}|\boldsymbol{\Gamma},\boldsymbol{R},\alpha)q(\alpha'|\alpha)}\right), \qquad (21)$$

where α is proposed from $q(\alpha'|\alpha)=\mathcal{N}(\alpha,\sigma_\alpha^2)$ with some variance σ_α^2.

As for the α parameter, we can update the latent variables $\boldsymbol{\Gamma}$ and $\boldsymbol{R}$ using M.-H. sampling. Setting the proposals to be the priors $q(\boldsymbol{\Gamma}'_n|\boldsymbol{\Gamma}_n)=p(\boldsymbol{\Gamma}_n)$ and $q(\boldsymbol{R}'_n|\boldsymbol{R}_n)=p(\boldsymbol{R}_n)$, the corresponding acceptance probabilities result in

$$\begin{aligned}
\rho(\boldsymbol{\Gamma}_n,\boldsymbol{\Gamma}'_n)&=\min\left(1,\frac{p(X_n|\boldsymbol{\Gamma}'_n,\boldsymbol{R}_n,\mu_W,\sigma_W^2,\alpha)}{p(X_n|\boldsymbol{\Gamma}_n,\boldsymbol{R}_n,\mu_W,\sigma_W^2,\alpha)}\right)\\
&=\min\left(1,\frac{\mathcal{N}(X_n|\mu'_{X_n},\sigma'_{X_n})}{\mathcal{N}(X_n|\mu_{X_n},\sigma_{X_n})}\right) \qquad (22)
\end{aligned}$$

combined with the subsequent M.-H. step for the residual terms, which are accepted with probability

$$\begin{aligned}
\rho(\boldsymbol{R}_n,\boldsymbol{R}'_n)&=\min\left(1,\frac{p(X_n|\boldsymbol{\Gamma}_n,\boldsymbol{R}'_n,\mu_W,\sigma_W^2,\alpha)}{p(X_n|\boldsymbol{\Gamma}_n,\boldsymbol{R}_n,\mu_W,\sigma_W^2,\alpha)}\right)\\
&=\min\left(1,\frac{\mathcal{N}(X_n|\mu'_{X_n},\sigma'_{X_n})}{\mathcal{N}(X_n|\mu_{X_n},\sigma_{X_n})}\right). \qquad (23)
\end{aligned}$$

Alternatively to the above presented M.-H. samplers we can use rejection sampling to obtain samples for $\boldsymbol{\Gamma}$ and $\boldsymbol{R}$. The rejection sampler is expected to be slower than the M.-H. sampler, since it proposes samples until one is accepted in each iteration. On the other hand, it provides samples from the exact full conditional while the M.-H. sampler might need some period to converge.

For the set $\boldsymbol{\Gamma}$ we sample $\boldsymbol{\Gamma}_n=\{\Gamma_{i,n}\}_{i=1}^{M_n}$ for the n-th observation from the full conditional distribution using rejection sampling with the envelope function,

$$\begin{aligned}
p\left(\boldsymbol{\Gamma}_n|X_n,\boldsymbol{R}_n,\mu_W,\sigma_W,\alpha\right)&\propto\mathcal{N}\left(X_n|\mu_{X_n},\sigma_{X_n}^2\right)p\left(\boldsymbol{\Gamma}_n\right)\\
&<\left(2\pi\sigma_W^2R_{2,n}\right)^{-1/2}p\left(\boldsymbol{\Gamma}_n\right).
\end{aligned} \qquad (24)$$

Residuals are updated using the same scheme as for the set $\boldsymbol{\Gamma}_n$ with the bounding envelope function

$$p(\boldsymbol{R}_n|X_n, \{\Gamma_{i,n}\}_{i=1}^{M_n}, \mu_W, \sigma_W, \alpha) < \left(2\pi\sigma_W^2 \sum_{i=1}^{M_n} \Gamma_{i,n}{}^{-2/\alpha}\right)^{-1/2} p\left(\boldsymbol{R}_n|\alpha\right). \tag{25}$$

For large observations the MCMC sampler might suffer from bad mixing or high rejection rates, since a very small value of $\Gamma_{1,n}$ is required to generate a large value of m_n. For these cases additional samplers have been developed. These rely on the near-deterministic relation $m_n^2 \approx s_n$ for large data points and allow for a joint move for the auxilary variables m, Γ_i and (R_1, R_2) leading to higher acceptance rates. A detailed describtion is omitted here due to space constraints (for a review see [13]).

5. NUMERICAL RESULTS

In order to validate the introduced inference methods for the α-stable distribution parameters α, β and σ, we generate 500 observations from some α-stable distribution, $S_\alpha(\sigma, \beta, 0)$, under the use of the Chambers-Mallows-Stuck [14] sampling method and run the according samplers for 10,000 iterations. We choose the starting points for α, μ_W and σ_W to be well away from the true values. Based on the traceplots of the sampled parameters in our studies we note that convergence seems to be attained when allowing for a burn-in period of 5,000 iterations. Thus, only the last 5,000 samples are used for the histrograms of the marginals of the parameters and the autocorrelation functions (ACFs).

We give an examplary simulation for a fairly heavy-tailed stable distribution with $\alpha = 0.8$. Skewness and scale are set to $\beta = -0.84$ and $\sigma = 1.71$, respectively, which corresponds to the parameter values $\alpha = 0.8$, $\mu_W = -1$ and $\sigma_W = 1$ in terms of the approximated PSR. $(\alpha, \mu_W, \sigma_W)$ are initialised with $(0.4, -3, 3)$. We choose the Gibbs sampler to include a M.-H. step for the set of Γs and residuals (R_1, R_2) for each observation as presented in (22) and (23).

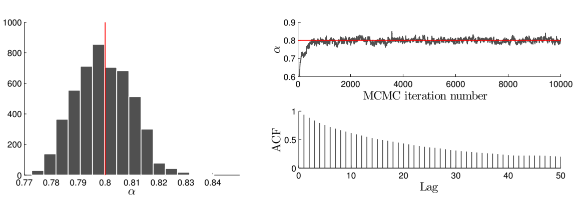

Fig. 1. Inference for the parameter α. Left: Histogram from the MCMC output. The true value is marked by the vertical line. Right, top: MCMC sampled parameter value α. The true value is marked by the horizontal line. Right, bottom: ACF as a function of lag.

Figures 1 and 2 show the MCMC sampled parameter values for α, μ_W and σ_W on the right-hand side with the ACFs below. On the left-hand side we see the unimodal histograms centred around the true values. With the MCMC samples

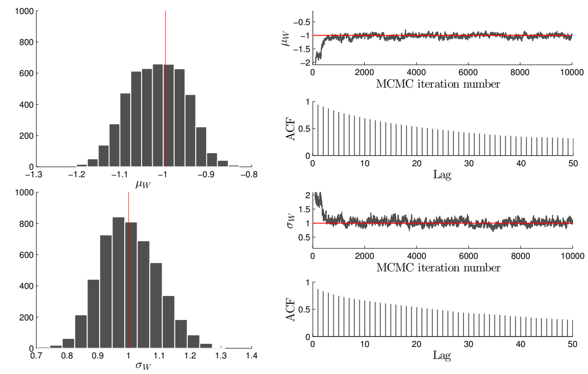

Fig. 2. Inference for the parameters μ_W and σ_W. Left: Histograms from the MCMC output. The true values are marked by the vertical lines. Right: MCMC sampled parameter values μ_W and σ_W with the ACFs as functions of lag below each. The true values are marked by the horizontal lines.

for the parameters α, μ_W and σ_W of the PSR we obtain the missing paths for the distribution parameters β and σ by reparametrising according to (4). The resulting traceplots, ACFs and histograms are shown in Figure 3. As with the parameters μ_W and σ_W, the samples for β and σ lead to unimodal histograms centred around the true values. The sample means yield the estimates $\hat{\alpha} = 0.8$, $\hat{\beta} = -0.84$ and $\hat{\sigma} = 1.32$ with standard deviations 0.01, 0.04 and 0.04, respectively.

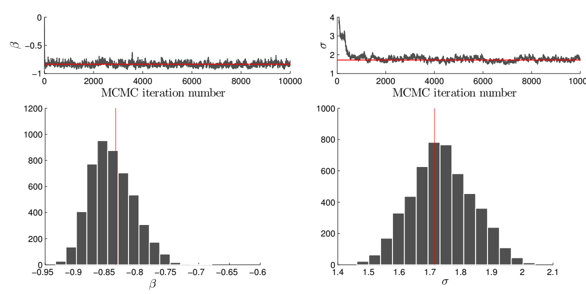

Fig. 3. Inference for the parameters β and σ. Top: MCMC sampled parameter values β and σ. The true values are marked by the horizontal lines. Bottom: Histograms from the MCMC output. The true values are marked by the vertical lines.

6. CONCLUSIONS

We have achieved satisfactory results for parameter estimation for an α-stable distribution applying a MCMC sampler, which is based on our conditionally Gaussian framework including the PSR and a novel RA method. The presented framework could serve as a basis for future research, when including our work in various models and applications.

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