POLARIMETRIC MIMO RADAR TARGET DETECTION BASED ON GLOWWORM SWARM OPTIMIZATION ALGORITHM

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ABSTRACT

Arbitrary transmit polarization waveforms are permitted to match the scattering properties of the target for polarimetric multiple-input multiple-output (MIMO) radar systems in order to improve the detection performance. In this paper, a polarization optimization method based on glowworm swarm optimization (GSO) algorithm is proposed to select the polarization waveforms of the transmit array to maximize detection probability. The signal model of polarimetric MIMO radar is described. A Neyman-Pearson criterion based detector is designed, and the closed expression of false alarm probability and detection probability in Gaussian clutter is elicited. Then, polarization waveform design based on GSO algorithm is presented. The algorithm performance is analyzed by simulation, and the detection probability for the solution of GSO is compared with other several transmit polarization schemes.

Index Terms— Polarimetric MIMO radar, target detection, glowworm swarm optimization, polarization waveform design

1. INTRODUCTION

Polarization diversity provides additional information which can significantly enhance the target detection capability [1], especially when the target fails to be detected using Doppler frequency effect. Also, it can improve the accuracy of multi-target identification and localization [2], which has attracted particular attention of many researchers.

A polarimetric multiple-input multiple-output (MIMO) radar system with distributed antennas can provide improvement of target detection performance compared with a conventional MIMO radar system. In [3], a stealth target detection for polarimetric MIMO radar was demonstrated and the formula on theoretics was analyzed. In [4, 5], the detection performance was investigated according to various correlation coefficients. By optimally selecting the polarization of the transmit waveforms that matches the target scattering characteristic can further improve the detection performance in distributed MIMO radar systems. The author in [6] performed a grid searching over the possible waveform polarizations across all the transmit antennas to obtain optimal detection performance in noise environment. However, grid searching is a time-consuming process. In [7], a polarimetric design scheme was proposed based on game theory. Unfortunately, it cannot provide optimal design and an accurate solution.

Glowworm swarm optimization (GSO) algorithm is a novel popular swarm intelligent optimization algorithm that has been used in many fields [8–10], which is inspired by glowworm recurring to its own luciferin to attract companions in the nature. The advantage of GSO algorithm lies in high speed to capture extreme range, multimodal functions processing and a strong commonality.

In this paper, we improve the detection performance in noise and clutter environment through polarization waveform design for MIMO radar based on GSO algorithm, which exploits the GSO multimodal search ability. The rest of the paper is organized as follows: In Sect. 2, we describe the signal model of polarimetric MIMO radar. In Sect. 3, we present the design of the target detector. In Sect. 4, the GSO principle and algorithm flow are elaborated. In Sect. 5, the validity of the algorithm is illustrated by numerical simulation. Finally, we draw conclusions in Sect. 6.

2. POLARIMETRIC MIMO RADAR MODEL

We consider a full polarimetric MIMO radar system with M transmit antennas and N receive antennas, which are widely spaced respectively. Assume that the target is point-like, whose scattering matrix depends on the viewing angle.

Define $\mathbf{t}^i = \left[t_h^i, t_v^i\right]^T = \left[cos\gamma^i, sin\gamma^i e^{j\eta^i}\right]^T$, where $i = 1, \cdots, M$, as the transmit polarization Jones vector of the i-th transmitter, where t_h^i and t_v^i respectively represent the horizontal component and the vertical one with polarization angle $\gamma^i \in [0, \pi/2]$ and polarization phase difference $\eta^i \in [0, 2\pi]$. $[\cdot]^T$ means the transpose of $[\cdot]$. Let $\mathbf{T}^i = \left[\begin{array}{c} t_h^i, t_v^i, 0, 0 \\ 0, 0, t_h^i, t_v^i \end{array}\right]$ denote the polarization waveform matrix for the i-th transmit antenna.

The linear output of the entire matched filters at the re-

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ceiver can be described as

$$\mathbf{z} = \mathbf{Hs} + \mathbf{Hc} + \mathbf{n} \tag{1}$$

where

$$\mathbf{H} = diag\left(\mathbf{T}^1, \cdots, \mathbf{T}^1, \cdots, \mathbf{T}^M, \cdots, \mathbf{T}^M\right) \in \mathbb{C}^{2MN \times 4MN} \tag{2}$$

The elements of $\mathbf{H}$ depend on the transmit waveform polarizations. $\mathbf{s} = [\mathbf{s}^{11}, \cdots, \mathbf{s}^{1N}, \cdots, \mathbf{s}^{M1}, \cdots, \mathbf{s}^{MN}]^T \in \mathbb{C}^{4MN\times 1}$ denotes the target scattering matrix, where $\mathbf{s}^{ij} = [s_{hh}^{ij}, s_{hv}^{ij}, s_{vh}^{ij}, s_{vv}^{ij}]$, $i = 1, \cdots, M$, $j = 1, \cdots, N$, denotes the column vector of target scattering matrix, which is the polarization change of the signal from the i-th transmit antenna to the j-th receive antenna. The matrix $\mathbf{s}$ follows independent identically distributed (i.i.d) circularly symmetric complex Gaussian random variables, as $\mathbf{s} \sim CN(\mathbf{0}, \mathbf{\Sigma}_s)$, where $\mathbf{\Sigma}_s$ is the covariance matrix of $\mathbf{s}$, and $\mathbf{z}$ is the observation vector with dimension of $2MN \times 1$.

Similarly, let $\mathbf{c} = \left[\mathbf{c}^{11}, \cdots, \mathbf{c}^{1N}, \cdots, \mathbf{c}^{M1}, \cdots, \mathbf{c}^{MN}\right]^T \in \mathbb{C}^{4MN\times 1}$ denote the clutter scattering matrix, where $\mathbf{c}^{ij} = \left[c_{hh}^{ij}, c_{hv}^{ij}, c_{vh}^{ij}, c_{vv}^{ij}\right]$ corresponds to i.i.d circularly symmetric zero mean complex Gaussian random variables, namely each component of $\mathbf{c}^{ij}$ satisfies $CN\left(0, \sigma_c^2\right)$, where $\mathbf{c} \sim CN(\mathbf{0}, \mathbf{\Sigma}_c)$. $\mathbf{n} \in \mathbb{C}^{2MN\times 1}$ is the noise vector, each component also corresponds to circularly symmetric zero mean complex Gaussian random variables with variance σ_n^2, where $\mathbf{n} \sim CN(\mathbf{0}, \mathbf{\Sigma}_n)$. We assume that σ_n^2, σ_c^2 and $\mathbf{\Sigma}_s$ are known or can be estimated in advance.

3. DETECTOR DESIGN

In this section, we design a detector according to the above condition. What we need to do is to detect whether the target is present in the range cell. If the target is absent, the observation consists of the clutter and the noise. The problem becomes a binary hypothesis test as

$$\begin{aligned} H_0 &: \mathbf{z} = \mathbf{Hc} + \mathbf{n} \\ H_1 &: \mathbf{z} = \mathbf{Hs} + \mathbf{Hc} + \mathbf{n} \end{aligned} \tag{3}$$

When the target exists, the vector $\mathbf{z}$ follows complex Gaussian distribution with zero mean and covariance matrix $\mathbf{H}(\mathbf{\Sigma}_s + \mathbf{\Sigma}_c)\mathbf{H}^H + \mathbf{\Sigma}_n$. Otherwise, $\mathbf{z}$ follows a zero mean complex Gaussian distribution, whose covariance matrix is $\mathbf{H\Sigma}_c\mathbf{H}^H + \mathbf{\Sigma}_n$, where $[\cdot]^H$ denotes the conjugate transpose of $[\cdot]$.

Under the assumptions above, the probability density functions (pdf) of the observation vector can be described as

$$\begin{aligned} f(\mathbf{z} \mid H_0) &\propto \frac{1}{\det(\mathbf{H\Sigma}_c\mathbf{H}^H + \mathbf{\Sigma}_n)} e^{\left(-\mathbf{z}^H\left(\mathbf{H\Sigma}_c\mathbf{H}^H + \mathbf{\Sigma}_n\right)^{-1}\mathbf{z}\right)} \\ f(\mathbf{z} \mid H_1) &\propto \frac{1}{\det\left(\mathbf{H}(\mathbf{\Sigma}_s + \mathbf{\Sigma}_c)\mathbf{H}^H + \mathbf{\Sigma}_n\right)} \\ &\quad \cdot e^{\left(-\mathbf{z}^H\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H + \mathbf{\Sigma}_n\right)^{-1}\mathbf{z}\right)} \end{aligned} \tag{4}$$

where $\det(\cdot)$ denotes the determinant of the matrix $(\cdot)$.

In this paper, we adopt the Neyman-Pearson (NP) criterion, which has been widely applied in MIMO radar, to design the polarization detector, i.e.,

$$\begin{aligned} \frac{f(\mathbf{z}|H_0)}{f(\mathbf{z}|H_1)} &= \frac{\det\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)}{\det\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)} \\ &\cdot e^{\left(-\mathbf{z}^H\left(\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)^{-1}-\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)^{-1}\right)\mathbf{z}\right)} \end{aligned} \tag{5}$$

The logarithmic NP statistic is

$$\begin{aligned} L(\mathbf{z}) = -\mathbf{z}^H &\Big[\left(\mathbf{H\Sigma}_c\mathbf{H}^H + \mathbf{\Sigma}_n\right)^{-1} \\ &- \left(\mathbf{H}(\mathbf{\Sigma}_s + \mathbf{\Sigma}_c)\mathbf{H}^H + \mathbf{\Sigma}_n\right)^{-1}\Big]\mathbf{z} \end{aligned} \tag{6}$$

Then, the detection can be defined as

$$L(\mathbf{z}) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{th} \tag{7}$$

where γ_{th} is the threshold.

We observe that the detection statistic has a quadratic form of the complex Gaussian random $\mathbf{z}$. Since there is no placed restrictions on $\mathbf{H\Sigma}_c\mathbf{H}^H$, $\mathbf{H\Sigma}_s\mathbf{H}^H$ and $\mathbf{\Sigma}_n$, it is difficult to achieve the exact pdf for the detection statistic. Thus, we use the approximating form with the Gamma density [6]. The probabilities of detection and false alarm can be respectively expressed as

$$P_d = \int_\gamma^\infty t^{\alpha_{H_1}-1}\frac{e^{-\frac{t}{\beta_{H_1}}}}{\beta_{H_1}^{\alpha_{H_1}}\Gamma(\alpha_{H_1})}dt \tag{8}$$

$$P_{fa} = \int_\gamma^\infty t^{\alpha_{H_0}-1}\frac{e^{-\frac{t}{\beta_{H_0}}}}{\beta_{H_0}^{\alpha_{H_0}}\Gamma(\alpha_{H_0})}dt \tag{9}$$

where α_{H_1}, β_{H_1}, α_{H_0} and β_{H_0} are the approximate parameters of the Gamma distribution, which are given by

$$\alpha_{H_0} = \left(\frac{\left(\sum\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}{\sum\left(\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}\right) \tag{10}$$

$$\beta_{H_0} = \left(\frac{\sum\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)}}{\sum\left(\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H}(\mathbf{\Sigma}_s+\mathbf{\Sigma}_c)\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}\right)^{-1} \tag{11}$$

$$\alpha_{H_1} = \left(\frac{\left(\sum\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}{\sum\left(\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}\right) \tag{12}$$

$$\beta_{H_1} = \left(\frac{\sum\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)}}{\sum\left(\frac{diag\left(\mathbf{H\Sigma}_s\mathbf{H}^H\right)}{diag\left(\mathbf{H\Sigma}_c\mathbf{H}^H+\mathbf{\Sigma}_n\right)}\right)^2}\right)^{-1} \tag{13}$$

In the above formulas, $diag\left(\cdot\right)$ denotes the diagonal elements of the matrix $(\cdot)$. We can easily calculate the threshold γ_{th} for a given P_{fa}, then calculate P_d. It's not difficult to see that the values of γ_{th} and P_d depend on $\mathbf{H}\mathbf{\Sigma}_c\mathbf{H}^H$ and $\mathbf{H}\mathbf{\Sigma}_s\mathbf{H}^H$. Therefore, we can design and optimize the transmit polarization waveforms to improve the detection performance. Next, we propose a novel swarm intelligence optimal algorithm, i.e., glowworm swarm optimization algorithm, to design the polarization parameters of the transmit waveforms.

4. GLOWWORM SWARM OPTIMIZATION ALGORITHM

In GSO algorithm, glowworm attracting to each other depends on its own luciferin and attractiveness. Its luciferin value lies on the target value of its own location. The brighter its luciferin is, the better its location is, namely the better the target value is. The attractiveness relates to its brightness. The brighter its luciferin value is, the more its attractiveness is, which can attract the glowworm with lower appeal to move to it. If glowworms are with the same luciferin value, they move randomly. We use a point in the search space to analog glowworm individual in nature. Searching and optimization processing are emulated as attracting and moving processing [11]. Luciferin value reflects the merits of its location and determines its direction of motion. The attractiveness determines its displacement distance. By updating the luciferin value and attractiveness, the optimization can be implemented.

4.1. Mathematical Description

In order to achieve the optimization, GSO algorithm first initializes a group of random individuals in the feasible solution domain. Each denotes a glowworm carrying in luciferin. All the glowworms carry the equal quantity of luciferin. Each iteration contains two procedures: luciferin updating phase and glowworm movement phase.

4.1.1. Luciferin updating phase

In this stage, glowworm updating its luciferin value follows

$$l_p\left(t\right) = (1-\rho)l_p\left(t-1\right) + \gamma f\left(x_p(t)\right) \tag{14}$$

where $l_p\left(t\right)$ represents the luciferin value of the p-th glowworm at time t, $\rho \in (0,1)$ is the parameter which control the luciferin value, γ denotes the parameter evaluating the target function value, and $f\left(x\right)$ stands for the objective function. $x_p(t)$ denotes the location of the p-th glowworm at time t.

4.1.2. Glowworm movement phase

In this stage, the p-th glowworm selects the q-th glowworm in the local-decision range with a certain probability and then moves to it.

a) The selection formula of path probability is given by

$$P_{pq}(t) = \frac{l_q\left(t\right) - l_p\left(t\right)}{\sum\limits_{k\in N_p(t)} l_k(t) - l_p(t)} \tag{15}$$

b) The local-decision domain radius updating rule is

$$r_d^p(t+1) = \min\{r_s, \max\left\{0, r_d^p(t) + \beta\left(n_t - \left|N_p\left(t\right)\right|\right)\right\}\} \tag{16}$$

where r_s denotes the radial sensor range of the glowworms. $r_d^p(t)$ represents local-decision domain radius of the glowworms at current iteration t, where $0 < r_d^p(t) \leq r_s$. β is the proportionality constant. n_t is neighborhood threshold which is in charge of the number of glowworms in the local-decision range, and $N_p(t)$ denotes the set of neighbors of the p-th glowworm at t.

$$N_p(t) = \{q : ||x_q\left(t\right) - x_p\left(t\right)|| < r_d^p(t); l_p\left(t\right) < l_q\left(t\right)\} \tag{17}$$

c) Location updating follows the rule as

$$x_p\left(t+1\right) = x_p\left(t\right) + s\frac{x_q\left(t\right) - x_p\left(t\right)}{||x_q\left(t\right) - x_p\left(t\right)||} \tag{18}$$

where s is the step-size.

4.2. Implementation Step

We design the polarization waveform for MIMO radar through the GSO optimization algorithm based on detection probability maximization. In other words, detection probability formula is the objective function. Thus, $x_p\left(t\right)$ corresponds to the polarization parameters of the p-th glowworm at time t.

$$x_p\left(t\right) = \left[\gamma_p^1\left(t\right), \eta_p^1\left(t\right), \gamma_p^2\left(t\right), \eta_p^2\left(t\right), \cdots, \gamma_p^M\left(t\right), \eta_p^M\left(t\right)\right] \tag{19}$$

where $\gamma_p^i\left(t\right)$ and $\eta_p^i\left(t\right)$ are the two polarization parameters of the p-th glowworm and the i-th transmit antenna. GSO algorithm finds the optimal solution through the next procedures.

Step 1. Initialize the parameter values for $r_d^p(t)$ and a group of n individuals in the feasible solution domain, which are shown in Table 1.

Table 1. Initialization

ρ	γ	β	s	l_0	r_s	r_0	n_t	n	N_{it}
0.4	0.6	0.08	0.3	5	$\frac{\pi}{3}$	$\frac{4}{3}\pi$	5	100	1000

In Table 1, l_0 is the initial luciferin value, and N_{it} is iterations. r_0 is the initial local-decision range, which depends on the Euclidean norm of solution space (usually selected such that $r_0 = \frac{2}{3} \times ||solution \quad space||$).

Step 2. Update the luciferin value according to (14), where the target function is replaced by (8). The threshold has been achieved by the known value of P_{fa} .

Step 3. Go to movement stage, each glowworm selects a neighbor whose luciferin value is higher than its own in the local-decision range $r_d^p(t)$ to compose $N_p(t)$, where $r_d^p(t)$ and $N_p(t)$ are obtained by (16) and (17), respectively.

Step 4. Employ (15) to calculate the path selection probability that the p-th glowworm moves to the q-th glowworm.

Step 5. By the use of the roulette method, let the p-th glowworm select a neighbor, namely the q-th glowworm, and move toward it, then update the location of the p-th glowworm according to (18).

Step 6. Update the value of the local-decision domain by (16).

Step 7. Complete this iteration, go to step 2 until meeting the maximum iterations or searching precision.

5. SIMULATION AND ANALYSIS

The effectiveness of GSO algorithm has been proved by several common functions [12]. In this paper, we design the polarization waveforms according to the practical condition in a polarimetric MIMO radar system. Based on GSO algorithm, the optimal polarization parameters can be adaptively choose under clutter and noise environment. For simplification, we discuss the system for $M = 2$, $N = 2$.

First, we suppose the diagonal elements of $\mathbf{\Sigma}_n$ and $\mathbf{\Sigma}_c$ respectively as $\sigma_n^2 = 0.2$, $\sigma_c^2 = 0.2$, and $\mathbf{\Sigma}_s^{11}$, $\mathbf{\Sigma}_s^{12}$, $\mathbf{\Sigma}_s^{21}$, $\mathbf{\Sigma}_s^{22}$ are selected from [6]. Given P_{fa}, we can obtain optimal polarization transmit waveforms adaptively according to the target and the environment information when $\mathbf{\Sigma}_s$ is known in advance.

To show the validity of GSO algorithm, we compare the detection probability using the optimally designed polarization waveforms with that using a few traditional polarization transmit waveforms as follows. $\mathbf{t}^1 = \mathbf{t}^2 = [1, 0]^T$ and $\mathbf{t}^1 = \mathbf{t}^2 = [0, 1]^T$ respectively represent the two transmit antennas adopting horizontal polarization and vertical polarization synchronously. $\mathbf{t}^1 = [1, 0]^T$ and $\mathbf{t}^2 = [0, 1]^T$ denote the two transmit antennas employing orthogonal polarization.

As shown in Fig 1, we vary the value of P_{fa} to plot the curves of P_d for different transmit polarization schemes. It reveals that using GSO-based polarization design, the detection performance of the proposed algorithm outperforms that of the other several transmit waveform polarization schemes.

Second, we analyze the variation on P_d for different signal-clutter-noise radio (SCNR) in order to further illustrate the detection performance using GSO algorithm. We define the SCNR as

$$SCNR = \frac{||\mathbf{\Sigma}_s||_2}{\sigma_c^2 + \sigma_n^2} \tag{20}$$

where $||\cdot||_2$ denotes the 2-norm for matrix $(\cdot)$. We assume that $\mathbf{\Sigma}_s$ is the same as before, $\sigma_c^2 = 0.2$, the value of $\mathbf{\Sigma}_n$ varies for different SCNR. We plot the curves of P_d versus SCNR under different P_{fa}, i.e., $P_{fa} = 0.1, 0.01$ and 0.001, respectively.

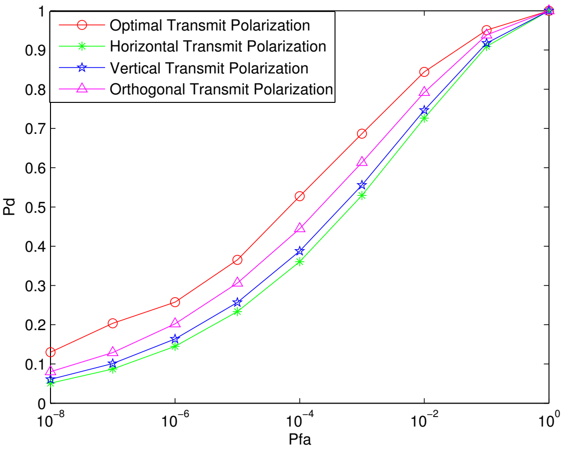

Fig. 1. P_d versus P_{fa} under different transmit polarizations.

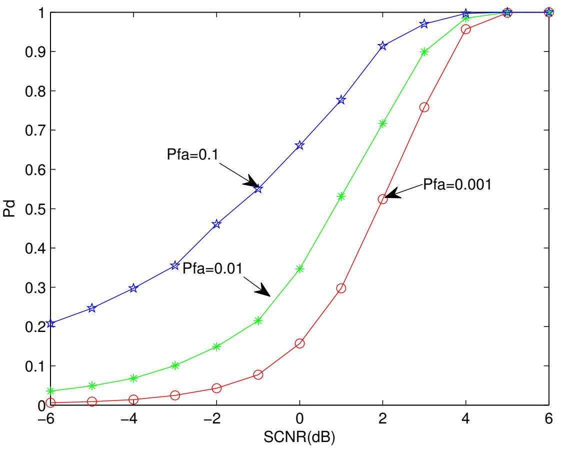

Fig. 2. P_d versus SCNR under different P_{fa}.

It has been shown that P_d increases with the increasing of SCNR, as shown in Fig 2. Also, we can find how P_{fa} effects the detection performance for different value of SCNR.

6. CONCLUSIONS

In this paper, we investigate the problem of polarization waveform design to maximize the probability of detection in polarimetric MIMO radar. Instead of traditional way of training data, we adopt a novel swarm intelligent optimization and save the time for optimization. For MIMO radar with two distributed transmit antennas, the algorithm can handle in parallel and optimize four polarization parameters at the same time. From this, we can solve the challenge of nested loop which is hardly able to achieve, whereas the shortcomings of low convergence speed in classical GSO algorithm inevitably exist, which we will overcome in the next work.

7. REFERENCES

[1] M. Hurtado and A. Nehorai, "Polarimetric detection of targets in heavy inhomogeneous clutter," *IEEE Transactions on Signal Processing*, vol. 56, pp. 1349–1361, April 2008.

[2] H. Jiang, Y. Zhang, C. Liu, J. Li, and H.J. Cui, "A PARAFAC-based algorithm for multidimensional parameter estimation in polarimetric bistatic MIMO radar," *EURASIP Journal on Advances in Signal Processing*, vol. 2013, August 2013.

[3] Y.Y. Liao and Z.S. He, "Study on stealth target detection performance of polarimetric MIMO radar," *Application Research of Computers(in Chinese)*, vol. 29, pp. 246–249, January 2012.

[4] G.L. Cui, L.J. Kong, X.B. Yang, and J.Y. Yang, "Adaptive polarimetric MIMO radar detection," in *International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS)*, Chengdu, December 2010, pp. 1–4.

[5] G.L. Cui, L.J. Kong, X.B. Yang, and J.Y. Yang, "GLRT design for polarimetric MIMO radar in non-Gaussian clutter," in *IEEE Radar Conference (RADAR)*, Kansas City, MO, May 2011, pp. 106–108.

[6] S. Gogineni and A. Nehorai, "Polarimetric MIMO radar with distributed antennas for target detection," in *IEEE Transactions on Signal Processing*, March 2010, vol. 58, pp. 1689–1697.

[7] S. Gogineni and A. Nehorai, "Game theoretic design for polarimetric MIMO radar target detection," *Signal Processing*, vol. 92, pp. 1281–1289, May 2012.

[8] I. Aljarah and S.A. Ludwig, "A new clustering approach based on glowworm swarm optimization," in *2013 IEEE Congress on Evolutionary Computation (CEC)*, Cancun, June 2013, pp. 2642–2649.

[9] D.X. He, H.Z. Zhu, and G.Q. Liu, "Glowworm swarm optimization algorithm for solving multi-constrained QoS multicast routing problem," in *2011 Seventh International Conference on Computational Intelligence and Security*, Hainan, December 2011, pp. 66–70.

[10] K.N. Krishnanand and D. Ghose, "Detection of multiple source locations using a glowworm metaphor with applications to collective robotics," in *IEEE Swarm Intelligence Symposium*, USA, June 2005, pp. 84–91.

[11] K.N. Krishnanand and D. Ghose, "Glowworm swarm optimization for searching higher dimensional spaces," *Innovations in Swarm Intelligence Studies in Computational Intelligence*, vol. 248, pp. 61–65, January 2009.

[12] K.N. Krishnanand and D. Ghose, "Glowworm swarm optimization for simultaneous capture of multiple local optima of multimodal functions," *Swarm Intelligence*, vol. 3, pp. 87–124, June 2009.