# DISTRIBUTED PARTICLE FILTERING FOR BLIND EQUALIZATION IN RECEIVER NETWORKS USING MARGINAL NON-PARAMETRIC APPROXIMATIONS

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## ABSTRACT

This paper introduces a novel distributed particle filtering algorithms for the blind equalization of frequency-selective channels in a setup where a single transmitter broadcasts to multiple remote receivers. The algorithm computes particle-independent non-parametric approximations of some posterior probability functions, which are propagated between nodes via minimum-consensus iterations. We verify via numerical simulations that the proposed algorithms exhibit bit error rate (BER) performances markedly superior to that of particle-filtering-based isolated receivers with communication requirements far inferior to that of previous distributed algorithms.

***Index Terms***— Distributed Algorithms, Particle Filters, Blind Equalization.

## 1. INTRODUCTION

Distributed estimation has attracted much interest due to the widespread availability of computing devices capable of sensing and communicating. We tackle in this paper a distributed blind equalization problem, in which a network of remote sensors cooperate to detect a sequence of discrete-valued symbols broadcast by a single transmitter through independent frequency-selective channels. Our proposed method drops the need for a fusion center, since the different nodes process their local observations independently and cooperate to evaluate an approximation to the joint optimal estimate of the transmitted data given all observations in the network.

Recently, a multitude of distributed particle filtering methods was developed (see [1] for a review). These methods circumvent restrictions on the signal model imposed by linear estimation techniques such as the Kalman [2] or adaptive [3] filters at the cost of increased computational and communication complexity. However, they generally lead to superior estimation results for nonlinear or nongaussian models.

Distributed particle filters must deal with the fact that particle-dependent quantities must be propagated across nodes [4] unless additional approximations the empirical posterior densities are made. Due to the generally large number of particles required for proper operation of these methods, this is an undesirable feature in many scenarios with communication or power constraints.

In this paper we introduce a new distributed particle filtering algorithm that evaluates non-parametric approximations to some node dependent probabilities across the receiver network. The algorithm has a communication complexity an order of magnitude smaller than that of previous methods [5], comparable to that of methods based on parametric approximations [6]. This complexity can be further alleviated by replacing broadcast-dependent quantities with minimum-consensus derived ones [5], which drops the need for one node to communicate beyond its neighborhood.

The remaining text is organized as follows: in Sec. 2 we describe the problem setup, introducing in Sec. 3 a centralized particle filter approach to its solution. In Sec. 4 we describe a distributed solution based on parametric approximations and, later, in Sec. 5, we introduce the new reduced complexity method, whose is performance assessed in Sec. 6. Our conclusions are summarized in Sec. 7.

## 2. PROBLEM SETUP

Denote by $\{b_n\}$ an independent, identically distributed (i.i.d.) binary bit sequence and by $\{x_n\}$, $x_n \in \{\pm 1\}$, the corresponding differentially encoded symbols. We assume that the observations $y_{r,0:n} \triangleq \{y_{r,0}, \ldots, y_{r,n}\}$ at the $r-$th node of a network of $R$ receivers are obtained as the output of the additive noise frequency-selective finite impulse response (FIR) channel

$$y_{r,n} = \mathbf{h}_r^T \mathbf{x}_n + v_{r,n} \ , \tag{1}$$

where $\mathbf{h}_r \in \mathbb{R}^{L\times 1}$ is a vector with the (time-invariant) channel impulse response terms, $\mathbf{x}_n \triangleq [x_n \ldots x_{n-L+1}]^T$, and $v_{r,n}$ represents an i.i.d. zero-mean Gaussian random process of known variance $\sigma_r^2$.

The *unknown, random* parameters $\mathbf{h}_r$, $1 \le r \le R$, are assumed to be independent for $r \ne s$, and distributed a priori as $\mathbf{h}_r \sim \mathcal{N}_L(\mathbf{h}_r|0; I/\varepsilon^2)$, where $\mathcal{N}_L$ denotes an $L-$variate Gaussian p.d.f., and $\varepsilon$ is the model's hyper-parameter.

Under these hypotheses, we aim at developing a recursive method for obtaining MAP estimates $\hat{b}_n = \arg\max_{b_n} p(b_n| y_{1:R,0:n})$, where $y_{1:R,0:n} \triangleq \{y_{1,0:n} \cdots y_{R,0:n}\}$.

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## 3. CENTRALIZED SOLUTION VIA PARTICLE FILTERS

Particle filters allow one to approximate the posterior probability mass function (p.m.f.) of the transmitted bits as

$$p(b_n|y_{1:R,0:n}) \approx \sum_{q=1}^{Q} w_n^{(q)} \mathcal{I}\left\{b_n = b_n^{(q)}\right\}, \quad (2)$$

where $\mathcal{I}\{\cdot\}$ denotes the indicator function, $Q$ the number of *particles* $b_n^{(q)}$ sampled from the importance function $\pi(\mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n})$, and $w_n^{(q)}$ are the importance weights. For simplicity, we employ the *prior* importance function, so that $\pi(\mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}^{(q)}, y_{1:R,0:n}) \triangleq p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}^{(q)})$. The resulting *importance weights'* update expression is then given by

$$w_n^{(q)} \;\propto\; w_{n-1}^{(q)}\, p(y_{1:R,n}|\mathbf{x}_{0:n}^{(q)}, y_{1:R,0:n-1}). \quad (3)$$

From the a priori independence of the unknown parameters for each receiver's channel, it can be easily shown that the quantity on the right-hand side (r.h.s.) of (3) can be factored as

$$p(y_{1:R,n}|\mathbf{x}_{0:n}^{(q)}, y_{1:R,0:n-1}) \propto \prod_{r=1}^{R} \lambda_{r,n}(\mathbf{x}_{0:n}^{(q)}), \quad (4)$$

where $\lambda_{r,n}(\mathbf{x}_{0:n}^{(q)}) \triangleq p(y_{r,0:n}|\mathbf{x}_{0:n}^{(q)}, y_{r,0:n-1})$.
Under the assumptions of Sec. 2, one can verify from Kalman filter theory that

$$\begin{aligned} & p(y_{r,0:n}|\mathbf{x}_{0:n}^{(q)}, y_{r,0:n-1}) = \\ = \;& \int_{\mathbb{R}^L} p(y_{r,0:n}, \mathbf{h}_r|\mathbf{x}_{0:n}^{(q)}, y_{r,0:n-1})\, d\mathbf{h}_r & (5)\\ = \;& \int_{\mathbb{R}^L} p(y_{r,n}|\mathbf{h}_r, \mathbf{x}_n^{(q)}) p(\mathbf{h}_r|\mathbf{x}_{0:n-1}^{(q)}, y_{r,0:n-1})\, d\mathbf{h}_r & (6)\\ = \;& \mathcal{N}\left(y_{r,n}|\; \hat{\mathbf{h}}_{r,n-1}^{(q)T}\mathbf{x}_n^{(q)} \;; \gamma_{r,n}^{(q)}\right), & (7) \end{aligned}$$

where $\hat{\mathbf{h}}_{r,n-1}^{(q)T}$ and $\gamma_{r,n}^{(q)}$ can be computed via the Kalman filter recursions

$$\begin{aligned} \gamma_{r,n}^{(q)} &\triangleq \sigma_r^2 + \mathbf{x}_n^{(q)T}\mathbf{\Sigma}_{r,n-1}^{(q)}\mathbf{x}_n^{(q)}\;, & (8)\\ e_{r,n}^{(q)} &\triangleq y_{r,n} - \hat{\mathbf{h}}_{r,n-1}^{(q)T}\mathbf{x}_n^{(q)}\;, & (9)\\ \hat{\mathbf{h}}_{r,n}^{(q)} &= \hat{\mathbf{h}}_{r,n-1}^{(q)} + \mathbf{\Sigma}_{r,n-1}^{(q)}\mathbf{x}_n^{(q)} e_{r,n}^{(q)}/\gamma_{r,n}^{(q)}\;, & (10)\\ \mathbf{\Sigma}_{r,n}^{(q)} &= \mathbf{\Sigma}_{r,n-1}^{(q)} - \mathbf{\Sigma}_{r,n-1}^{(q)}\mathbf{x}_n^{(q)}\mathbf{x}_n^{(q)T}\mathbf{\Sigma}_{r,n-1}^{(q)}/\gamma_{r,n}^{(q)}\;, & (11) \end{aligned}$$

with $\hat{\mathbf{h}}_{r,-1}^{(q)} = \underline{\mathbf{0}}$ and $\mathbf{\Sigma}_{r,-1}^{(q)} = \mathbf{I}\varepsilon^{-2}$.

The weight update rule can be derived by replacing (4) into (3), so that

$$w_n^{(q)} \propto w_{n-1}^{(q)} \prod_{r=1}^{R} \lambda_{r,n}(\mathbf{x}_{0:n}^{(q)}). \quad (12)$$

## 4. PARAMETRIC APPROXIMATIONS

The exact distributed computation of (12) requires that each node broadcasts $Q$ real numbers to all remaining nodes, which may be undesirable in many scenarios. To reduce this communication burden, the $r-$th node may employ parametric approximations to $\lambda_{s,n}(\mathbf{x}_{0:n}^{(q)})$, $\forall s \neq r$. In the sequel, we describe a version of the algorithm introduced in [6], adapted for the signal model at hand, that performs such approximations.

First, note that (10) implies that

$$p(\mathbf{h}_s|\mathbf{x}_{0:n-1}^{(q)}, y_{s,0:n-1}) = \mathcal{N}_L\left(\mathbf{h}_s|\hat{\mathbf{h}}_{s,n-1}^{(q)}; \mathbf{\Sigma}_{s,n-1}^{(q)}\right), \quad (13)$$

so that

$$\begin{aligned} p(\mathbf{h}_s|y_{r,0:n-1}) &= \sum_{\mathbf{x}_{0:n-1}} p(\mathbf{h}_s|\mathbf{x}_{0:n-1}, y_{s,0:n-1})\cdot \\ \cdot p(\mathbf{x}_{0:n-1}|y_{s,0:n-1}) &\approx \sum_{q} w_{s,n-1}^{(q)} p(\mathbf{h}_s|\mathbf{x}_{0:n-1}^{(q)}, y_{s,0:n-1}). \end{aligned} \quad (14)$$

A parametric approximation to $\lambda_{s,n}(\mathbf{x}_{0:n}^{(q)})$ can be determined by replacing the rightmost density in (6) with

$$p(\mathbf{h}_s|y_{s,0:n-1}) \approx \mathcal{N}_L\left(\mathbf{h}_s|\tilde{\mathbf{h}}_{s,n-1}; \tilde{\mathbf{\Sigma}}_{s,n-1}\right), \quad (15)$$

where the parameters $\tilde{\mathbf{h}}_{s,n}$ and $\tilde{\mathbf{\Sigma}}_{s,n}$ are determined so as to match the moments of the gaussian mixture on the left-hand side (l.h.s.) of (14). After evaluating the integral in (6) using (15), we get that

$$\lambda_{s,n}(\mathbf{x}_{0:n}^{(q)}) \approx \mathcal{N}\left(y_{s,n}|\; \tilde{\mathbf{h}}_{s,n-1}^{T}\mathbf{x}_n^{(q)} \;; \tilde{\gamma}_{s,n}^{(q)}\right). \quad (16)$$

where $\tilde{\gamma}_{n,s}^{(q)} \triangleq \sigma^2 + (\mathbf{x}_n^{(q)})^T\tilde{\mathbf{\Sigma}}_{s,n-1}\mathbf{x}_n^{(q)}$, and $\mathbf{x}_n^{(q)}$ are particles of the $r-$th node, which need not be synchronized [4] with the particles of the remaining nodes.

The inherent phase ambiguity that affects the blind equalization problem causes $\mathbf{x}_n^{(q)}$ and $\hat{\mathbf{h}}_{s,n-1}^{(q)}$ to be affected by *random* multiplication terms $\pm 1$. To circumvent this problem, which otherwise was experimentally verified to lead to poor performance, we introduce a phase insensitive approximation

$$\begin{aligned} \tilde{\lambda}_{s,n}(\mathbf{x}_n^{(q)}) = \frac{1}{2}&\mathcal{N}\left(y_{s,n}|\tilde{\mathbf{h}}_{s,n-1}^{T}\mathbf{x}_n^{(q)}; \tilde{\gamma}_{s,n}^{(q)}\right) + \\ &+\frac{1}{2}\mathcal{N}\left(y_{s,n}| -\tilde{\mathbf{h}}_{s,n-1}^{T}\mathbf{x}_n^{(q)}; \tilde{\gamma}_{s,n}^{(q)}\right), \end{aligned} \quad (17)$$

whose parameters are estimated, by the $s-$th node, according to the modified moment matching expressions

$$\begin{aligned} \tilde{\mathbf{h}}_{s,n} &= \sum_{q} w_{s,n}^{(q)}\, \alpha_{s,n}^{(q)}\hat{\mathbf{h}}_{n,s}^{(q)} & (18)\\ \tilde{\mathbf{\Sigma}}_{s,n} &= \sum_{q} w_{s,n}^{(q)}\Big(\mathbf{\Sigma}_{s,n}^{(q)} + (\tilde{\mathbf{h}}_{s,n} - \alpha_{s,n}^{(q)}\hat{\mathbf{h}}_{s,n}^{(q)})\;\cdot \\ &\qquad \cdot\,(\tilde{\mathbf{h}}_{s,n} - \alpha_{s,n}^{(q)}\hat{\mathbf{h}}_{s,n}^{(q)})^T\Big) & (19) \end{aligned}$$

where $\alpha_{s,n}^{(q)} \in \{\pm 1\}$ are constants dependent on $\mathbf{x}_n^{(q)} = \mathbf{x}$, determined so that $\alpha_{s,n}^{(q)} = -\alpha_{s,n}^{(q')}$ if $\mathbf{x}_n^{(q)} = -\mathbf{x}_n^{(q')}$.

In summary, the proposed parametric algorithm propagates the particles weights at the $r-$th node as

$$w_{r,n}^{(q)} \propto w_{r,n-1}^{(q)} \lambda_{r,n}(\mathbf{x}_{0:n}^{(q)}) \prod_{\substack{s=1 \\ s \neq r}}^{R} \tilde{\lambda}_{s,n}(\mathbf{x}_n^{(q)}). \tag{20}$$

Each remote node $s$ must then *broadcast* at each time step $\tilde{\lambda}_{r,n}(\mathbf{x}_n^{(q)})$, $\forall \mathbf{x}^{(q)}$, a total of $2^L$ real numbers.

## 5. NON-PARAMETRIC APPROXIMATIONS

We introduce here a novel algorithm that approximates (3) at the $r-$th node as

$$w_{r,n}^{(q)} \underset{\sim}{\propto} w_{r,n-1}^{(q)} \, p(y_{r,n}|\mathbf{x}_{0:n}^{(q)}, y_{r,0:n-1}) \prod_{\substack{s=1 \\ s \neq r}}^{R} \check{\lambda}_{s,n}(\mathbf{x}_n^{(q)}), \tag{21}$$

where $\check{\lambda}_{s,n}(\mathbf{x}_n^{(q)}) \triangleq p(y_{s,n}|\mathbf{x}_n^{(q)}, y_{1:R,0:n-1})$. The proposed approximation leads to a communication complexity smaller than that of the optimal solution, e.g. Eq. (12), because $\check{\lambda}_{s,n}(\mathbf{x}_n^{(q)})$ only depends on $\mathbf{x}_n^{(q)}$, which can assume $2^L \ll Q$ states.

To evaluate the terms on the right-hand side (r.h.s.) of (21), first note that

$$\check{\lambda}_{s,n}(\mathbf{x}_n^{(q)}) = \frac{p(y_{s,n}, \mathbf{x}_n^{(q)}|y_{1:R,0:n-1})}{p(\mathbf{x}_n^{(q)}|y_{1:R,0:n-1})} \tag{22}$$

The denominator of (22) evaluates to

$$\begin{aligned}
&\sum_{\mathbf{x}_{0:n-1}} p(\mathbf{x}_n^{(q)}, \mathbf{x}_{0:n-1}|y_{1:R,0:n-1}) \\
&= \sum_{\mathbf{x}_{0:n-1}} p(\mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) p(\mathbf{x}_{0:n-1}|y_{1:R,0:n-1}) \\
&\approx \sum_{p} p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}^{(p)}) w_{s,n-1}^{(p)} \ ,
\end{aligned} \tag{23}$$

since the weights $w_{s,n-1}^{(p)}$ (and particles $\mathbf{x}_{0:n-1}^{(p)}$) are a Monte Carlo representation to $p(\mathbf{x}_{0:n-1}|y_{1:R,0:n-1})$.

To compute the numerator of (22), first observe that

$$\begin{aligned}
&p(y_{s,n}, \mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) = \\
&= p(y_{s,n}|\mathbf{x}_n^{(q)}, \mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) p(\mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) \\
&= p(y_{s,n}|\mathbf{x}_n^{(q)}, \mathbf{x}_{0:n-1}, y_{s,0:n-1}) p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}),
\end{aligned} \tag{24}$$

since, due to the underlying assumptions, $y_{s,n}$ is conditionally independent of the other nodes' past observations given its own past observations and the state sequence; see also [6, App. B]. Therefore,

$$\begin{aligned}
&p(y_{s,n}, \mathbf{x}_n^{(q)}|y_{1:R,0:n-1}) \\
&= \sum_{\mathbf{x}_{0:n-1}} p(y_{s,n}, \mathbf{x}_n^{(q)}, \mathbf{x}_{0:n-1}|y_{1:R,0:n-1}) = \\
&= \sum_{\mathbf{x}_{0:n-1}} p(y_{s,n}, \mathbf{x}_n^{(q)}|\mathbf{x}_{0:n-1}, y_{1:R,0:n-1}) p(\mathbf{x}_{0:n-1}|y_{1:R,0:n-1}) \\
&\approx \sum_{p} p(y_{s,n}|\mathbf{x}_n^{(q)}, \mathbf{x}_{0:n-1}^{(p)}, y_{s,0:n-1}) p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}^{(p)}) w_{s,n-1}^{(p)} \ .
\end{aligned} \tag{25}$$

Plugging back (23) and (25) into (22), we get that

$$\check{\lambda}_{s,n}(\mathbf{x}_n^{(q)}) = \frac{\sum_p p(y_{s,n}|\mathbf{x}_{0:n-1}^{(p)}, \mathbf{x}_n^{(q)}, y_{s,0:n-1}) p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}^{(p)}) w_{s,n-1}^{(p)}}{\sum_p p(\mathbf{x}_n^{(q)}|\mathbf{x}_{n-1}^{(p)}) w_{s,n-1}^{(p)}}. \tag{26}$$

The proposed algorithm propagates the weights $w_{r,n}^{(q)}$ similarly to (20); each node $s$ broadcasts $\tilde{\lambda}_{r,n}(\mathbf{x}_n^{(q)})$, at each time step, for each possible state, the same communication requirements of the parametric algorithm of Sec. 4.

### 5.1. Communication Complexity Reduction via Minimum-Consensus

It was verified in [5] that the centralized weight update expression (12) can be approximated via

$$w_n^{(q)} \propto w_{n-1}^{(q)} \min_r \lambda_{r,n}(\mathbf{x}_{0:n}^{(q)}), \tag{27}$$

without incurring in large performance penalties. This is advantageous because (27) can be evaluated via the *minimum-consensus* [7] algorithm, which is guaranteed to converge in a finite, topology-dependent number of iterations and only requires that each node be able to communicate with its immediate neighbors. When $\lambda_{r,n}(\mathbf{x}_{0:n}^{(q)})$ are equal variance independent gaussian densities, as in [5], the approximation in (27) is tantamount to replacing the $L$-2 norm in the expression of the gaussians with the $L-\infty$ norm.

Applying the approximation of [5] to (21) leads to

$$w_{r,n}^{(q)} \underset{\sim}{\propto} w_{r,n-1}^{(q)} \min \left\{ p(y_{r,n}|\mathbf{x}_{0:n}^{(q)}, y_{r,0:n-1}); \min_{s \neq r} \check{\lambda}_{s,n}(\mathbf{x}_n^{(q)}) \right\}, \tag{28}$$

which can be evaluated via $R{\cdot}2^L$ parallel minimum-consensus iterations. Note that each node performs a distinct minimization, for each of the $2^L$ values of $\mathbf{x}_n^{(q)}$.

## 6. SIMULATION RESULTS

The steady state performance of the described algorithms was assessed via simulations consisting of 2500 independent

Monte Carlo runs. In each realization, we estimated the mean bit error rate (BER) as a function of $E_B/N_0$ transmitting a random sequence of 300 i.i.d bits, being the first 200 bits discarded to allow for convergence.

The simulated system has $R = 4$ receiving nodes and the filters employed $Q = 300$ particles. All algorithms perform systematic resampling [4] at all steps.

The transmission channels $\mathbf{h}_r$ have $L = 3$ coefficients, and were obtained by sampling independently in each realization and for each receiver from a Gaussian p.d.f. $\mathcal{N}(\mathbf{0};\mathbf{I})$ and normalized so that $\|\mathbf{h}_r\|^2 = 1$. The noise variances were determined as $\sigma^2 = \|\mathbf{h}_r\|^2 N_0/E_B$. The model hyper-parameter was set to $\varepsilon = 1$.

In Fig. 1, we show the performance of the proposed non-parametric algorithm ($*$) employing minimum-consensus (Equation 28) and of the parametric algorithm ($\nabla$) (Equation 20). For comparison, we ran with the same setup the optimal joint particle-filter-based algorithm that exactly computes (4) ($\bigcirc$) and its approximation via minimum-consensus [5] ($\square$). Fig. 1 also displays the performance of isolated receivers ($+$) that do not cooperate.

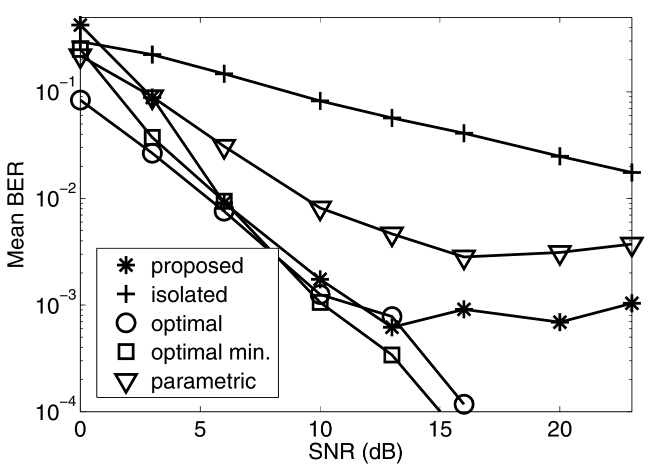


**Fig. 1**. Mean bit error rate (BER) estimated over 2500 independent runs.

As one may verify, the non-parametric algorithm performs better than the parametric algorithm. Both algorithms are outperformed by the optimal centralized particle filter estimator, but surpass the isolated receivers by a great margin.

## 7. CONCLUSIONS

We described in this paper a new distributed particle filtering algorithm based on non-parametric approximations and minimum consensus. The algorithm determines approximations to some required posterior probability functions that converge within a finite number of iterations per time step. Compared to previous parametric approaches, the proposed method attains better performances with equivalent communication and computational requirements. The method introduced in this paper can be applied to any filtering problem with conditionally independent linear Gaussian observations and discrete-valued variables.

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