Over-the-Horizon Radar Potential Signal Parameter Estimation Accuracy in Harsh Sensing Environments

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***Abstract*—Modern high frequency (HF) over-the-horizon Radar's (OTHR's) that perform parametric sensing over a huge coverage of several million square kilometers operate in harsh sensing environments consisting of strong interference and clutter. The highly dynamic nature of such an environment governed by harsh ionosphere propagation conditions and a highly occupied HF spectrum mandate the application of adaptive signal processing architectures enabled by the emerging direct digital receiver technology. In this paper, we investigate the potential accuracy of OTHR signal parameter estimation, in particular, direction-of-arrival (DOA) estimation of target returns and/or HF signals masked by both interference and background noise. Recently, the concept of two-dimensional (2D) receive arrays has been introduced for skywave OTHR, along with a more accurate model of the background noise spatial distribution. In this prior work it was shown that with 2D spatial sampling, the external background noise covariance is no longer white. In this paper we introduce the Cramer-Rao bound for angle estimates using an arbitrary array geometry and arbitrary known colored noise spatial covariance. We will show through numerical simulation that improved DOA performance is possible with 2D array geometries that provide both improved signal-to-external noise ratio and more highly curved array manifolds. The particular array geometries discussed exploit the colored noise spatial covariance.**

***Index Terms*—Interference, over-the-horizon Radar, parameter estimation, harsh sensing environment, Cramer-Rao bounds.**

I. Introduction

Over-the-horizon Radar (OTHR), both skywave and surface wave, provides a unique wide area surveillance capability that few if any contemporary sensing modalities can match in terms of near simultaneous coverage area [3]. These Radar systems are required to operate throughout the 3-30MHz HF spectrum; the actual operational center frequency is dictated by coverage area requirements, ionospheric propagation conditions, and current HF spectrum usage conditions. In fact, OTHR is one of the only high-power Radar systems restricted to operate not with primary spectrum usage authorization, but with secondary not to interfere restrictions. Proper operation of an OTHR system is thus complicated by the requirement to constantly assess ongoing usage of the HF spectrum, and to select operational frequencies that might cause the least co-user interference. During nighttime conditions when the support for ionospheric propagation decreases to under 15MHz and daytime D-layer absorption disappears, it can become nearly impossible to select a single frequency band, i.e. 10kHz free from interference. In addition to co-user interference, which is relatively narrowband, OTHR systems must deal with impulsive lighting noise which is wideband, and finally they must also cope with residual HF background noise.

In a recent paper discussing potential 2D OTHR receive array geometries [1], a 2D spatial noise model was proposed for nighttime background HF noise. The authors showed that improved signal-to-external noise ratio (SENR) gains could be achieved with 2D oversampled array geometries that exploited the colored spatial noise covariance through fully optimal processing. In addition, the authors showed that standard uniform linear array (ULA) geometries resulted in array spatial samplings that restricted the spatial noise covariance to be white and thus limited the potential optimal signal processing improvement gains. The key realization in this prior work was the SENR gains were achieved through optimal superdirective beamforming [5]. Such beamforming methods actually result in more narrow beams than traditional beams. The obvious implication, which we will discuss in this paper, is that these narrow beams provide improved target/signal angle estimation accuracy in addition to improved SENR. Thus, in this paper we will show that within the regime of OTHR operation during the nighttime harsh HF environment, there is the potential to realize improved radar system performance from 2D oversampled array geometries and optimal signal processing. There are of course other regimes in which 2D array architectures provide benefits in comparison 1D apertures, over those are beyond the scope of this paper.

The primary metric which will be utilized to demonstrate improved array DOA estimation accuracy will be the Cramer-Rao lower bound (CRB). This metric provides a lower bound on the variance of any unbiased parameter estimate. Much work in the literature has been devoted to discussing issues related to the CRB [2]. Our contribution in this area relates to connection between improved DOA performance using superdirective beamforming and its quantifiable behavior through the CRB which analytically expresses DOA performance through improved SENR and array manifold curvature. We will introduce the expression for a stochastic single source CRB and demonstrate several numerical simulations that quantify the improved DOA performance.

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II. Signal and Noise Model

The signal and noise model used throughout this paper is as follows. Let an array of N sensors receive M multiple narrowband signals from unknown directions $\{(\theta, \phi)\} = \{(\theta_1, \phi_1), \ldots, (\theta_M, \phi_M)\}$ corrupted by noise. The total number of array snapshots is K, with each snapshot k given as

$$\mathbf{z}(k) = \mathbf{A}(\theta, \phi)\mathbf{s}(k) + \mathbf{n}(k),\ k = 1, \ldots, K. \tag{1}$$

The $N \times M$ matrix $\mathbf{A}(\theta, \phi)$ is a matrix of array manifold vectors parametrized by unknown source directions (θ, ϕ). The quantities $\mathbf{s}(k)$ and $\mathbf{n}(k)$ are circular complex zero mean Gaussian random vectors, which are independent, temporally uncorrelated, and have known spatial covariances $E[\mathbf{s}(k)\mathbf{s}^H(k)] = \mathbf{S}$, and $E[\mathbf{n}(k)\mathbf{n}^H(k)] = \mathbf{R}_n$. Furthermore, we can define the full data covariance as,

$$\mathbf{R}_z = \mathbf{A}\mathbf{S}\mathbf{A}^H + \mathbf{R}_n. \tag{2}$$

This paper explores potential performance capability of particular 2D array geometries. It is therefore sufficient to assume all noise and signal covariance properties are known. It later papers we will asses the impact of treating these model parameters as unknowns and assessing their contribution to the DOA accuracy performance degradation. The noise spatial covariance matrix can be more explicitly expressed as,

$$\mathbf{R}_n = \mathbf{R}_{ext} + \sigma_n^2 \mathbf{I} \tag{3}$$

where the noise covariance has been expressed as the sum of both an external colored component $\mathbf{R}_{ext}$ and a white internal component $\sigma_n^2\mathbf{I}$. In the following section we will discuss the details of the external noise covariance matrix. We will define the external-to-internal noise ratio (EINR) as

$$EINR = \mathbf{R}_{ext}(1,1)/\sigma_n^2. \tag{4}$$

In the above definition it is assumed that all array sensors have equal external noise power levels, or $\mathbf{R}_{ext}(1,1) = \mathbf{R}_{ext}(i,i)$ for $i = 1, \ldots, N$.

III. HF Background Noise Model

In this section we briefly introduce the HF background noise model that will be utilized in the study of array DOA performance to follow. We considered multi-element antenna arrays consisting of short non-resonant monopoles. The element pattern for such an element may be expressed as

$$b_H(\theta, \phi) = 1/4(2\pi h/\lambda)^4 \sin^2(\phi)\cos^\eta(\phi),\ 0 \le \eta \le 1. \tag{5}$$

The element pattern incorporates an adjustable factor parametrized by η to control the pattern cutback encountered by a non-ideal ground plane. Numerical electromagnetic simulations suggest that $\eta = 0.7$ provides close agreement with a 16 radial type ground plane, each with length $\lambda/4$. In this development we utilize a spherical coordinate system with the angle from the x-axis to the y-axis denoted as θ and the angle from the z-axis denoted as ϕ. The distribution of external noise considered in this paper is one in which the noise is homogeneous in azimuth, but tapered in elevation.

$$f^\mu(\theta, \phi) = \frac{\Gamma(\mu+2)}{2^{\mu+1}\pi\Gamma^2(1+\mu/2)}\sin^\mu(\phi). \tag{6}$$

More complex noise spatial distribution models can be derived, i.e. inhomogeneity in azimuth or complex temporal correlation. These more complex noise distributions are more applicable to specific environmental scenarios. For example [7] illustrates a technique for computing atmospheric noise spatial distributions based on ray-tracing and lighting maps. The simple model introduced above illustrates the majority of the array geometry and noise field interaction. Combining the element pattern, noise pattern, and array geometry, the array external noise spatial covariance can be expressed as

$$\begin{aligned}\mathbf{R}_{ext} = \sigma_{ext}^2 \int_0^\pi \int_0^{2\pi} &\mathbf{a}(\theta, \phi)\mathbf{a}^H(\theta, \phi)\cdot \\ &b_H(\theta, \phi) f^\mu(\theta, \phi)\sin(\phi)d\theta d\phi,\end{aligned} \tag{7}$$

where $\mathbf{a}(\theta, \phi)$ are the array steering vectors. In the general case for arbitrary (μ, η), the elements of the spatial noise covariance can be expressed as,

$$\begin{aligned}[\mathbf{R}_{ext}]_{i,j} = &\frac{\Gamma(\mu+2)\Gamma(\frac{\eta+1}{2})\Gamma(2+\mu/2)}{2^\mu\Gamma(1+\mu/2)\Gamma(\frac{\eta+\mu+1}{2}+2)}\cdot \\ &{}_1F_2\left[\mu/2+2; \frac{\eta+\mu+1}{2}+2, 1; -\frac{\pi^2}{\lambda^2}\rho_{i,j}^2\right].\end{aligned} \tag{8}$$

The function ${}_1F_2$ denotes a generalized hypergeometric function and the parameter $\rho_{i,j}$ is the distance between array elements i and j in meters. The above expression can be simplified to one involving bessel functions if the parameter μ is chosen to be an even integer. The generalized hypergeometric formulation allows for arbitrary μ.

IV. Cramer-Rao Bound DOA Performance Analysis

In this section we introduce the stochastic signal model Cramer-Rao bound (CRB) for DOA performance in the case of known signal spectrum and known noise covariance. We will further simplify the expression to one for which a single source is present. The basic form of the CRB can be found directly from [4] (8.199).

$$\begin{aligned}C_{CR}(\theta, \phi) = \frac{1}{2K}\Big\{\Re\Big\{&\big(\mathbf{S}\mathbf{A}^H\mathbf{R}_z^{-1}\mathbf{A}\mathbf{S}\big)\odot\big(\mathbf{D}^H\mathbf{R}_z^{-1}\mathbf{D}\big) \\ &+ \big(\mathbf{S}\mathbf{A}^H\mathbf{R}_z^{-1}\mathbf{D}\big)\odot\big(\mathbf{S}\mathbf{A}^H\mathbf{R}_z^{-1}\mathbf{D}\big)^T\Big\}\Big\}^{-1}\end{aligned} \tag{9}$$

Note that here the matrix $\mathbf{D}$ is defined as the partial derivatives of the array manifold matrix $\mathbf{A}$ with respect to the DOA parameters (θ, ϕ). We will now focus on the case when only a single source is present with power σ_s^2.

As was mentioned in the introduction, the purpose of this paper is to examine the DOA performance gains of newly proposed 2D oversampled receive array architectures. We would like to demonstrate the DOA performance gain achieved through the benefit of optimal beamforming versus

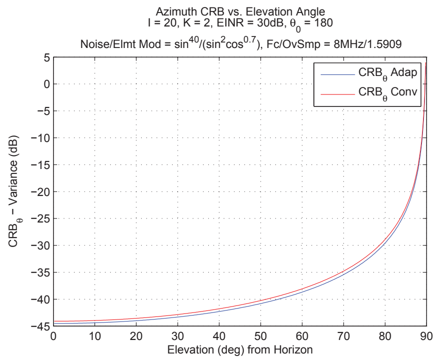

Fig. 1. Azimuth CRB as a function of elevation angle for optimal beamforming vs. conventional beamforming at broadside.

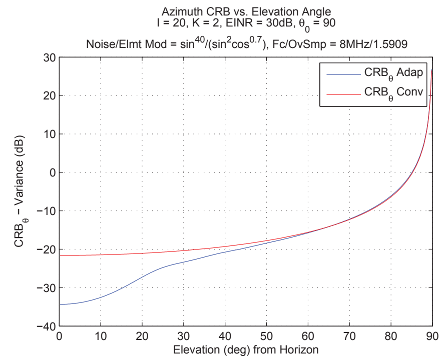

Fig. 2. Azimuth CRB as a function of elevation angle for optimal beamforming vs. conventional beamforming at endfire.

non-optimal beamforming. The non-optimal (conventional) beamforming case will be simulated by modifying the data model noise covariance to be white only (diagonal with equal power). This data model modification has the effect of computing a performance bound with respect an optimal solution that will be a conventional beam (optimal for white noise plus single source). The net effect is that one may asses how much better the optimal solution with respect to colored noise is versus the optimal solution with respect to white noise. Of course in practice other factors are involved with the implementation of the optimal solution and therefore one may argue over the "tightness" of the colored noise optimal CRB, but that discussion is beyond the scope of this paper. Here we seek to demonstrate the potential performance. In the simulations that follow we have constructed a 2D array geometry composed on 40 array elements arranged in a 20 x 2 hexagonal structure. The array is oriented such that the broadside direction is perpendicular to the long axis of the array. In this manner, the array is roughly two rows of 20 elements each.

We show two simulation examples both run at a frequency corresponding to a spatial oversampling rate of 1.59:1. The first example demonstrates the variation of the CRB with respect to the azimuth parameter as a function of the source elevation position. Two curves are plotted in figure (1). The blue curve demonstrates the bound using the optimal beamformer at each elevation angle and the red curve shows the bound when using the conventional beamformer. There is not much difference, however the optimal beamformer bound is always lower than the conventional beamformer bound. If we look at the azimuthal CRB as a function of elevation, but now at the endfire azimuth, we can see a remarkable difference between the optimal beamformer and the conventional beamformer. Figure (2) shows this case. To further examine the cause of the dramatic decrease in the azimuthal CRB using the optimal beamformer vs. the conventional beamformer it is useful to examine the beampattern responses. Figure (3) shows the two beampatterns. This demonstrates that the enhanced accuracy is due to the increased curvature of the optimal beamformer response.

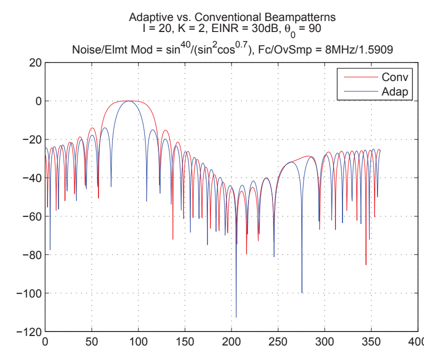

Fig. 3. Optimal vs. conventional beampatterns at endfire and 0 elevation angle.

As another example to exclude the notion that this is simply a discussion regarding the traditional use of superdirectivity, we examine a more square oversampled geometry. We now show the azimuthal CRB for a 6x5 hexagonally sampled array. Figure (4) shows the CRB for both the conventional and optimal beamformer. The optimal beamformer shows improved performance at all elevation angles.

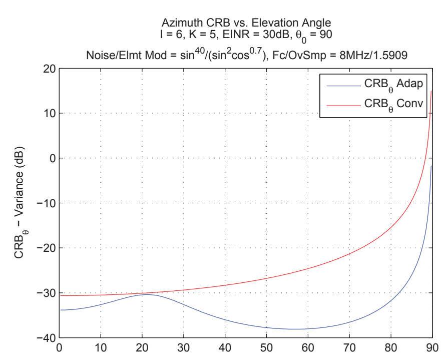

Fig. 4. Azimuth CRB as a function of elevation angle for optimal beamforming vs. conventional beamforming at endfire, 6x5 array

V. Conclusion

In this paper, we conducted a simplified analysis of the potential DOA accuracy improvement that can be realized with the use of 2D oversampled receive apertures when coupled to optimal spatial processing. We provided the stochastic Cramer-Rao bound on azimuth and elevation angles in the presence of known signal and colored noise covariance properties. The DOA improvement gains are due to the use of optimal superdirective beamforming provided by the spatial oversamping array architecture and the colored background noise model. Future work will focus on the practical performance gains achievable in the presence of unknown noise covariance structure. Other authors have published works in related areas [6], but have not explicitly investigated the behavior of 2D spatially oversampled geometries.

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