AN UNMIXING-BASED METHOD FOR THE ANALYSIS OF THERMAL HYPERSPECTRAL IMAGES

Manuel Cubero-Castan^{1,2}, Jocelyn Chanussot^{1,3}, Xavier Briottet², Michal Shimoni⁴, Véronique Achard²

¹ GIPSA-Lab, 11 rue des Mathématiques, F-38402 Saint Martin d'Hères, France
² ONERA/DOTA, 2 Avenue Edouard Belin, F-31055 Toulouse, France
³ Faculty of Electrical and Computer Engineering, University of Iceland
⁴ Signal and Image Centre, Belgian Royal Military Academy (SIC-RMA)

ABSTRACT

The estimation of surface emissivity and temperature from thermal hyperspectral data is a challenge. Methods that estimate the temperature and emissivity on a pixel composed by one single material exist. However, the estimation of the temperature on a mixed pixel, i.e. a pixel composed by more than one material, is more complex and has scarcely been investigated in the literature. This paper addresses this issue by proposing an estimator which linearizes the Black Body law around the mean temperature of each material. The performance of this estimator is studied using simulated data with different hyperspectral sensor configurations and under various noise conditions. The obtained results are encouraging and show an accuracy on the estimated temperature of 0.5 K while using high spectral resolution sensor.

Index Terms— Temperature & Emissivity Separation (TES), Linear Unmixing, Hyperspectral Sensors, Cramer-Rao Lower Bound (CRLB)

1. INTRODUCTION

Hyperspectral image sensors provide images with a large number of spectral bands in the reflective and/or the thermal infrared domain. These images enable information about different materials within the pixels and especially the abundances, i.e. the proportion of each materials composing the pixels.

The abundances are retrieved using unmixing models that have been extensively investigated over the last decade [1, 2]. With an image acquired in the reflective domain, many unmixing methods have been designed (statistical [3, 4, 5] or geometric [6, 7]) using various model of the radiance mixing (linear [8, 9] or non-linear [10, 11]).

However, in the thermal infrared domain, the radiance is function of the optical properties of the materials, like in the reflective domain, but also of the temperature of materials. The main goal in thermal hyperspectral image analysis is to retrieve both of these parameters: the emissivity and the temperature. Due to the spatial variability of the temperature, these unmixing methods do not perform well in the thermal infrared domain.

The estimation of temperature and emissivity, an issue refered as Tempereature and Emissivity Separation (TES), can be interpreted as an illposed semi-blind source separation problem. Many TES methods are available but they assume that either the pixel is pure [12, 13], i.e. composed by only one material, or it is isothermal [14], i.e. composed by one or more materials but at the same temperature; while in real data, many pixels are mixed with various materials at different temperatures [15, 16].

In this study, we developed a new method to estimate temperatures on mixed pixels of a thermal hyperspectral image using a three-steps procedure. The first step estimates the abundance using a co-registered hyperspectral image acquired in the reflective domain. Secondly, the emissivity and the mean temperature of pure pixels selected from the abundance map are estimated using conventional TES methods [12, 13]. Eventually, we propose a new approach to retrieve the temperature on mixed pixels thanks to the previous estimation of the abundances and the emissivities.

The paper is structured as follows : the problem of estimating the temperature on mixed pixels is introduced in section 2 and the proposed estimator is detailed in section 3. The first and the second order performances of the proposed estimator are studied using simulated data in the section 4.

2. THE SPECIFIC CASE OF MIXED PIXELS

The first step consists in separating pure pixels (i.e. pixels made of one single material, even if with non uniform temperature) from mixed pixels (i.e. pixels made of different materials). The formula of the input signal with a mixed pixel introduced in section 2.1 allows to write the Cramer-Rao Lower Bound, presented in section 2.2.

Acknowledgement: This study is funded by the French Defence Agency (DGA) and the French Aerospace Laboratory (ONERA) and partially supported by ERC CHESS (ERC-2012-AdG-320684).

2.1. The Radiative Transfert Modeling on mixed pixels

At sensor level, the instrument measures a radiance $R^{\lambda}(x, y)$ which combines the atmosphere terms (the atmospheric upwelling transmittance $\tau^{\lambda}_{atm,\uparrow}$ and the atmospheric upwelling radiance $R^{\lambda}_{atm,\uparrow}$) and the at-ground leaving radiance $R^{\lambda}_{BOA}(x, y)$ (also called Bottom-Of-Atmosphere (BOA) Radiance) :

$$R^{\lambda}(x,y) = R^{\lambda}_{BOA}(x,y) \cdot \tau^{\lambda}_{atm,\uparrow} + R^{\lambda}_{atm,\uparrow} + w^{\lambda}(x,y)$$
(1)

where $w^{\lambda}(x, y)$ represents the at-sensor noise. This noise covers different kinds of noise such as the inaccuracies in the instrumental calibration, the thermal noise, the atmospheric correction noise and the truncation noise (signal accuracy at $0.01 \ W.m^{-2}.sr^{-1}.\mu m^{-1}$). Generally, it also includes the spatial and the spectral variation on the atmospheric terms and on the emissivity and the temperature inside of the pixel. This noise is designed as an additive zero-mean gaussian noise with a standard deviation at σ^{λ} .

Under the assumption that the linear mixing model well approximates flat ground scenes at radiance level, the BOA radiance may be written as follows:

$$R_{BOA}^{\lambda}(x,y) = \sum_{i=1}^{M} (\varepsilon_i^{\lambda} \cdot B^{\lambda}(T_i) + (1 - \varepsilon_i^{\lambda}) \cdot R_{atm,\downarrow}^{\lambda}) \cdot S_i \quad (2)$$

where the measured radiance is a function of the optical, the thermal and the geometrical properties of the ground material i. ε_i^{λ} is the emissivity, T_i is the temperature, S_i represents the abundance of the material in the pixel (x, y) and M is the number of materials composing the pixel (x, y). In this work we assume that $\tau_{atm,\uparrow}^{\lambda}$, $R_{atm,\uparrow}^{\lambda}$ and $R_{atm,\downarrow}$ the atmospheric downwelling radiance, are known. $B^{\lambda}(T)$ is the Planck spectral law at temperature T that is written as:

$$B^{\lambda}(T) = \frac{C_1/\lambda^5}{exp(C_2/(\lambda \cdot T)) - 1}$$
(3)

where $C_1 \approx 1.19 \cdot 10^8 W.m^{-2}.sr^{-1}.\mu m^4$ and $C_2 \approx 1.44 \cdot 10^4 K.\mu m$.

2.2. The Cramer-Rao Lower Bound

The parameter to be estimated is the vector temperature $T \in \mathbb{R}^M$ of the materials composing the pixel. The signal used to estimate this parameter is the BOA radiance after atmospheric correction. Equations (1) and (2) lead to the formulation of the equivalent noise w_{BOA}^{λ} , a zero-mean gaussian noise with standard deviation $\sigma_{BOA}^{\lambda} = \sigma^{\lambda} / \tau_{atm,\uparrow}^{\lambda}$.

The estimation theory [17] shows that a lower bound of the variance of any unbiased estimator \hat{T}_i exists if the Probability Density Function (PDF) p(x; T) satisfies the regularity condition:

$$\frac{\partial ln(p(x, \boldsymbol{T}))}{\partial T_i} = 0 \qquad \forall T_i \tag{4}$$

This bound is called the Cramer-Rao Lower Bound (CRLB) and is written as:

$$var(\hat{T}_i) \ge [\boldsymbol{I}^{-1}(\boldsymbol{T})]_{ii} \tag{5}$$

where $I(T)|_{l,k} = -E\left[\frac{\partial^2 lnp(x;T)}{\partial T_l \partial T_k}\right]$ is the Fisher Information Matrix (FIM), l and k are any of the materials composing the pixel. $[I^{-1}(T)]_{ii}$ is the i^{th} component of the inverse of the FIM.

Equation (2) leads to this formulation of the FIM:

$$\boldsymbol{I}(\boldsymbol{T})|_{l,k} = \sum_{\lambda=1}^{N} \alpha_l^{\lambda} \cdot \alpha_k^{\lambda}$$
(6)

with $\alpha_i^{\lambda} = \frac{1}{(\sigma_{BOA}^{\lambda})} \cdot S_i \cdot \varepsilon_i \cdot \frac{\partial B^{\lambda}(T)}{\partial T} \Big|_{T_i}$ for the *i*th material.

When the pixel is composed of only two materials, the analytical expression of the CRLB (for \hat{T}_1) is:

$$Var(\hat{T}_{1}) \geq \frac{\sum_{\lambda=1}^{N} (\alpha_{2}^{\lambda})^{2}}{\sum_{\lambda=1}^{N} (\alpha_{1}^{\lambda})^{2} \cdot \sum_{\lambda=1}^{N} (\alpha_{2}^{\lambda})^{2} - \left(\sum_{\lambda=1}^{N} \alpha_{1}^{\lambda} \cdot \alpha_{2}^{\lambda}\right)^{2}} \quad (7)$$

3. THE ESTIMATION OF TEMPERATURES

The proposed approach is to linearize the Black Body law $B(T_i^{x,y})$ around the mean temperature of each material.

The resulting estimator is introduced in section 3.1 and the Condition Number of the FIM, presented in section 3.2, is used to assess the stability of the solution.

3.1. The linearized estimation problem

The Black Body law is formulated in equation (3). Assuming that the temperature of material *i* in a mixed pixel is close to the mean temperature $\overline{T_i}$ estimated previously on pure pixels, the Black Body law in the BOA equation (2) can be linearized around this mean temperature. The centered radiance $\Delta R_{\lambda_j} = R_{BOA}(T_i) - R_{BOA}(\overline{T_i})$ writes:

$$\begin{pmatrix} \Delta R_{\lambda_1} \\ \vdots \\ \Delta R_{\lambda_N} \end{pmatrix} = \begin{pmatrix} A_{\lambda_1}^{\overline{T_1}} & \cdots & A_{\lambda_1}^{\overline{T_M}} \\ \vdots & \ddots & \vdots \\ A_{\lambda_N}^{\overline{T_1}} & \cdots & A_{\lambda_N}^{\overline{T_M}} \end{pmatrix} \cdot \begin{pmatrix} \Delta T_1 \\ \vdots \\ \Delta T_M \end{pmatrix}$$
(8)

with $A_{\lambda_j}^{\overline{T_i}} = \varepsilon_i \cdot S_i \cdot \frac{\partial B^{\lambda_j}(T)}{\partial T} \Big|_{\overline{T_i}}$ and $\Delta T_i = \overline{T_i} - T_i$. Note that N is the number of sensor spectral bands. With N equations, $M \leq N$ represents an overdetermined problem (with real data, $M \ll N$).

The linear unbiased estimator that minimizes the variance of the estimation is commonly called the Best Linear Unbiased Estimator (BLUE)[17] and writes:

$$\Delta T = (A^t \cdot C^{-1} \cdot A)^{-1} \cdot A^t \cdot C^{-1} \cdot \Delta R \qquad (9)$$

with C the noise covariance matrix. Note that the matrix $(A^t \cdot C^{-1} \cdot A)$ is the FIM of the equation (2).

3.2. The Condition Number of the problem

The condition number (CN) evaluates the impact of a variation on ΔR on the estimation of ΔT . With a high CN, the estimation is ill-conditioned, i.e. a little variation of the BOA radiance creates a high variation on the estimated ΔT .

Considering the Frobenius norm, the CN is the ratio between the maximum and the minimum of the eigenvalues of the FIM. With a scene composed by 2 materials, an analytical expression of the eigenvalues is found and the CN writes:

$$CN = \frac{\sum (\alpha_1^{\lambda})^2 + \sum (\alpha_2^{\lambda})^2 + \sqrt{D}}{\sum (\alpha_1^{\lambda})^2 + \sum (\alpha_2^{\lambda})^2 - \sqrt{D}}$$
(10)

with : $D = \left(\sum (\alpha_1^{\lambda})^2 - \sum (\alpha_2^{\lambda})^2\right)^2 + 4 \cdot \left(\sum \alpha_1^{\lambda} \cdot \alpha_2^{\lambda}\right)^2$. From (10), if $\alpha_2^{\lambda} \to \alpha_1^{\lambda}$, then the minimum of the eiguen-

values tends to 0 and the $CN \rightarrow \infty$. This may happen when for example the emissivity and the temperature are the same for both materials (this pixel is then seen as a pure pixel).

The CN decreases as the difference between emissivities decreases. Of course, it is much easier to estimate the temperature of a material when its emissivity has discriminent absorption features.

Also note that the CN increases if the different abundances are unevenly distributed within a pixel. Indeed, estimating the characteristics (temperature) of one given material from the global measurement is difficult if this given material only marginally impacts this measurement.

4. THE STUDY OF THE ESTIMATOR BEHAVIOR

An expected error of 1.2 K to 1.4 K on the estimation of temperature was obtained while applying the standard MMD/TES method [13] for pure pixel of multispectral image data and 0.3 K with hyperspectral image data [18, 15]. Unfortunately, it is not the case for mixed pixels with a lower number of bands (N small) or with an ill-conditioned problem (CN high).

The error E studied in this section is the Root Mean Square Error (RMSE) between the input temperature T_i and the estimated temperature \hat{T}_i for all the materials i within a given pixel.

$$E = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (T_i - \hat{T}_i)^2}$$
(11)

All the simulations are done on mixed pixels with 2 materials. By adding more materials in the pixel, the CN increases; therefore the error E increases too.

The bias and the variance are studied in section 4.1 and 4.2 when the parameters (ε_i , S_i , atmosphere) are well known. However, in real conditions, some uncertainties remain on



Fig. 1. Evolution of the estimation error E in a mixed pixel without instrumental noise. The error increases with CN and with the input ΔT .

these parameters. The impact of a wrong estimation of those parameters on the estimated temperatures is studied in section 4.3.

4.1. The bias of the estimator

Equations (1) and (2) illustrate that the measured at-ground leaving radiance is a function of the atmospheric terms and the ground parameters (ε ,T,S). Even by knowing the filtered atmosphere, the temperature, the abundance of the materials in the pixels and the *N*-bands filtered emissivity, the atmospheric correction is not perfect and the sensors accuracy is finite. Furthermore with the linearization (Eq. (2)), there is a residual error induced by the Taylor approximation. It results in an approximation of the BOA radiance, hence an approximation of ΔT .

These errors have small amplitudes and impact on the estimated temperatures when the CN is high. An error on the estimation with a mixed pixel composed by 2 materials is simulated using the TASI sensor characteristics [19]. The mean temperature is simulated between 280K and 310K, the abundance between [25%/75%] to [75%/25%] and the emissivities are extracted from the ASTER emissivity of man-made materials, vegetation and soils database [20]. Figure 1 shows the error *E* of the estimation with $\Delta T_{1,2}$ varying between 0K and 10K.

With $\Delta T = 0$ K, the obtained error is below 2K in 96% of the cases. This proportion drops to 55% when ΔT is between 5 and 10 K. This is limiting the validity of the proposed estimator to reasonable situations, with $\Delta T \leq 5$ K, typically.

To summarize, this estimator performs well with a ΔT varying from 0 K to 5 K. Please note that this study of the bias was conducted without considering the instrumental noise effect. This effect is assessed in the following sections.



Fig. 2. Evolution of the CRLB with the CN for different sensors. The CRLB increases exponentially with the CN log.

4.2. The Cramer-Rao Lower Bound of the estimator

As explained in section 2.2, the CRLB is a lower bound of the variance. For small variations of ΔT , equation (8) is a good approximation. [17] shows that with a linear model and a gaussian noise, the BLUE is the Minimum Variance Unbiased estimator and the CRLB is reached. The study of the variance of the estimator is then reduced to the study of the CRLB.

Figure 2 represents the CRLB for 3 sensors : the AHS [21], with 9 bands between 8 to 12.5 μm and a standard deviation of the instrumental noise of $\sigma^{\lambda} \approx .04 W.m^{-2}.sr^{-1}.\mu m^{-1}$, the TASI sensor [19], with 64 bands between 8 to 11.5 μm and $\sigma^{\lambda} \approx .025 W.m^{-2}.sr^{-1}.\mu m^{-1}$, and the SEBASS sensor [22], with 115 bands between 7.5 to 13.5 μm and $\sigma^{\lambda} \approx 0.01 W.m^{-2}.sr^{-1}.\mu m^{-1}$. The other simulated parameters (the mean temperature, the abundance, the emissivity) are the same as in the section 4.1.

By increasing the quality of sensor (from AHS with low SNR and low number of bands to SEBASS with high SNR and high number of bands), a decrease of the CRLB is observed. To separate the influence of the SNR and the number of bands, simulations have been conducted with the TASI bands filter and the SNR of the 2 others instruments. The results show that with low SNR, the number of bands impacts more than with high SNR.

The residual errors are assumed to be spectrally independant. It means that by increasing the number of spectral bands, more equations are adding to the estimation without increasing the noise level, as illustrated in figure 2 when only the number of bands increases.

Another point is that, in real condition, the emissivity and the abundance are not well retrieved. The impact of uncertainties on those two parameters is investigated in section 4.3.

4.3. The influence of inaccurate estimation of S & ε

To investigate the impact of these uncertainties, the simulations are made on mixed pixels as described in section 4.1



Fig. 3. Evolution of the mean and three times the standard deviation of the error E with the shift on the abundance. The general observed trend is the increase of E with the increase of the shift on the abundance.

with the TASI sensor and $\Delta T < 5$ K. The input abundances have been shifted with ΔS varying between -20% and 20%. The input emissivity is the estimated emissivity after TES method [13]. The standard deviation of the E < 0.5 K. 50 repetitions of each simulation have been done and the mean of E is studied.

If the TES method does not well estimate the emissivity, there is no impact on the estimation of the temperatures. Indeed, with no error on the abundance, the error E is 0.5 K. When the error on the abundance increases, it highly degrades the temperature estimation by more than 5 K with a shift on the abundance of 10%. This is illustrated in figure 3.

From this study we can draw two conclusions. First, the results validate the TES methods that estimate the emissivity with an error of .02 [15]. Secondly, a shift on the abundance estimation has higher impact on the estimated temperature than emissivity. It means that the success of the method relies on a very accurate estimation of the abundance.

5. CONCLUSION AND PERSPECTIVES

This paper presents a new method to estimate the temperature in mixed pixels by first estimating the abundance on visible images and then the emissivity and the mean temperature on pure pixel. This method, based on a linearization of the Planck law around the mean temperature of each materials present in the scene, shows encouraging results provided that the abundance map of the materials is accurately estimated.

Further work will focus on the application of this method using real data and the study of the blind approach, i.e. without any prior knowledge about the materials abundance or the emissivities.

6. REFERENCES

- N. Keshava and J.F. Mustard, "Spectral unmixing," *Signal Processing Magazine*, *IEEE*, vol. 19, pp. 44 57, 2002.
- [2] J. M. Bioucas-Dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader, and J. Chanussot, "Hyperspectral unmixing overview: Geometrical, statistical and sparse regression-based approaches," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 5, no. 5, pp. 354–379, April 2012.
- [3] N. Dobigeon, S. Moussaoui, M. Coulon, J.-Y. Tourneret, and A.O. Hero, "Joint bayesian endmember extraction and linear unmixing for hyperspectral imagery," *Signal Processing, IEEE Transactions on*, vol. 57, pp. 4355 – 4368, 2009.
- [4] K.E. Themelis, A.A. Rontogiannis, and K.D. Koutroumbas, "A novel hierarchical bayesian approach for sparse semisupervised hyperspectral unmixing," *Signal Processing, IEEE Transactions on*, vol. 60, pp. 585 – 599, 2012.
- [5] R. Mittelman, N. Dobigeon, and A.O. Hero, "Hyperspectral image unmixing using a multiresolution sticky hdp," *Signal Processing, IEEE Transactions on*, vol. 60, pp. 1656 – 1671, 2012.
- [6] J. Li and J.M. Bioucas-Dias, "Minimum volume simplex analysis: A fast algorithm to unmix hyperspectral data," in *Geoscience and Remote Sensing Symposium*, 2008. IGARSS 2008. IEEE International, 2008.
- [7] R. Heylen, D. Burazerovic, and P. Scheunders, "Nonlinear spectral unmixing by geodesic simplex volume maximization," *Selected Topics in Signal Processing*, *IEEE Journal of*, vol. 5, pp. 534 – 542, 2011.
- [8] N. Dobigeon, J.-Y. Tourneret, and Chein-I Chang, "Semi-supervised linear spectral unmixing using a hierarchical bayesian model for hyperspectral imagery," *Signal Processing, IEEE Transactions on*, vol. 56, pp. 2684 – 2695, 2008.
- [9] Tsung-Han Chan, Chong-Yung Chi, Yu-Min Huang, and Wing-Kin Ma, "A convex analysis-based minimumvolume enclosing simplex algorithm for hyperspectral unmixing," *Signal Processing, IEEE Transactions on*, vol. 57, pp. 4418 – 4432, 2009.
- [10] Jie Chen, C. Richard, and P. Honeine, "Nonlinear unmixing of hyperspectral data based on a linearmixture/nonlinear-fluctuation model," *Signal Processing, IEEE Transactions on*, vol. 61, pp. 480 – 492, 2013.

- [11] Y. Altmann, N. Dobigeon, S. McLaughlin, and J.-Y. Tourneret, "Nonlinear spectral unmixing of hyperspectral images using gaussian processes," *Signal Processing, IEEE Transactions on*, vol. 61, pp. 2442 – 2453, 2013.
- [12] C.C. Borel, "Artemiss an algorithm to retrieve temperature and emissivity from hyper-spectral thermal image data," in 28th Annual GOMACTech Conference, 2003.
- [13] A. Gillespie, S. Rokugawa, T. Matsunaga, J.S. Cothern, S. Hook, and A.B. Kahle, "A temperature and emissivity separation algorithm for advanced spaceborne thermal emission and reflection radiometer (aster) images," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 36, pp. 1113–1126, 1998.
- [14] E. F. Collins, D. A. Roberts, and C. C. Borel., "Spectral mixture analysis of simulated thermal infrared spectrometry data: An initial temperature estimate bounded tessma search approach," *Geoscience and Remote Sensing, IEEE Transactions on*, vol. 39, pp. 1435 – 1446, 2001.
- [15] V. Payan and A. Royer, "Analysis of temperature emissivity separation (tes) algorithm applicability and sensitivity," *International Journal of Remote Sensing*, vol. 25, pp. 15–37, 2004.
- [16] M. F. Mccabe, L. K. Balick, J. Theiler, A. R. Gillespie, and A. Mushkin, "Linear mixing in thermal infrared temperature retrieval," *International Journal of Remote Sensing*, vol. 29, pp. 5047–5061, 2008.
- [17] S. M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Englewood Cliffs, NJ, 1993.
- [18] J. Sobrino, J. C. Jiménez-Muñoz, P. J. Zarco-Tejada, G. Sepulcre-Cantó, and E. de Miguel, "Land surface temperature derived from airborne hyperspectral scanner thermal infrared data," *Remote Sensing of Environment*, vol. 102, pp. 99–115, 2006.
- [19] ITRES, TASI-600 An Airborn Hyperspectral Thermal Imager (8-11.5 microns).
- [20] A. M. Baldridge, S.J. Hook, C.I. Grove, and G. Rivera, "The aster spectral library version 2.0.," *Remote Sensing* of *Environment*, vol. 113, 2009.
- [21] Caravan International Corp, AHS-160 Airborne Hyperspectral Scanner.
- [22] J. A. Hackwell, D. W. Warren, R. P. Bongiovi, S. J. Hansel, T. L. Hayhurst, D. J. Mabry, M. G. Sivjee, J. A. J. W. Skinner al., "LWIR/MWIR imaging hyperspectral sensor for airborne and ground-based remote sensing," *Imaging Spectrometry II.*, 1996.