

INTEGRATING ENERGY STORAGE INTO THE SMART GRID: A PROSPECT THEORETIC APPROACH

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ABSTRACT

In this paper, the interactions and energy exchange decisions of a number of geographically distributed storage units are studied under decision-making involving end-users. In particular, a noncooperative game is formulated between customer-owned storage units where each storage unit's owner can decide on whether to charge or discharge energy with a given probability so as to maximize a utility that reflects the tradeoff between the monetary transactions from charging/discharging and the penalty from power regulation. Unlike existing game-theoretic works which assume that players make their decisions rationally and objectively, we use the new framework of prospect theory (PT) to explicitly incorporate the users' subjective perceptions of their expected utilities. For the two-player game, we show the existence of a proper mixed Nash equilibrium for both the standard game-theoretic case and the case with PT considerations. Simulation results show that incorporating user behavior via PT reveals several important insights into load management as well as economics of energy storage usage. For instance, the results show that deviations from conventional game theory, as predicted by PT, can lead to undesirable grid loads and revenues thus requiring the power company to revisit its pricing schemes and the customers to reassess their energy storage usage choices.

Index Terms—Smart grid, game theory, prospect theory, energy storage.

I. INTRODUCTION

Customer participation in energy management is seen as an integral feature of the smart grid [1]. In particular, the introduction of customer-owned storage units will provide the means for active user participation in managing energy transactions in the grid. For instance, these storage units provide the grid with the opportunity of storing energy at customer premises and they also allow customers to sell any surplus of energy available at their premises [1]. This represents a key feature for deploying smart grid applications such as demand response [2]–[6].

The integration of storage units into the smart grid, particularly at the customer side, requires overcoming many technical challenges [7]–[13]. The authors in [7] addressed the problem of intermittent renewable energy generation by using energy storage to deal with dynamic loads and sources. In [8], the authors studied the use of storage units as a means for complementing the stochastic generation of wind farms. In this work, the authors also investigate the impact of the presence of such storage units on the market price. Other related game-theoretic solutions for smart grid pricing and energy management are discussed in [10]–[13].

Game theory has been a popular tool for smart grid design. However, most existing works assume that customers will abide by the rules of the game and act in a rational manner [2], [3], [10]–[15]. Indeed, none of these works incorporates the realistic behavior of the users which, in practice, can deviate from the conventional, rational norm set by game theory as observed in [16]–[18]. In this respect, prospect theory (PT), a Nobel-prize winning theory, provides the needed tools to explain how real-life user decisions can deviate from those predicted by conventional game theory [19]–[21]. In particular, PT has shown that, in real life, users often act irrationally when faced with risk and uncertainty of outcome, as is the case in the smart grid, where the decisions of the customers are largely interdependent leading to risky outcomes. These irrational decisions can stem not only from

the users' behavior but also from computational errors occurring at the smart grid devices that are often resource-constrained. There are many studies that have applied PT to solve problems in the social sciences [19], [22], [23] as well as recent efforts to study the influence of end-users on wireless networks [24]–[28]. However, our work here is the first to break new ground in using PT to study end-user influence on the workings of the smart grid.

The main contribution of this paper is to propose a new framework for energy management in the smart grid using the tools of prospect theory. In particular, we formulate a noncooperative game between the customer-owned storage units, in which the decision of each customer explicitly incorporates its *subjective* perception on the actions taken by other customers. In this game, each customer can decide whether to charge its storage unit or sell the available surplus to the grid, while optimizing a utility that captures the associated costs and benefits, under a subjective observation of the other customers' actions. Compared to related works on smart grid markets and demand response [2]–[5], [7]–[15], our paper has several new contributions: 1) in contrast to the conventional expected utility theory (EUT), we develop a novel PT-based framework that allows proper modeling of realistic user behavior during energy management; 2) we design a novel game-theoretic model that allows incorporation of both economic (pricing) and power factors (grid regulation); and 3) we show the existence of a mixed Nash equilibrium for the proposed game under PT considerations. Extensive simulation results show that deviations from the rational, EUT behavior can lead to unexpected, and possibly undesirable performance, in terms of power company revenues and average grid load.

The remainder of the paper is organized as follows: Section II presents the studied system model and formulates the problem as a game with PT considerations. In Section III, we analyze the equilibrium for the two-player case. Simulation results are presented in Section IV, while conclusions are drawn in Section V.

II. SYSTEM MODEL AND PT GAME FORMULATION

Consider a smart grid in which N customers are present. Let \mathcal{N} be the set of all N customers. Under normal operating conditions, we assume that each customer $i \in \mathcal{N}$ constitutes a constant load D_i on the grid. Among all N customers, a subset $\mathcal{K} \subseteq \mathcal{N}$ of K customers is assumed to be "active". Here, an active customer refers to a user equipped with a smart home and able to actively participate in the energy management of the smart grid, as allowed by the power company. Every customer $k \in \mathcal{K}$ owns a storage unit that initially stores an amount of energy $S_k < D_k$. At a given period of time, we assume that the participation of each customer $k \in \mathcal{K}$ is restricted to one of two actions: a) charge the needed amount D_k (act as load) or b) discharge/sell the surplus S_k to the other customers (act as source).

Naturally, any given action by a customer $k \in \mathcal{K}$ will affect both the power system (needed generation, losses, etc.) and the market economics (prices). We assume that the power company allows the customers to charge or discharge, but it requires that the total generation power remains within a nominal, desirable value to maintain the power system's stability [29]. In this studied scenario, all K participating users use storage units to charge and discharge so as to optimize their overall monetary benefits. The decisions of the customers are, however, largely coupled, which leads to a game-theoretic setting as discussed next.

II-A. Noncooperative Game Model

We analyze the interactions between the active customers using noncooperative game theory [30]. As the strategy choices of the customers are largely *interdependent*, we can formulate a strategic noncooperative game $\Xi = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$, that is characterized by three main elements: *a*) the players are the active customers in the set \mathcal{K} , *b*) the action $a_k \in \mathcal{A}_k := \{D_k, S_k\}$ of each player is to either charge/buy a total amount of energy D_k ($a_k = D_k$) or discharge/sell the available surplus S_k ($a_k = S_k$), and *c*) the utility function u_k of each player k which captures the benefit-cost tradeoffs associated with the different choices. Each customer k is assumed to have enough storage capacity to handle an amount $D_k + S_k$. Here, we note that, although the customers may have other demands, our model is solely focused on the discharge/charge actions and their impact on the grid and customers. The utility function achieved by a player $k \in \mathcal{K}$ that chooses an action a_k is given by

$$u_k(a_k, \mathbf{a}_{-k}) = -\alpha(a_k, \mathbf{a}_{-k}) \left(D_k + L_k(a_k, \mathbf{a}_{-k}) \right) + \gamma(a_k, \mathbf{a}_{-k}) S_k - \beta \left(G(a_k, \mathbf{a}_{-k}) - \hat{G} \right)^2, \quad (1)$$

where $\mathbf{a}_{-k} = [a_1, a_2, \dots, a_{k-1}, a_{k+1}, \dots, a_K]$ is the vector of action choices of all players other than k , $L_k(a_k, \mathbf{a}_{-k})$ are the total losses over the distribution/transmission lines which depend on the total demand and are computed using conventional optimal power flow algorithms [29], $G(a_k, \mathbf{a}_{-k})$ denotes the total generation by the *power company* (not the customers) under current action choices, and β is a regulation penalty factor, that allows the power company to maintain a regulated power supply, i.e. \hat{G} . Maintaining such a regulation is important for many operational aspects of the grid, such as the conversion between AC and DC. We note that, in our game, the actions are positive and we have positively/negatively defined charging/discharging unit payments α and γ in (1). Here, we define the charging price and the discharging price, respectively, set by the power company and participating users as follows:

$$\alpha(a_k, \mathbf{a}_{-k}) = \begin{cases} c(a_k, \mathbf{a}_{-k}) & \text{if } a_k = D_k, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

with $c(a_k, \mathbf{a}_{-k})$ being the unit price in the energy market which follows the pricing strategy of the power company. Moreover,

$$\gamma(a_k, \mathbf{a}_{-k}) = \begin{cases} b_k & \text{if } a_k = S_k, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

with b_k being the unit price at which a certain customer k would sell its surplus S_k . We assume that each customer can set its own price, but the power company will impose a pricing restriction B , such that $b_k < B$, $\forall k \in \mathcal{K}$.

The utility function in (1) captures both the economic benefits of customer participation as well as the impact on the power system (via the regulation term). Here, while the power company allows the K active customers to actively decide on whether to buy or sell energy, it mandates that the generated power in the considered geographical area remains within desired, stable operating conditions. Also, we note that both demand and line loss determine the total generation level and, excessive charging or discharging might damage the generator due to a frequency variation thus requiring regulation [31]. Without loss of generality, we assume that the normal, stable operating conditions correspond to the case in which all N customers act as loads and we let $\hat{G} = \sum_{i \in \mathcal{N}} (D_i + L_i)$ denote the total generated power required for this distribution area during normal operation. L_i represents the losses incurred over the distribution/transmission lines for delivering D_i to customer i which depend on the total demand and are computed using power flow algorithms. Therefore, for the case in which $a_k = D_k, \forall k \in \mathcal{K}$, we have $G(\mathbf{a}) = \hat{G}$ (with \mathbf{a} being the vector of all strategies). Consequently, any actions taken by a certain customer that shifts the generated power from its nominal value \hat{G} will require the power company to regulate the generation. The need for this regulation indirectly yields a cost penalty on the active participants as captured in (1).

II-B. Expected Utility Theory

In a smart grid, owing to uncertainty in power generation as well as the fact that the customers can make certain decisions (such as whether to allow the use of their storage device or not) with different frequency over time, it is reasonable to assume that customers make probabilistic choices. Therefore, we are interested in studying the game under *mixed strategies* [30]. As customers are often uncertain when presented with different choices in practice, a mixed-strategy solution can better capture their realistic behavior. Let $\mathbf{p} = [p_1, \dots, p_K]$ be the vector of all mixed strategies, where, for every customer $k \in \mathcal{K}$, $p_k(a_k)$ is the probability distribution over the pure strategies $a_k \in \mathcal{A}_k$.

Under the conventional EUT model, the utility of each user is simply the expected value over its mixed strategies. Thus, the EUT utility of a player k is given by

$$U_k^{\text{EUT}}(\mathbf{p}) = \sum_{\mathbf{a} \in \mathcal{A}} \left(\prod_{l=1}^K p_l(a_l) \right) u_k(a_k, \mathbf{a}_{-k}), \quad (4)$$

where \mathbf{a} is the vector of all players' strategies and $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \dots \times \mathcal{A}_K$.

II-C. Prospect Theory

As previously mentioned, EUT evaluates an objective expected utility in which users are assumed to act rationally and objectively. However, it has been observed that, in real life, users' behavior deviates considerably from the rational path predicted by EUT. For the proposed game, a customer k has to decide on its action, in the face of uncertainty induced by the mixed strategies of its opponents, which impact directly its utility as in (4). In order to capture such behavioral factors in the proposed energy trading game, we turn to the framework of prospect theory [19].

One important PT notion is the so-called *weighting effect*. In particular, in PT [32] it is observed that in real-life decision-making, people tend to subjectively weight uncertain outcomes. In the proposed game, this weighting effect allows capture of each user's subjective evaluation on the mixed strategy of its opponents. Thus, under PT, instead of objectively observing the mixed strategy vector \mathbf{p}_{-k} chosen by the other players, each user perceives a weighted version of it, $w_k(\mathbf{p}_{-k})$. Here, $w_k(\cdot)$ is a nonlinear transformation that maps an objective probability to a subjective one. PT studies have shown that most people could often overweight low probability outcomes and overweight high probability outcomes [19]. Hereinafter, we assume that all players utilize a similar weighting approach, such that $w_k(\cdot) = w(\cdot), \forall k \in \mathcal{K}$. While many weighting functions exist in the PT literature, we choose the popular Prelec function (for a given probability σ) [32]:

$$w(\sigma) = \exp(-(-\ln \sigma)^\alpha), \quad 0 < \alpha \leq 1, \quad (5)$$

where α is a parameter used to characterize the distortion between subjective and objective probability. Note that when $\alpha = 1$, (5) is reduced to the conventional EUT probability.

Under PT, the expected utility achieved by a player k , given the weighting effect, is

$$U_k^{\text{PT}}(\mathbf{p}) = \sum_{\mathbf{a} \in \mathcal{A}} \left(p_k(a_k) \prod_{l \in \mathcal{K} \setminus \{k\}} w(p_l(a_l)) \right) u_k(a_k, \mathbf{a}_{-k}). \quad (6)$$

Here, we assume that a player uses a subjective evaluation only on the other players' strategy probabilities. Thus, customer k 's subjective evaluation of its own probability is equal to its objective probability. Given the set of probability distributions \mathcal{P}_k over its set of strategies \mathcal{A}_k , the solution of the game can be found via the notion of a mixed-strategy Nash equilibrium:

Definition 1: A mixed strategy profile $\mathbf{p}^* \in \mathcal{P} = \prod_{k=1}^K \mathcal{P}_k$ is a mixed strategy Nash equilibrium if, for each player $k \in \{1, 2, \dots, K\}$, we have (for either PT or EUT)

$$U_k(p_k^*, \mathbf{p}_{-k}^*) \geq U_k(p_k, \mathbf{p}_{-k}^*), \quad \forall p_k \in \mathcal{P}_k. \quad (7)$$

III. SOLUTION: THE TWO-PLAYER CASE

To gain greater insight into the solution of the proposed game, we analyze a case study for the scenario in which only $K = 2$ customers are active. In particular, we are interested in analyzing the *proper mixed Nash equilibrium* of the game. A proper mixed-strategy Nash equilibrium is the solution in which each player chooses a certain action a_k with probability $0 < p_k < 1$. While

the existence of a mixed-strategy Nash equilibrium is well-known for conventional EUT games [30], it is of interest to study whether the PT game admits such an equilibrium. Moreover, for both the EUT and PT games, we are interested in guaranteeing a *proper* mixed strategy Nash equilibrium, in which the users will indeed mix between their strategies. With this in mind, we can state the following result:

Theorem 1: For the proposed two-player smart grid game $\Xi = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$, there exists a unique, proper mixed Nash equilibrium for both the EUT and PT games if $-c(D_k, D_{-k})D_k + \beta(D_k + S_k)^2 < b_k S_k < -c(D_k, S_{-k})D_k + \beta(D_k + S_k)^2 + 2\beta \prod_{l=1}^2 (D_l + S_l)$, where $k = \{1, 2\}$.

Proof: In the proposed model, there always exists at least one mixed NE under EUT as guaranteed by Nash's result [30]. Thus, our proof mainly focus on finding a condition to guarantee 1) there exists a *proper* mixed NE under EUT and PT, and 2) such a proper mixed NE is unique. By using the indifference principle under EUT, a proper mixed-strategy Nash equilibrium, (p_1^*, p_2^*) , exists when the average charging utility is equal to the average discharging utility. For example, computing customer 1's average utility by p_2^* , we have $p_2^* u_1(D_1, D_2) + (1 - p_2^*) u_1(D_1, S_2) = p_2^* u_1(S_1, D_2) + (1 - p_2^*) u_1(S_1, S_2)$; that is,

$$p_2^* = \frac{u_1(S_1, S_2) - u_1(D_1, S_2)}{u_1(D_1, D_2) - u_1(S_1, D_2) + u_1(S_1, S_2) - u_1(D_1, S_2)}. \quad (8)$$

A sufficient condition to have a proper mixed strategy Nash equilibrium, such that $0 < p_2^* < 1$, is to have

$$\text{sgn}\left(u_1(S_1, S_2) - u_1(D_1, S_2)\right) = \text{sgn}\left(u_1(D_1, D_2) - u_1(S_1, D_2)\right), \quad (9)$$

where $\text{sgn}(\cdot)$ denotes the algebraic sign of its argument and

$$\begin{cases} u_1(D_1, D_2) = -c_{11}(D_1 + L_1(D_1, D_2)), \\ u_1(D_1, S_2) = -c_{12}(D_1 + L_1(D_1, S_2)) - \beta(G(D_1, S_2) - \hat{G})^2, \\ u_1(S_1, D_2) = b_1 S_1 - \beta(G(S_1, D_2) - \hat{G})^2, \\ u_1(S_1, S_2) = b_1 S_1 - \beta(G(S_1, S_2) - \hat{G})^2. \end{cases} \quad (10)$$

On the other hand, we assume that player i 's subjective evaluation of its own probability is equal to its objective probability, such that $w_1(p_1) = p_1$ and $w_2(p_2) = p_2$. Then, using the indifference principle under PT, player 1's average utility of charging $w_1(p_2^*) u_1(D_1, D_2) + w_1(1 - p_2^*) u_1(D_1, S_2)$ is equal to its average discharging utility $w_1(p_2^*) u_1(S_1, D_2) + w_1(1 - p_2^*) u_1(S_1, S_2)$; that is,

$$\frac{w_1(p_2^*)}{w_1(1 - p_2^*)} = \frac{u_1(S_1, S_2) - u_1(D_1, S_2)}{u_1(D_1, D_2) - u_1(S_1, D_2)} > 0, \quad (11)$$

which is analogous to the condition (9) under EUT. Computing player 2's average utility by p_1^* , we also have the condition

$$\text{sgn}\left(u_2(S_1, S_2) - u_2(S_1, D_2)\right) = \text{sgn}\left(u_2(D_1, D_2) - u_2(D_1, S_2)\right).$$

To solve (9), we need to simplify

$$\begin{aligned} & u_1(D_1, D_2) - u_1(S_1, D_2) \\ &= -c_{11}(D_1 + L_1(D_1, D_2)) - b_1 S_1 + \beta(G(S_1, D_2) - \hat{G})^2, \\ &= -c_{11}(D_1 + L_1(D_1, D_2)) - b_1 S_1 + \beta\{[D_2 - S_1 + D_{\text{others}} \\ &\quad + L(S_1, D_2)] - [D_1 + D_2 + D_{\text{others}} + L(D_1, D_2)]\}^2, \\ &= -c_{11}D_1 - b_1 S_1 + \beta(D_1 + S_1)^2, \end{aligned} \quad (12)$$

where D_{others} represents the total constant demand of non-participating users. Here, we have assumed that the losses $L(\cdot)$ are negligible with respect to the demand, which is a reasonable assumption when dealing with two players only, i.e., $L_k \ll D_k$, $k = 1, 2$. Similarly,

$$\begin{aligned} & u_1(S_1, S_2) - u_1(D_1, S_2) \\ &= b_1 S_1 - \beta(D_1 + S_1 + D_2 + S_2)^2 + c_{12}D_1 + \beta(D_2 + S_2)^2, \\ &= c_{12}D_1 + b_1 S_1 - \beta(D_1 + S_1)^2 - 2\beta(D_1 + S_1)(D_2 + S_2). \end{aligned} \quad (13)$$

If (12) is greater than 0, (13) cannot be greater than 0 due to the fact that, in practice, as the locational marginal pricing (LMP) [33] increases with the generated power, the price at a lower generation level cannot exceed that charged at a higher level; thus, mathematically, $c_{12} \leq c_{11}$. Thus, both sides of (9) have to

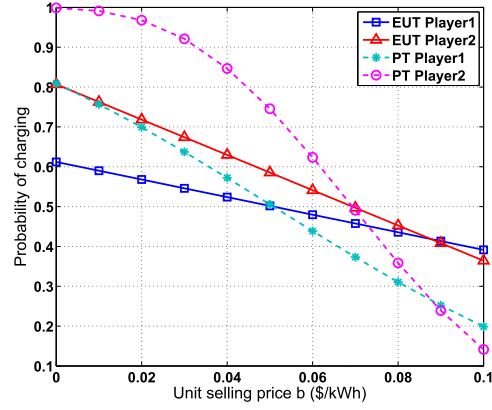


Fig. 1. Probability of charging under EUT and PT as b varies.

be negative and then, we obtain the range of $b_k S_k$ in Theorem 1. ■

Under given loads and surpluses, Theorem 1 provides a relationship between the unit selling price b_k of each player, the LMP price $c(a_1, a_2)$, and the penalty factor for regulation β , such that we could obtain a proper mixed strategy equilibrium. From the utility functions in (4) and (6), we can mathematically see the difference between EUT and PT. Here, given the players' mixed strategies, we define the company's expected revenues under the equilibrium probabilities, (p_1^*, p_2^*) for EUT and $(p_1^{*,PT}, p_2^{*,PT})$ for PT. The power company generates a revenue depending on the energy sold to the two customers, although the customers' probability of charging or discharging can be different between EUT and PT. Thus, the power company revenues obtained from customers 1 and 2 are as follows:

$$\begin{aligned} R_{EUT} &= p_1^* p_2^* c_{11}(D_1 + D_2 + L_{1,2}) + p_1^* (1 - p_2^*) c_{12}(D_1 + L_1) \\ &\quad + (1 - p_1^*) p_2^* c_{21}(D_2 + L_2), \\ R_{PT} &= p_1^{*,PT} p_2^{*,PT} c_{11}(D_1 + D_2 + L_{1,2}) + p_1^{*,PT} (1 - p_2^{*,PT}) c_{12} \\ &\quad (D_1 + L_1) + (1 - p_1^{*,PT}) p_2^{*,PT} c_{21}(D_2 + L_2), \end{aligned} \quad (14)$$

where $L(\cdot)$ is the loss in power flow (1). R_{EUT} is the expected revenue obtained by the power company. And R_{PT} is the PT revenue obtained by the power company, in which player 1 and player 2 use their subjective perspectives.

IV. SIMULATION RESULTS AND ANALYSIS

For simulating the proposed system, we consider a geographical region in which two active customers equipped with storage units exist. We choose typical values for the demand and surplus: $D_1 = 20$ kWh, $D_2 = 15$ kWh, $S_1 = 10$ kWh, $S_2 = 5$ kWh, $\alpha = 0.25$, $\beta = 0.0018$. The constant load is set as 200 kWh, and power line parameters are set from a typical 4-bus system [34]. The following examples assume that the generation power (kW) is numerically equal to the energy (kWh) in a one-hour time unit. For pricing, we assume that $c(a_1, a_2)$ follows a conventional LMP scheme, such as the following:

$$c = \begin{cases} \$0.05/\text{kWh} & \text{power company generation is } \leq 200 \text{ kWh,} \\ \$0.10/\text{kWh} & \text{power company generation between } 200\text{--}250 \text{ kWh,} \\ \$0.15/\text{kWh} & \text{power company generation between } 250\text{--}300 \text{ kWh,} \\ \$0.20/\text{kWh} & \text{power company generation is } > 300 \text{ kWh.} \end{cases}$$

In Fig. 1, we depict the impact of the unit selling price on the behavior of the customers. Without loss of generality, we assume that both customers use the same price $b_1 = b_2 = b$ and we vary the price within the range in which the equilibrium exist as per Theorem 1. Fig. 1 shows how the probability of charging for both players varies as b increases, for both the EUT and PT cases. Clearly, as the selling price increases, both players would have more incentive to discharge than to charge, as the benefit would start outweighing the regulation penalty. More interestingly, Fig. 1 shows that, for both customers, the PT behavior significantly differs from the EUT behavior. For example, for customer 2, below a selling price of $b = \$0.07$ per kWh, the probability of charging at the equilibrium for PT is much higher than EUT. This implies that for low gains, each customer follows a more *conservative, risk-averse* strategy under PT and is less interested

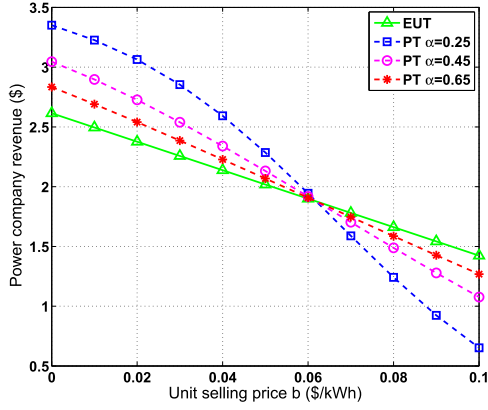


Fig. 2. Total revenue for the power company under EUT and PT as the individual customer selling price b varies.

in reaping the benefits of selling energy than in the EUT case. However, as the selling price crosses the threshold, the probability of charging for customer 2 under PT becomes much smaller than under EUT. This implies that once the selling benefits are significant (and the risks decrease), customer 2 starts selling more aggressively under PT than under EUT. A similar behavior can be observed for customer 1, although the benefit threshold of customer 1 is smaller ($b = \$0.05$), since customer 1 has more energy to sell/buy.

Fig. 2 evaluates the total revenues of the power company in (14) as the customers' unit selling price b increases, for both PT and EUT. Fig. 2 clearly shows that, as the unit selling price of the customers increases, the total revenue of the power company will decrease, as the customers start to sell more and buy less. Further, we can clearly see how the deviations from the EUT behavior, as predicted by PT can have a major impact on the market. First, as the customers' unit selling price is below about \$0.06 per kWh, under PT, the total revenue collected would be much higher than that expected under EUT. In contrast, if the customers are allowed to set prices that are higher than \$0.06 per kWh (and basically higher than the minimum unit price of the company's LMP model), PT predicts that the total revenue will be much smaller than in the EUT case. In this case, it is more beneficial for the power company to regulate the customers' unit selling price to be below \$0.06 per kWh (which is comparable to the minimum LMP price of \$0.05 per kWh). Fig. 2 demonstrates the importance of incorporating the customers' behavior into the analysis of the power market. In particular, if the power company utilizes EUT to regulate the customers' selling price, in practice, this may incur losses in revenues (relative to EUT) if realistic user behavior models are not accounted for. Finally, we note that the "crossing point" between PT and EUT in Fig. 2 depends largely on β . As β becomes higher, a higher unit selling price would be required for the customers to more aggressively sell energy.

In Fig. 3, we show that the expected load on the grid significantly differs between PT and EUT. For PT, when the unit price for buying energy is small, the customers are less interested (compared to EUT) in selling energy now. However, as the unit price crosses a threshold, the customers will sell more aggressively and, thus, the overall load on the grid will be smaller than expected. Fig. 3 can provide important guidelines for demand-side management in the smart grid. For example, assume the power company wants to increase its price to drive customers to sell more and reduce their average load to about 10 kWh while keeping the generation regulation within limits. Based on EUT, the company would have to increase the minimum LMP price to roughly \$0.077 per kWh. In reality, because users behave subjectively when faced with risk, the company does not need to introduce such a high price increase. In contrast, it can increase it to about \$0.06 per kWh and obtain the desired load reduction. On the other hand, if the company wants to reduce its price to sustain up to 23 kWh of load (from the two customers in question), based on EUT, it would have to offer a relatively low price of \$0.035 per kWh. In contrast, based on PT, a price of about \$0.047 per kWh can achieve the same impact yet yield

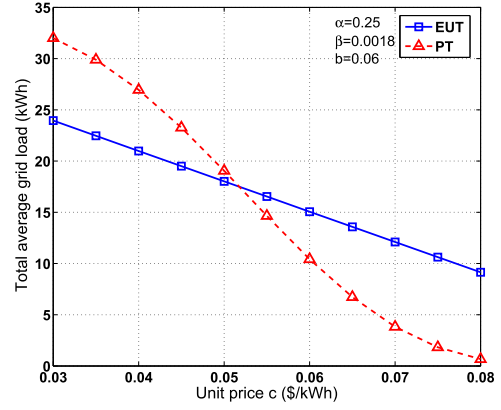


Fig. 3. The total average grid load for participating customers under EUT and PT as the power company's unit price c varies.

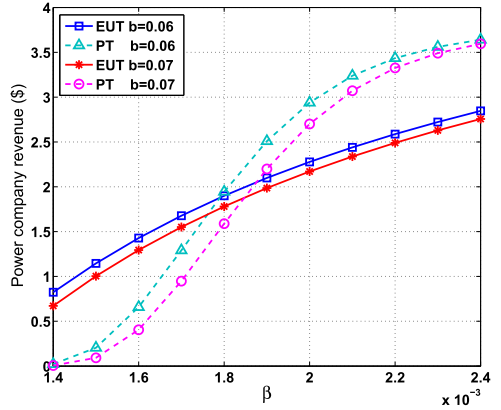


Fig. 4. Total revenue for the power company under EUT and PT as the regulation parameter β varies.

more profits. Clearly, ignoring the fact that users' behavior can deviate from the rational EUT path can yield undesirable loads on the grid which further motivates the need for PT analysis.

In Fig. 4, we show how the power company revenues under EUT and PT vary as the regulation parameter β increases. In particular, we vary β from 0.0014 to 0.0024 while satisfying the existence of a proper mixed Nash equilibrium. First, the solid lines show that the revenue under EUT is concave. This is due to the fact that the objective probability of charging is computed from a nonlinear utility that integrates power regulation. As the parameter β increases, both players want to store/charge more, since discharging increases the penalty of power regulation. Also, we can see that, after the crossing point (i.e. $\beta = 0.0018$ when $b = 0.06$), the power company would obtain a high revenue from the PT actions of players. This is because players are more likely to charge (act more conservatively) at a high β comparable to their objective action. Thus, the power company must choose an optimal β while balancing the tradeoff between its own revenues and effective customer participation via discharging.

V. CONCLUSIONS

In this paper, we have introduced a novel approach for studying the problem of customer-owned energy storage integration in the smart grid. We have developed a novel game-theoretic approach, based on prospect theory, using which each player subjectively observes and determines its actions so as to optimize a utility function that captures the benefit from selling energy as well as the associated regulation penalty. For the two-player scenario, we have shown the existence of an equilibrium for both EUT and PT. Simulation results have shown that prospect theory enables the power company to better decide on its pricing parameters, given realistic behavior of the users which deviate considerably from conventional EUT behavior. This paper only scratches the surface of prospect theory, which is expected to become a key technique in the design and analysis of a user-centric smart grid.

VI. REFERENCES

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