Intertemporal Trading Economy Model for Smart Grid Household Energy Consumption

Jayaprakash Rajasekharan and Visa Koivunen

SMARAD CoE, Department of Signal Processing and Acoustics, Aalto University, Finland.
Email: {jayaprakash.rajasekharan, visa.koivunen}@aalto.fi

Abstract—**In this paper, we propose to model the energy consumption of smart grid households with energy storage systems (ESS) as an intertemporal trading economy. Intertemporal trade refers to transaction of goods across time when an agent, at any time, is faced with the option of consuming and/or saving with the aim of using the savings in the future and/or spending the savings from the past. Smart homes define optimal consumption as balancing/leveling their consumption profile such that the utility company is presented with a more uniform demand. Due to the varying nature of energy requirements of household and market energy prices over different time periods in a day, households face a trade-off between consuming to meet their current energy requirements and/or storing energy for future consumption and/or spending energy stored in the past. These trade-offs or consumption preferences of the household are modeled as a Cobb-Douglas utility function using consumer theory. This utility function is maximized subject to budget and storage constraints to solve for the optimal consumption profile. We graphically illustrate the process of computing the optimal consumption point when a day is divided into two or three time periods. For higher dimensional multi-period models, we formulate the optimization problem as a geometric program (GP). Simulation results show that the proposed approach is able to achieve a uniform consumption profile with extremely low peak to average ratio (PAR) close to 1 in addition to reducing consumption costs for the household by about 6%.**

Keywords*—*Smart grids, demand side management, constrained optimization, geometric programming, time-varying systems.

I. Introduction

Energy storage systems (ESS) has recently gained attention due to the integration of fluctuating and intermittent renewable energy sources and plug-in hybrid electric vehicles (PHEVs) into smart grid systems [1]. Demand side management (DSM) commonly refers to programs that control the energy consumption of households. DSM programs such as residential load management aim at reducing, shifting and/or scheduling consumption at the household level to off-peak hours by means of smart pricing options such as critical peak pricing (CPP) [2], time-of-use pricing (ToUP) [3], etc. Smart pricing combined with fluctuating renewable energy production makes energy consumption schedulers (ECS) and ESS indispensable in smart homes. Though scheduling itself is successful to a certain extent in reducing the peak to average ratio (PAR) of consumption and yielding some cost savings, there is a limit to the amount of household energy requirements that can be scheduled without causing excessive discomfort. Even though scheduling is implicitly supported by the utility company through smart pricing, scheduling alone cannot guarantee consumption PAR minimization for the utility company. ESS provide smart homes with an attractive option to balance/level their consumption such that their consumption costs are reduced and the utility company is presented with a more uniform demand.

Residential energy storage is enabled by dedicated battery systems, supercapacitors, PHEVs, etc. [4]. Vehicle to home (V2H) and vehicle to grid (V2G) technologies [5] have already enabled bidirectional transfer of energy between the grid or home and the battery system in PHEVs with the aim of selling demand response services back to the grid or home. Though energy storage using batteries has been traditionally considered lossy, difficult and expensive, it is expected to be a key component of smart homes in future smart grid systems. Affordable home battery back-up systems with storage capacities around 3-6 KWh are available at the retail level due to wide spread adoption of renewable energy production sources. Moreover, households with battery systems have the advantage of generating additional income by selling surplus stored energy during peak periods to neighbors without storage facilities [6]. Along with pricing incentives, scheduling capabilities, renewable energy source integration, consumption balancing/leveling and cost minimization options, home battery systems are not only cost-effective for the households in the long run, but also increase the social welfare for the entire energy generation and distribution system.

Saving goods or money for future use is an inherent characteristic of *Homo economicus*. In macroeconomic theory, intertemporal trade is defined as the transaction of goods or money across time when an agent is faced with the option of consuming and/or saving in the present with the aim of using the savings in the future [7]. Optimal energy consumption for a household could be defined as minimizing consumption costs which would involve storing energy during off-peak hours when prices are lower and using it during peak hours when prices are higher. However, in this scheme, the household is the sole beneficiary and there is no direct incentive for the utility company to support this scheme as the resulting consumption profile, if not worse, is as non-uniform as the actual energy requirements of the household. Therefore, we define the optimal consumption of the household as a balanced consumption profile that is as uniform as possible in addition to reduction in consumption costs as well. Thus, there is an incentive for both the household and the utility company to support the consumption balancing/leveling scheme (reduction in consumption costs for the former and uniform demand response or balanced overall load for the latter).

Using micro/macro economic concepts to study and model the dynamics of smart grid systems is a fairly recent approach. A market clearing auctioning approach for buying and selling demand response as a public good has been studied in [8]. Deployment of optimal and autonomous incentive based ECS algorithm for smart grids without energy storage devices is discussed in [9]. A non-cooperative game-theoretic approach to modeling DSM with energy storage devices for a whole locality is discussed in [10], which studies the effect of multiple households with battery systems in the same neighborhood simultaneously opting for cost minimization scheme that could lead to extremely non-uniform demand resulting in grid failure and suggests a game-theoretic and machine learning based approach to arrive at a Nash equilibrium consumption point.

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

The contributions of this paper are as follows. We model the energy consumption of smart homes with ESS as an intertemporal trading economy. Optimal consumption is defined as balancing/leveling the household consumption such that the utility company is presented with a demand that is a as uniform as possible. The trade-off between consuming energy to meet current energy requirements versus storing energy for future consumption is represented by a Cobb-Douglas utility function using consumer theory. The process of computing the optimal consumption point when a day is divided into two or three time periods is graphically illustrated. For higher dimensional models, the optimization problem is formulated as a geometric program and solved subject to budget and storage constraints. For a given set of hourly day-ahead market energy prices, daily energy requirements and operational parameters of a battery system, the proposed model is able to achieve extremely low consumption PAR close to 1 in addition to reducing consumption costs by about 6%.

The rest of this paper is organized as follows. The system model is briefly described in Section II. An introduction to intertemporal trade and consumer theory is given in Section III and the optimization problem is formulated as geometric program. In Section IV, examples are given for graphically solving the optimal consumption profiles of two- and three-period models and the optimization problem for a 24-period model is solved. Section V concludes the paper.

II. System Model

Consider a smart grid system where households are served by multiple utility companies that exogenously provide energy. Additionally, households may also generate energy by means of privately-owned renewable sources. Households are equipped with ESS and a smart meter with ECS capabilities. Each household has access to day-ahead hourly prediction prices issued by their utility companies so that they can schedule their appliances accordingly and choose an optimum strategy for charging and discharging their batteries. Each household also has accurate knowledge of its energy requirements during every time period of the day. We make a simplistic assumption that there are no externalities in the market, i.e., each household cares only about the amount of energy that it consumes and is not concerned with the consumption of other households even though it may indirectly affect the market. Finally, we assume that households are price takers, so that they take the prices in the market as fixed and act accordingly, and have no power (or at least believe that they have no power) to change the market prices.

We define a N-period model for the household as a 24 hour day that is equally split into N intervals and each period is indexed by $\{1, 2, \cdots, N\}$. Household defined time periods are synchronized with the periods set by the utility company for their dynamic pricing model. The price, energy requirement, consumption and state of battery storage (charge levels at the end of a period) in periods 1 through N are denoted by p_1, l_1, c_1, b_1 through p_N, l_N, c_N, b_N. Let b_0 denote the initial state or charge level of the battery before the beginning of the first period, b_N the final state of the battery at the end of N periods and b_{max} its maximum charge levels or its capacity. Without any prior knowledge about the state of the battery before period 1 and after period N, we can set both b_0 and b_N to zero. However, any arbitrary value for b_0 and b_N can be set in this model without loss of generality. Let r be the rate of storage loss per period in the battery that accounts for unavoidable self-discharge and other loss factors, meaning E Wh of energy stored in one period is worth $E(1-r)$ Wh of energy in the next period and $E(1-r)^2$ after two periods, i.e., $(1-r)$ is the per period storage efficiency of the battery.

III. Intertemporal Trade

In order to describe the proposed approach, let us assume that households face only two time periods in a day, where period 1 occurs during the off-peak hours when prices are low and period 2 occurs during peak hours when prices are high, and that the energy requirements and prices within these periods are constant. More variations in energy requirements and prices can be accommodated into the model by expanding the number of time periods in a day such that the energy requirements and prices are constant within those periods. In period 1, the household consumes an amount equal to its energy requirements in that period with the option of using any stored energy from the previous period and since the prices are lower than in next period, the household also chooses to store energy by charging its batteries. Thus consumption in period 1 is given by, $c_1 = l_1 + b_1 - b_0(1-r) = l_1 + b_1$. In period 2, the household can use stored energy from period 1 to partially or fully meet its energy requirements by discharging the batteries and hence consumption is given by, $c_2 = l_2 + b_2 - b_1(1-r) = l_2 - b_1(1-r)$. Rearranging, we arrive at the household budget constraint as shown in Eq. (1).

$$c_1 + \frac{c_2}{(1-r)} = l_1 + \frac{l_2}{(1-r)}. \tag{1}$$

The budget constraint of the household gives the present value (i.e., w.r.t period 1) of total consumption in terms of its present value of total energy requirement. This is illustrated graphically in Fig. 1. Let $(c_1, c_2) \in \mathrm{R}^2$ be the consumption space. The budget constraint

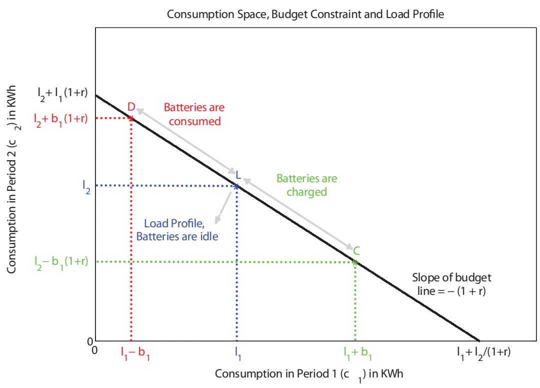

Fig. 1. Consumption space, budget constraint and energy requirement profile of a household. The black line gives the budget constraint and its intercepts represent extreme points of consumption in different periods. Point L is the energy requirement profile of the household when there is no storage and consumption is same as the energy requirement. Points C and D represent the levels to which the batteries are charged or discharged in different periods.

is a line in the consumption space whose intercepts are determined by the energy requirement profile of the household. The horizontal intercept $l_1 + l_2/(1+r)$ gives the amount that would be consumed in period 1 if there would be no consumption in period 2. The vertical intercept $l_2 + l_1(1+r)$ gives the amount that would be consumed in period 2 if there was no consumption in period 1. The slope of the budget line is given by $-(1-r)$, or the negative of storage efficiency. The household can operate at any point or consumption profile (c_1, c_2) that is on the line (efficient) or in the region below (inefficient), but not above (unattainable). The point L on the budget line is the household energy requirement profile (l_1, l_2) and at this point, consumption is equal to energy requirement and the batteries are idle and not used. If period 1 occurs during the off-peak hours when market prices are low, the household operates at point C on the budget line, where, in addition to the energy requirement l_1, the consumption is $l_1 + b_1$ and the batteries are charged to b_1 to be used in period 2 when the market prices are higher. Similarly, during peak hours when the market prices are high, the household operates at point D where the batteries are consumed to reduce the consumption and costs. Extending recursively to a N-period model, we can derive the general budget constraint which is a hyperplane in an N-dimensional space as shown in Eq. (2).

$$c_1 + \frac{c_2}{(1-r)} + \frac{c_3}{(1-r)^2} + \cdots + \frac{c_N}{(1-r)^{N-1}} = l_1 + \frac{l_2}{(1-r)} + \frac{l_3}{(1-r)^2} + \cdots + \frac{l_N}{(1-r)^{N-1}}. \quad (2)$$

The problem faced by households can now be stated as follows: Given N periods in a day $\{1, 2, \cdots, N\}$, market prices $\mathbf{p} = [p_1, p_2, \cdots, p_N]^T$, energy requirements $\mathbf{l} = [l_1, l_2, \cdots, l_N]^T$, battery capacity b_{max} and battery loss rate r per period, at which point on the budget hyperplane should the household operate, i.e., how to choose an optimal consumption profile $\mathbf{c}^* = [c_1^*, c_2^*, \cdots, c_N^*]^T$, or in other words, when should the household charge or discharge its batteries and by how much ? The answer is given by consumer theory.

A. Consumer Theory

Preferences are used to model the way rational households make choices about their consumption. Preference relations are defined in the consumption space. Formulating appropriate utility functions that reflect the consumption preferences of users over different time periods is of vital importance in modeling intertemporal trade. Utility functions $u(c_1, c_2)$ with respect to consumption in time periods 1 and 2 for a two-period model are usually visualized as isoquants or contours in the two dimensional consumption space with each consumption period on each of the axes and contour lines linking points of equal utility. The constant utility contour lines are known as indifference curves as they link points of equal preference, in other words, linking consumption points that are indifferent. Some common utility functions as shown in Fig. 2 are,

$$u(c_1, c_2) = c_1 + c_2, \qquad \text{(Perfect Substitutes)}$$
$$u(c_1, c_2) = min(c_1, c_2), \qquad \text{(Perfect Complements)}$$
$$u(c_1, c_2) = {c_1}^{\alpha_1} * {c_2}^{\alpha_2}. \qquad \text{(Cobb-Douglas Utility)}$$

If a household does not care about consumption in individual periods, but is concerned only about the total consumption, the perfect substitute utility function captures this preference aptly as consumption in period 1 can be substituted for consumption in period 2. If the utility company employs a static pricing model, then households will tend to use this kind of utility function to optimize their consumption. If a household values consumption in period 1 with a certain minimum constraint on consumption in period 2, this preference is reflected in the perfect complement utility function. If the household has renewable energy sources with a fixed amount of energy production, then the perfect complement utility function would be ideal for modeling this scenario. However, if a household values a certain share of consumption in period 1 (α_1) in relation to consumption in period 2 (α_2), the Cobb-Douglas utility function is best suited for modeling this preference. This kind of utility function is applicable to households that try to balance/level their consumption to help the utility company by supplying a uniform demand. Since these utility functions capture the best possible trade-off between consuming and storing energy under different scenarios while taking into account the current and future energy requirement and market prices, the optimal consumption point of a household is achieved when its utility function is maximized subject to its budget constraint.

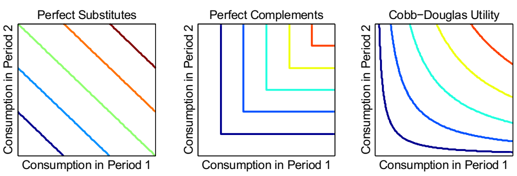

Fig. 2. Examples of common utility functions such as perfect substitutes, perfect complements and Cobb-Douglas utility function with $\alpha_1, \alpha_2 = 0.5$.

B. Optimal Consumption

The household aims to achieve optimal consumption by balancing/leveling its consumption such that the utility company is presented with a more uniform demand. The Cobb-Douglas utility function is apt for representing how households value a certain share of consumption in every period depending upon the energy requirements and market prices in order to even out overall consumption. The parameter α_i in the Cobb-Douglas utility function for period i is chosen such that it represents the normalized cost of consumption in all time periods excluding i and by constraining $\alpha_1 + \alpha_2 + \cdots + \alpha_n = 1$, the peaks in consumption are flattened. For example, in a two period model, $\alpha_1 = p_2 l_2/(p_1 l_1 + p_2 l_2)$ and $\alpha_2 = p_1 l_1/(p_1 l_1 + p_2 l_2)$. In addition to the budget constraint, we can also add a savings constraint that restricts the optimal consumption profile such that the household incurs no additional cost for balancing/leveling the consumption. The optimization problem can be formally stated as follows :

$$\text{Max } u(c_1, c_2, ..., c_N) = \prod_{i=1}^{N} c_i^{\alpha_i}, \text{ where}$$
$$\alpha_i = \frac{\sum_{j=1, j\neq i}^{N} p_j l_j}{(N-1)\sum_{i=1}^{N} p_i l_i} \text{ and } \sum_{i=1}^{N} \alpha_i = 1,$$
$$\text{s.t } \sum_{i=1}^{N} \frac{c_i}{(1-r)^{i-1}} = \sum_{i=1}^{N} \frac{l_i}{(1-r)^{i-1}}, \ \mathbf{p}^T\mathbf{c} \leq \mathbf{p}^T\mathbf{l}.$$

This class of optimization problems are referred to as a geometric programs (GPs) [11], where the objective function is a posynomial and the constraints are posynomial equalities and/or monomial inequalities. An optimal solution always exists for a GP, and the trick to solving it efficiently is to convert it to a non-linear but convex optimization problem by logarithmic change of variables. Computationally advanced methods such as primal-dual interior point algorithms can solve large-scale GPs extremely efciently and reliably.

IV. EXAMPLES

Let us assume the utility company charges households with energy prices based on the USA New England hourly real-time prices of January 1st, 2011 [12]. We model the daily energy requirement of households with usage-statistics-based load model proposed in [13]. This model simulates daily load with one hour time resolution through simulation of appliance use and also by taking into account simulated resident activity in households. The day-ahead hourly market energy prices and hourly energy requirements of household are shown in Fig. 5(a) and (b) respectively.

A. Two-period and Three-period Models

We divide the USA NE hourly prices into two time periods with period 1 running from midnight until noon and period 2 from noon until midnight and set market prices by averaging the prices over those time periods. Other time period divisions are also possible depending upon the utility company's definition of peak and off-peak hours without loss of generality. The simulated hourly energy requirement of the household is aggregated over the time periods. Similarly, for the three-period model, prices are averaged and energy requirements are aggregated over 3 periods of 8 hours each. The market energy prices and energy requirements of the household for two- and three-period models are shown in Fig. 3. Given this set

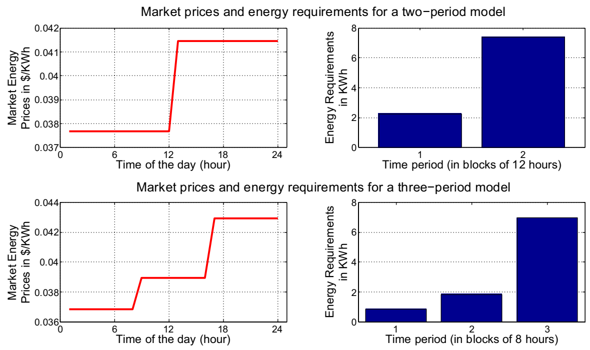

Fig. 3. Market energy prices and daily energy requirements of a household for two-period and three-period models respectively.

of market prices, energy requirement profile and storage loss rate $r = 0.01$, the optimization problem can be solved for two- and three-period models by maximizing the Cobb-Douglas utility function subject to the budget constraint. Graphically, the optimal point of

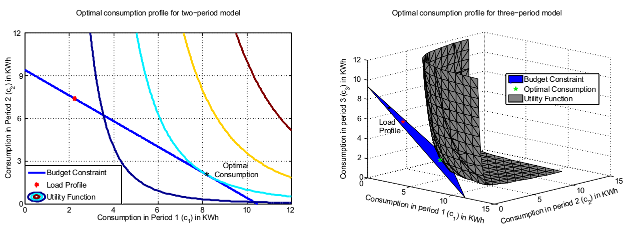

Fig. 4. Graphical representation of optimal consumption point for two-period and three-period models. The optimal point occurs when the budget hyperplane is tangential to the indifference surfaces of the utility function.

consumption occurs where the budget hyperplane is tangential to the indifference surfaces of the utility function as depicted in Fig. 4.

B. 24-period Model

We extend the two-period to a generic multi-period model. Given the day-ahead hourly market energy prices and hourly household energy requirements, we solve for the 24-dimensional optimization problem with battery loss rate of $r = 0.001$ and battery capacity of $b_{max} = 5$ KWh. The results are presented in Fig. 5. We see that the optimal consumption (c) of the household is very uniform over time with PAR = 1.0390, even though the energy requirement profile (b) is highly non-uniform. The battery profile (d) shows the levels to which the batteries are charged or discharged during each time period

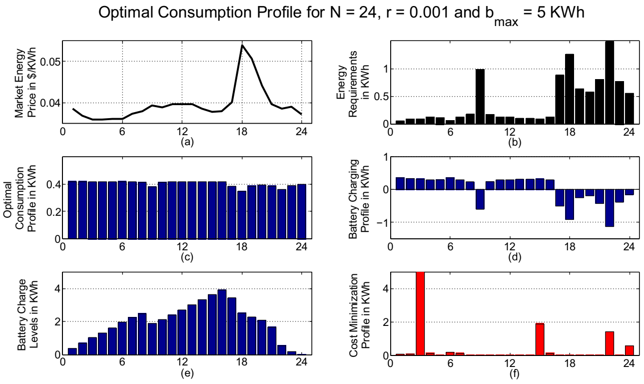

Fig. 5. Optimal consumption for a 24-period model with battery loss rate $r = 0.001$ and battery capacity $b_{max} = 5$ KWh. The day-ahead hourly market energy prices (a) and hourly energy requirements (b) are shown in black. The optimal consumption profile (c), battery charging/discharging profile (d) and battery charge levels at the end of each period (e) are shown in blue. The consumption profile resulting from cost minimization (f) is shown in red. We see that the optimal consumption profile (c) is very uniform (PAR = 1.0390) over all time periods with about 6% reduction in consumption costs.

and the battery state (e) shows the charge levels of the battery at the end of each time period. In order to achieve the optimal consumption profile, we see that the batteries are used continuously during all time periods to flatten the peaks in the energy requirements of the household. A reduction of about 6% in consumption costs is achieved. In comparison, under the cost minimization scheme, a reduction of about 12% in consumptions costs is achieved. However, this is achieved at the cost of a highly non-uniform consumption profile (f). Thus, for a given set of hourly day-ahead market energy prices, daily household energy requirements, battery capacity and loss rate, the optimal consumption profile is achieved by balancing/leveling the consumption of the household. In this scheme, both the household and the utility company benefit as the household enjoys reduction in consumption costs while the utility company is presented with demand that is as uniform as possible.

V. Conclusions

A framework for modeling the energy consumption of smart households with storage devices as an intertemporal trading economy is proposed. The model is also applicable for households with renewable energy production sources and energy storage systems such as dedicated batteries or PHEVs. Households define optimal consumption as as balancing/leveling their consumption profile such that the utility company is presented with a demand that is as uniform as possible. Due to the dynamic nature of market energy prices and demand, the household is faced with a choice between consuming in the present to fulfill its current energy requirements, storing energy for future use and spending energy stored in the past. The consumption preferences of the household are modeled as Cobb-Douglas utility function using consumer theory. The process of computing the optimal consumption point when a day is divided into two or three time periods is graphically illustrated. For higher dimensional models, the optimization problem is formulated as a geometric program and solved subject to budget and storage constraints. Simulation results show that the proposed model achieves an optimal consumption profile with extremely low consumption PAR values close to 1 in addition to reducing the household consumption costs by about 6%.

References

[1] I. N. E. Inc., “Overview of the smart grid: Policies, initiatives and needs,” Feb. 2009.

[2] K. Herter, “Residential implementation of critical-peak pricing of electricity,” *Energy Policy*, vol. 35, no. 4, pp. 2121 – 2130, 2007.

[3] P. Yang, G. Tang, and A. Nehorai, “A game-theoretic approach for optimal time-of-use electricity pricing,” *IEEE Transactions on Power Systems,*, vol. 28, no. 2, pp. 884–892, 2013.

[4] W. Zhu, D. Garrett, J. Butkowski, and Y. Wang, “Overview of distributive energy storage systems for residential communities,” in *IEEE Energytech 2012,*, 2012, pp. 1–6.

[5] C. Liu, K. Chau, D. Wu, and S. Gao, “Opportunities and challenges of vehicle-to-home, vehicle-to-vehicle, and vehicle-to-grid technologies,” *Proceedings of the IEEE*, vol. PP, no. 99, pp. 1–19, 2013.

[6] J. Rajasekharan, J. Lunden, and V. Koivunen, “Competitive equilibrium pricing and cooperation in smart grids with energy storage,” in *47th Annual Conf. on Info. Sciences and Systems (CISS)*, 2013, pp. 1–5.

[7] I. Fisher, *The Theory of Interest As Determined by Impatience to Spend Income and Opportunity to Invest It.* Martino Publishing, 2012.

[8] D. T. Nguyen, M. Negnevitsky, and M. de Groot, “Walrasian market clearing for demand response exchange,” *Power Systems, IEEE Transactions on*, vol. 27, no. 1, pp. 535–544, 2012.

[9] A.-H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, “Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid,” *Smart Grid, IEEE Transactions on*, vol. 1, no. 3, pp. 320–331, 2010.

[10] P. Vytelingum, T. D. Voice, S. D. Ramchurn, A. Rogers, and N. R. Jennings, “Agent-based micro-storage management for the smart grid,” in *9th International Conf. on Autonomous Agents and Multiagent Systems (AAMAS)*, 2010, pp. 39–46.

[11] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, “A tutorial on geometric programming,” *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, 2007.

[12] I. N. E. Inc., “Usa new england control area hourly day-ahead and real-time prices in 2011, available online at http://www.iso-ne.com.”

[13] I. Richardson, M. Thomson, D. Infield, and C. Clifford, “Domestic electricity use: A high-resolution energy demand model,” *Energy and Buildings*, vol. 42, no. 10, pp. 1878–1887, 2010.