AN OFF-LINE OPTIMIZATION APPROACH FOR ONLINE ENERGY STORAGE MANAGEMENT IN MICROGRID SYSTEM

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ABSTRACT

This paper investigates the real-time energy management in power system with distributed microgrids, which are independently operated and each is modeled to comprise of a renewable generation system, an energy storage system and an aggregated load. We jointly optimize the energy charged/discharged to/from the storage system and that drawn from the main grid over a finite horizon to minimize the total energy cost of conventional generation subject to given load and storage constraints. We assume that the renewable energy offset by the load over time, named *net energy profile*, is predictable but with finite errors. First, we consider the "off-line" optimization under an idealized assumption that the net energy profile is known ahead of time, and derive its optimal closed-form solution. Next, by applying the off-line solution combined with a sliding-window based sequential optimization, we propose a new "online" algorithm for real-time energy management under the practical setup with noisy predicted net energy profile subject to arbitrary errors. Finally, through simulations, we compare the performance of our proposed online algorithm against the conventional dynamic programming based solution as well as a heuristically designed myopic algorithm under a practical setup.

1. INTRODUCTION

Over the last few years, the worldwide extensively increasing electric energy consumption has become a serious concern. To reduce both the operational and environmental costs of conventional fossil fuel based energy generation, energy harvesting from renewable sources such as solar and wind over geographically distributed locations has emerged as a promising solution. Thus, the concept of microgrids becomes appealing for next generation power systems, where each microgrid consists of a networked group of renewable energy generators and storage systems to provide cheaper and green energy to users in a small geographical area [1].

However, the efficient and reliable operation of microgrid system faces new challenges due to the intermittent characteristics of renewable energy sources. To overcome this problem, various approaches such as using conventional generation as the supplement and enabling microgrids' energy cooperation [2, 3] have been proposed. Moreover, energy storage is a practically adopted solution as well since it helps smooth out the power fluctuations in the renewable energy supply and thus improves the grid's efficiency and reliability.

In this paper, we investigate the problem of real-time energy management for distributed microgrids, which are assumed to be independently operated and each is modeled to comprise of a renewable generation system, an energy storage system, and an aggregated load. We jointly optimize the energy charged/discharged to/from the storage system and that drawn from the main grid over a finite horizon to minimize the total energy cost (modeled as the sum of time-varying strictly convex functions) of conventional generation, subject to given load and storage constraints. We assume that the renewable energy generated in each microgrid offset by its load over time, named *net energy profile*, is practically predictable but with finite errors that are arbitrarily distributed. Under this setup, we propose a new off-line optimization approach to devise the online energy management algorithm. First, we consider the off-line optimization by assuming that the net energy profile is perfectly known *a priori*, and obtain its optimal closed-form solution. Next, we consider the practical setup with noisy predicted net energy profile subject to arbitrary errors, and develop a new online algorithm by combining the off-line solution with a sliding-window based sequential optimization. Finally, we conduct simulations to compare the performance of our proposed online algorithm with the conventional dynamic programming based solution and a heuristically designed myopic algorithm, for a specific setup where the prediction errors of the net energy profile follow a given stationary distribution.

It is worth noting that there have been a handful of prior studies [4–12] on energy management for microgrid systems. [4–7] studied the off-line energy scheduling problem under the assumption that the demand and renewable generation are either deterministic or known ahead of time. [8–11] investigated the online energy management problem under the stochastic demand and/or renewable energy by assuming either a simplified storage model [8, 9] or that the net energy profile follows a stationary stochastic process with known distributions [10, 11], which may not be valid for practical renewable sources. Furthermore, [12] proposed an optimal online energy storage management policy under the assumption of a simplified time-invariant linear energy cost function of conventional generation.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a power system consisting of the main grid and a set of distributed microgrids, which all connect to the main grid but operate independently to supply power to their respectively covered geographical areas. We assume that there is no cooperation among the microgrids and thus focus our study on one single microgrid in this paper. We will address the general case with microgrids' cooperation in our future work. The system model of our interest is depicted in Fig. 1, where a microgrid is shown to connect to the main grid and comprise of three basic elements, i.e., a renewable generation system, an energy storage system, and an aggregated load.

We assume a time-slotted system with slot index $i, 1 \leq i \leq N$, where N denotes the total number of slots for energy scheduling. We further assume a quasi-static time-varying model for the renewable energy, in which the energy rate is constant within each slot, but may change from one slot to another. We also assume that the duration of each slot is normalized to a unit time unless specified otherwise; thus, we can use power and energy interchangeably. Next, we define each element of the microgrid system in more detail as follows.

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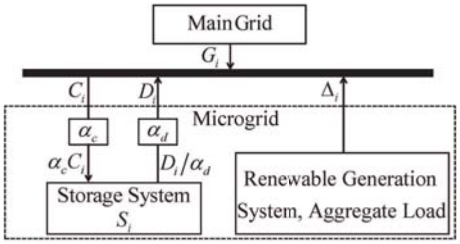

Fig. 1: System model.

Energy storage model: We denote the energy charged (discharged) to (from) the storage in slot i as $C_i \geq 0$ ($D_i \geq 0$). The charging and discharging efficiency parameters are denoted by $0 < \alpha_c < 1$ and $0 < \alpha_d < 1$, respectively. By denoting the state (stored energy) of the storage system at the beginning of each time slot i as $S_i \geq 0$, we have

$$S_{i+1} = S_i + \alpha_c C_i - D_i/\alpha_d,\ i = 1, \cdots, N. \tag{1}$$

Note that S_1 is the initial energy storage at the beginning of slot 1, while S_{N+1} is the final energy storage at the end of the N-slot scheduling period. Furthermore, practical energy storage systems should have a finite storage capacity $S_{\max} \geq 0$ and a minimum storage level $S_{\min} \geq 0$, i.e.,

$$S_{\min} \leq S_i \leq S_{\max},\ \ i = 2, \cdots, N+1, \tag{2}$$

where $S_{\min} \leq S_1 \leq S_{\max}$ is assumed by default. In addition, the final energy storage S_{N+1} needs to be kept above a given threshold $\overline{S}$ with $S_{\min} \leq \overline{S} \leq S_{\max}$, in order to ensure a cost-effective energy scheduling for the next N-slot horizon. Thus, we have:

$$S_{N+1} \geq \overline{S}. \tag{3}$$

Load and renewable energy model: In each slot i, the demand and the generated renewable energy are denoted as $DE_i \geq 0$ and $RE_i \geq 0$, respectively. We define the *net energy profile* over time as $\Delta_i = RE_i - DE_i, i = 1, \cdots, N$, which specifies the mismatch between the renewable energy supply and the demand. Note that Δ_i can be zero, positive (representing a supply energy surplus) or negative (representing a supply energy deficit). We assume that both RE_i's and DE_i's are predictable in practice but with finite errors, due to their randomness over time. Hence, we model the net energy profile as

$$\Delta_i = \overline{\Delta}_i + \delta_i,\ i = 1, \cdots, N, \tag{4}$$

where $\overline{\Delta}_i$ and δ_i denote the predictable component in the net energy profile and its corresponding prediction error in slot i, respectively. Under this model, we assume that at each slot $i \in \{1, \ldots, N\}$, the exact net energy profile over time $k \leq i$, i.e., $\Delta_1, \cdots, \Delta_i$, and the predictable net energy profile for time $k > i$, i.e., $\overline{\Delta}_{i+1}, \cdots, \overline{\Delta}_N$, are perfectly known to the microgrid, whereas the prediction errors for time $k > i$, i.e., $\delta_{i+1}, \cdots, \delta_N$, are unknown.

We further assume that the microgrid should always meet the load demand by discharging from its storage and/or drawing energy from the main grid. Let the energy drawn from the main grid in slot i be denoted by $G_i \geq 0$. We then have the *energy neutralization* constraints over time as $G_i + \Delta_i + D_i \geq C_i, i = 1, \cdots, N$.

Conventional generation cost: We consider a general time-varying energy cost model for conventional generation. We model the costs over time by a sequence of functions of G_i, denoted by $f_i(G_i), i = 1, \cdots, N$, each of which is assumed to be known to the microgrid and has the following properties:

- $f_i(G_i)$ is a strictly convex function over $G_i \geq 0$;
- $f_i(G_i)$ is a strictly positive and monotonically increasing function over $G_i \geq 0$;
- $f_i(G_i)$ is continuous and differentiable over $G_i \geq 0$, where $F_i(G_i) \triangleq f_i'(G_i)$ denotes the differential of $f_i(G_i)$ and $F_i^{-1}(\cdot)$ denotes the inverse function of $F_i(\cdot)$.

One commonly adopted function of $f_i(G_i)$ satisfying all the above properties is [13]

$$f_i(G_i) = a_i G_i^2 + b_i G_i + c_i, \tag{5}$$

where $a_i > 0$, $b_i \geq 0$, and $c_i \geq 0$ are given cost coefficients for slot i.

With the aforementioned models, we proceed to optimize the decision variables $\{C_i, D_i, G_i\}_{i=1}^N$ to minimize the cost of the total energy drawn from the main grid, i.e., $\sum_{i=1}^N f_i(G_i)$, while satisfying given storage and load constraints. We formulate the optimization problem as

$$(\text{P1}): \min_{\{C_i, D_i, G_i\}} \ \sum_{i=1}^N f_i(G_i)$$

$$\text{s.t.} S_1 + \alpha_c \sum_{k=1}^i C_k - 1/\alpha_d \sum_{k=1}^i D_k \geq S_{\min}, i = 1, \ldots, N \tag{6}$$

$$S_1 + \alpha_c \sum_{k=1}^i C_k - 1/\alpha_d \sum_{k=1}^i D_k \leq S_{\max}, i = 1, \ldots, N \tag{7}$$

$$S_1 + \alpha_c \sum_{k=1}^N C_k - 1/\alpha_d \sum_{k=1}^N D_k \geq \overline{S}, \tag{8}$$

$$G_i + \Delta_i + D_i \geq C_i, i = 1, \ldots, N \tag{9}$$

$$C_i \geq 0,\ D_i \geq 0,\ G_i \geq 0, i = 1, \ldots, N, \tag{10}$$

where (6) and (7) correspond to the storage constraints in (2), (8) represents the minimum storage requirement at slot $N+1$ in (3), and (9) stands for the energy neutralization constraints. Due to the unknown prediction error δ_k's in (4) at each slot i with $k > i$, (P1) is in general a challenging problem to solve. *Dynamic programming* technique is commonly used to solve problems of similar structures to (P1), which provides the optimal solutions if δ_i's are modeled as a stationary stochastic process with known distribution. However, due to the "curse of dimensionality" problem, the optimal solution by dynamic programming in general incurs an exponentially growing complexity in terms of the number of decision variables as N increases. Moreover, in practice, the renewable energy generated and/or load demand cannot be exactly modeled by stationary processes. Therefore, this motives our work to propose an alternative approach for solving (P1) in real life. First, we consider an *off-line* optimization of (P1), by assuming that the net energy profile, i.e., $\{\Delta_1, \ldots, \Delta_N\}$, is known ahead of time with no prediction errors, i.e., $\delta_i = 0, i = 1, \cdots, N$. We then propose an efficient algorithm to solve (P1) in the off-line case. Next, based on the optimal off-line solution, we further propose a new "online" algorithm for (P1) under the practical setup with noisy predicted net energy profiles, subject to arbitrary error sequence of δ_i's.

3. OFF-LINE OPTIMIZATION

In this section, we consider the off-line optimization for (P1) by assuming the net energy profile $\{\Delta_1, \ldots, \Delta_N\}$ are known at the beginning of slot $i = 1$. It is easy to verify that (P1) is a convex optimization problem [14], and thus can be solved by standard convex optimization techniques such as the interior point method. However, to draw more insights to the solution, we apply the Lagrange duality method to solve (P1) and obtain a closed-form optimal solution. Due to the space limitation, all proofs in this section are omitted and will be presented in the journal version of this paper.

Let $\underline{\nu}_i, \overline{\nu}_i, i = 1, \ldots, N$, and ω be the Lagrange dual variables associated with the constraints (6), (7), and (8), receptively. Define

$$\nu_i = \sum_{k=i}^{N} (\underline{\nu}_k - \overline{\nu}_k),\ i = 1, \ldots, N. \tag{11}$$

Then, the Lagrangian of (P1) is expressed as

$$\begin{aligned}&\mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i\}, \{D_i\}, \{G_i\})\\ =&\sum_{i=1}^{N} f_i(G_i) + \sum_{i=1}^{N} (\nu_i + \omega)(D_i/\alpha_d - \alpha_c C_i) - \omega S_1 + \omega \overline{S}\\ &- \Big(\sum_{i=1}^{N} \underline{\nu}_i\Big)(S_1 - S_{\min}) - \Big(\sum_{i=1}^{N} \overline{\nu}_i\Big)(S_{\max} - S_1). \end{aligned} \tag{12}$$

Accordingly, the dual function of $\mathcal{L}(\cdot)$ is given by

$$g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}) = \min_{\{C_i, D_i, G_i\}} \mathcal{L}(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}, \{C_i\}, \{D_i\}, \{G_i\}) \quad \text{s.t. } (9), (10). \tag{13}$$

As a result, the dual problem of (P1) is given by

$$\text{(D1)}: \max_{\omega \geq 0, \{\underline{\nu}_i \geq 0\}, \{\overline{\nu}_i \geq 0\}} g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\}). \tag{14}$$

Since (P1) is convex and satisfies the Slater's condition, strong duality holds between (P1) and (D1) [14]; thus, we can solve (P1) optimally by solving (D1) equivalently. In the following, we first obtain $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ with given $\omega \geq 0$, $\underline{\nu}_i \geq 0$, and $\overline{\nu}_i \geq 0$, $i = 1, \ldots, N$, by solving the minimization problem in (13), and then search over ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ to maximize $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ as shown in (14).

First, by denoting $\{C_i^*, D_i^*, G_i^*\}$ as the optimal solution for the problem in (13), we have the following two lemmas.

Lemma 3.1. *In order for $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ to be bounded from below, it must hold that $\nu_i \geq -\omega, i = 1, \ldots, N$.*

Lemma 3.2. *There always exists an optimal solution for problem (13) satisfying that $C_i^* \cdot D_i^* = 0, i = 1, \ldots, N$.*

Lemma 3.2 is intuitive since in general it is not optimal for the energy storage system to charge and discharge at the same time slot given efficiency factors $0 < \alpha_c < 1$ and $0 < \alpha_d < 1$.

With Lemmas 3.1 and 3.2 and by applying the Karush-Kuhn-Tucker (KKT) conditions [14], we obtain the optimal closed-form solution of (13) in the following proposition.

Proposition 3.1. *The optimal solution to problem (13) is given by*

$$C_i^* = \left[F_i^{-1}\left(\max(F_i(0), \alpha_c\omega + \alpha_c\nu_i)\right) + \Delta_i\right]^+, \tag{15}$$

$$D_i^* = \left[-F_i^{-1}\left(\max(F_i(0), \omega/\alpha_d + \nu_i/\alpha_d)\right) - \Delta_i\right]^+, \tag{16}$$

$$G_i^* = [C_i^* - D_i^* - \Delta_i]^+, \tag{17}$$

where $[x]^+ \triangleq \max(0, x)$.

From Proposition 3.1, we can obtain $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ with any set of $\omega \geq 0$, $\underline{\nu}_i \geq 0$, and $\overline{\nu}_i \geq 0$. Next, we maximize $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ over ω, $\{\underline{\nu}_i\}$, and $\{\overline{\nu}_i\}$ to solve the dual problem (D1) given in (14). Note that (D1) is always a convex optimization problem [14]; however, $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ is not necessarily differentiable. Nevertheless, it can be verified that the subgradient of $g(\omega, \{\underline{\nu}_i\}, \{\overline{\nu}_i\})$ always exists, which can be expressed as $\overline{S} - (S_1 + \alpha_c \sum_{k=1}^{N} C_k^* - 1/\alpha_d \sum_{k=1}^{N} D_k^*)$, $S_{\min} - (S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - 1/\alpha_d \sum_{k=1}^{i} D_k^*)$, and $(S_1 + \alpha_c \sum_{k=1}^{i} C_k^* - 1/\alpha_d \sum_{k=1}^{i} D_k^*) - S_{\max}$ at ω, $\underline{\nu}_i$, and $\overline{\nu}_i$, respectively, $i = 1, \ldots, N$. Therefore, (D1) can be solved by subgradient based methods such as the ellipsoid method [15], for which the optimal (dual) solution can be obtained as $\omega^\star$, $\{\underline{\nu}_i^\star\}$, and $\{\overline{\nu}_i^\star\}$.

Last, with the obtained $\omega^\star$, $\{\nu_i^\star\}$ and $\{\nu_i^\star\}$, the following proposition provides an optimal closed-form solution of (P1).

Proposition 3.2. *The optimal solution to (P1) is given by*

$$\begin{aligned}C_i^\star = \min\Big(&\big[F_i^{-1}\big(\max(F_i(0), \alpha_c\omega^\star + \alpha_c\nu_i^\star)\big) + \Delta_i\big]^+,\\ &(S_{\max} - S_i^\star)/\alpha_c\Big)\end{aligned} \tag{18}$$

$$\begin{aligned}D_i^\star = \min\Big(&\big[-F_i^{-1}\big(\max(F_i(0), \omega^\star/\alpha_d + \nu_i^\star/\alpha_d)\big) - \Delta_i\big]^+,\\ &\alpha_d(S_i^\star - S_{\min})\Big)\end{aligned} \tag{19}$$

$$G_i^\star = [C_i^\star - D_i^\star - \Delta_i]^+, \tag{20}$$

for $i = 1, \ldots, N$, where $\nu_i^\star$ is defined in (11) with the given $\{\underline{\nu}_i^\star\}$ and $\{\overline{\nu}_i^\star\}$, and $S_i^\star = S_1 + \alpha_c \sum_{k=1}^{i-1} C_k^\star - 1/\alpha_d \sum_{k=1}^{i-1} D_k^\star$.

Note that in Proposition 3.2, (18), (19) and (20) need to be computed iteratively from $i = 1$ to $i = N$.

4. ONLINE ALGORITHM

In the preceding section, we have derived the optimal off-line energy scheduling solution that provides a performance upper bound (or lower bound on the cost objective function in (P1)) for all online energy management algorithms without assuming a perfectly known net energy profile. In this section, based on the off-line solution, we propose a new online algorithm for (P1) under the practical setup of noisy net energy profile prediction, i.e., at each slot i, only the past and current net energy profile, i.e., $\Delta_1, \ldots, \Delta_i$, and the predicable part in the future net energy profile, i.e., $\overline{\Delta}_{i+1}, \ldots, \overline{\Delta}_N$, are known to the microgrid, whereas the future prediction errors $\delta_{i+1}, \ldots, \delta_N$, are unknown. Furthermore, under the special setup where the prediction errors follow a stationary stochastic process with known distribution, we propose the dynamic programming based algorithm to solve (P1), which is optimal in this particular case and thus serves as the performance upper bound for our proposed online algorithm. Nevertheless, note that our proposed online algorithm works even with non-stationary or unknown prediction errors, while dynamic programming approach in general does not.

4.1. Proposed Sliding-Window Based Online Algorithm

In this subsection, we propose our real-time energy management algorithm by applying the off-line solution for (P1) together with a "sliding-window" based sequential optimization. We define a parameter M with $1 \leq M \leq N$ as the size of the sliding window. At each slot i, we regard the online optimization as a finite-horizon off-line energy management problem over a window of M slots, with an initial energy state given by S_i, and an available net energy profile over this window as $\Delta_i, \overline{\Delta}_{i+1}, \ldots, \overline{\Delta}_{i+M-1}$.[1] For the online optimization at slot i, we denote the decision variables over the window of size M as $\{C_j^{(i)}, D_j^{(i)}, G_j^{(i)}\}_{j=1}^{M}$. Then, we formulate the online optimization problem at slot i similar to (P1), by replacing N and S_1 in (P1) by M and S_i, $f_1(\cdot), \ldots, f_N(\cdot)$ in (P1) by $f_i(\cdot), \ldots, f_{i+M-1}(\cdot)$, $\Delta_1, \Delta_2 \ldots, \Delta_N$ in (P1) by $\Delta_i, \overline{\Delta}_{i+1} \ldots, \overline{\Delta}_{i+M-1}$, $\overline{S}$ in (P1) by $\overline{S}_{i+M-1}$, and finally $\{C_i, D_i, G_i\}_{i=1}^{N}$ in (P1) by $\{C_j^{(i)}, D_j^{(i)}, G_j^{(i)}\}_{j=1}^{M}$. We set

[1] Note that this window will exceed the N-slot horizon if $i + M - 1 > N$. In this case, we use the predicted net energy profiles of the next N-slot horizon.

$\overline{S}_{i+M-1} = \overline{S}$ if $i+M-1 = N$ and $\overline{S}_{i+M-1} = 0$ otherwise. More specifically, we formulate the online problem at slot i as

$$\begin{aligned}
\min_{\{C_j^{(i)},D_j^{(i)},G_j^{(i)}\}_{j=1}^M} & \sum_{j=1}^{M} f_{i+j-1}(G_j^{(i)}) \\
\text{s.t. } & S_i + \alpha_c \sum_{k=1}^{j} C_k^{(i)} - 1/\alpha_d \sum_{k=1}^{j} D_k^{(i)} \geq S_{\min},\ j=1,\ldots,M \\
& S_i + \alpha_c \sum_{k=1}^{j} C_k^{(i)} - 1/\alpha_d \sum_{k=1}^{j} D_k^{(i)} \leq S_{\max},\ j=1,\ldots,M \\
& S_i + \alpha_c \sum_{k=1}^{M} C_k^{(i)} - 1/\alpha_d \sum_{k=1}^{M} D_k^{(i)} \geq \overline{S}_{i+M-1}, \\
& G_1^{(i)} + \Delta_i + D_1^{(i)} \geq C_1^{(i)}, \\
& G_j^{(i)} + \overline{\Delta}_j + D_j^{(i)} \geq C_j^{(i)},\ j=2,\ldots,M \\
& C_j^{(i)} \geq 0,\ D_j^{(i)} \geq 0,\ G_j^{(i)} \geq 0,\ j=1,\ldots,M. \qquad (21)
\end{aligned}$$

Problem (21) can be solved for each time slot i by the similar algorithm for solving (P1) in Section 3, via a change of variables/parameters as specified above. We denote its optimal solution as $\{C_j^{(i)\star}, D_j^{(i)\star}, G_j^{(i)\star}\}_{j=1}^M$. Accordingly, the proposed online algorithm sets the decision variables at each slot i as $C_i^{\text{online}} = C_1^{(i)\star}$, $D_i^{\text{online}} = D_1^{(i)\star}$, and $G_i^{\text{online}} = G_1^{(i)\star}$, $i = 1,\ldots,N$.

Remark 4.1. *The sliding-window length M is a key design parameter for the proposed online algorithm. Specifically, larger M is desirable for the case with small prediction error δ_i's to fully exploit the benefit of long-term prediction, while smaller M is preferable when the prediction errors are large so that the predicable net energy profile is rendered less useful as the window length is increased.*

4.2. Dynamic Programming based Online Algorithm

For comparison, we consider the conventional dynamic programming method to solve (P1) by assuming a special case where the prediction errors, $\delta_1,\ldots,\delta_N$, follow a stationary stochastic process with known distribution. Indeed, the dynamic programming based online algorithm minimizes the expected cost of the total energy drawn from the main grid, i.e., $\sum_{i=1}^N \mathbb{E}[f_i(G_i)]$, subject to (6)-(10). We thus have the following proposition.

Proposition 4.1. *Given Δ_1 and S_1, the optimal value achieved by minimizing $\sum_{i=1}^N \mathbb{E}[f_i(G_i)]$ subject to (6)-(10), is given by $J_1(\Delta_1, S_1)$, which is computed recursively based on the Bellman equations, starting from $J_N(\Delta_N, S_N)$, $J_{N-1}(\Delta_{N-1}, S_{N-1})$, and so on until $J_1(\Delta_1, S_1)$:*

$$\begin{aligned}
J_N(\Delta_N, S_N) = & \min_{C_N,D_N,G_N} f_N(G_N) \\
& \text{s.t.} \overline{S} \leq S_N + \alpha_c C_N - 1/\alpha_d D_N \leq S_{\max}, \\
& G_N + \Delta_N + D_N \geq C_N, \\
& C_N \geq 0,\ D_N \geq 0,\ G_N \geq 0. \qquad (22a)
\end{aligned}$$

$$\begin{aligned}
J_i(\Delta_i, S_i) = & \min_{C_i,D_i,G_i} f_i(G_i) + \overline{J}_{i+1}(S_i + \alpha_c C_i - 1/\alpha_d D_i) \\
& \text{s.t.} S_{\min} \leq S_i + \alpha_c C_i - 1/\alpha_d D_i \leq S_{\max}, \\
& G_i + \Delta_i + D_i \geq C_i, \\
& C_i \geq 0,\ D_i \geq 0,\ G_i \geq 0, \qquad (22b)
\end{aligned}$$

for $i = 1,\ldots,N-1$, where $\overline{J}_{i+1}(S_i + \alpha_c C_i - 1/\alpha_d D_i) = \mathbb{E}_{\Delta_{i+1}}[J_{i+1}(\Delta_{i+1}, S_i + \alpha_c C_i - 1/\alpha_d D_i)]$, and $\mathbb{E}_{\Delta_i}[\,\cdot\,]$ denotes the expectation over Δ_i. An optimal policy is accordingly given by $\pi^* = \left\{C_i^{\text{DP}}(\Delta_i,S_i), D_i^{\text{DP}}(\Delta_i,S_i), G_i^{\text{DP}}(\Delta_i,S_i)\right\}_{i=1}^N$, where $C_i^{\text{DP}}(\cdot)$, $D_i^{\text{DP}}(\cdot)$, and $G_i^{\text{DP}}(\cdot)$ is the optimal solution to (22).

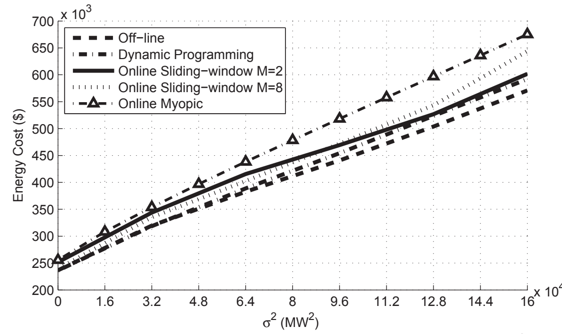

Fig. 2: Energy cost versus the prediction error variance σ^2.

Note that the dynamic programming based algorithm serves as a upper bound (or lower bound on the total cost) for any other online algorithms, provided that δ_i's follow a stationary stochastic process with known distribution as assumed.

5. NUMERICAL RESULTS

In this section, we provide numerical results to evaluate the performance of our proposed algorithms. We also consider a heuristically designed myopic online algorithm, as proposed in [12], for comparison. In this algorithm, at each slot i, the decision variables C_i ,D_i and G_i are determined based on only the energy state S_i and the net energy profile Δ_i at the current slot. Specifically, for $\Delta_i > 0$, the energy storage is charged up to $S_{\max}$; whereas for $\Delta_i < 0$, the energy storage is first discharged to meet the load demand until it reaches its minimum level ($S_{\min}$ for $i < N$ and $\overline{S}$ for $i = N$), and then the residual load (if any) is compensated by the main grid.

We consider a horizon of one week by setting $N = 168$ with each slot representing one hour. We assume a quadratic time-invariant cost function given in (5), where $a_i = 0.03125$ \$/MW^2, $b_i = 1$ \$/MW, and $c_i = 0, \forall i \in \{1,\ldots,N\}$. We also set $\alpha_c = 0.7$, $\alpha_d = 0.8$, $S_1 = 0$, $S_{\min} = 0$, $\overline{S} = 0$, and $S_{\max} = 400$ MW. The predictable net energy profile $\{\overline{\Delta}_i\}$ is taken as the hourly predicted wind energy generation over one week period (27 June, 2013 to 3 July, 2013) in the Ireland power grid [16] offset by a time-invariant demand load of 600 MW. Furthermore, we assume that δ_i's follow independent and identical Gaussian distributions with zero mean and variance σ^2. For the proposed sliding-window based online algorithm, we consider two window sizes of $M = 2$ and $M = 8$.

Fig. 2 shows the average energy cost versus the prediction error variance σ^2. It is observed that the energy cost of all considered algorithms increases with increasing σ^2, since larger σ^2 results in more substantial power fluctuations and thus the deficit in the net energy profile may not be fully compensated by the surplus due to limited storage capacity. It is also observed that our proposed sliding-window based online algorithm achieves a cost very close to the minimum cost by the optimal dynamic programming based algorithm, and also outperforms notably over the myopic online algorithm. Furthermore, it is observed that the case of $M = 8$ outperforms that of $M = 2$ when σ^2 is small, while the opposite is true when σ^2 is large. This is expected, as explained in Remark 4.1.

6. CONCLUSION

This paper studies the finite-horizon real-time energy scheduling for storage-capable microgrids to minimize the energy cost of conventional generation. Based on the optimal solution in the case of off-line optimization, we develop a new online algorithm for real-time energy management under the practical setup with noisy predicted net energy profiles subject to arbitrary errors. It is hoped that our results provide new insight to practically optimally integrating renewable energy and deploying energy storage in microgrid systems.

7. REFERENCES

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