JOINT DAY-AHEAD POWER PROCUREMENT AND LOAD SCHEDULING USING STOCHASTIC ALTERNATING DIRECTION METHOD OF MULTIPLIERS

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ABSTRACT

In this work, we consider the joint day-ahead power bidding and load scheduling problem for the smart grid system, in the presence of uncertain energy demand and renewable energy generation. We formulate the problem as a convex stochastic program in which the renewable energy generation and energy demand are modeled as random variables. The objective is to minimize the cost in the day-ahead market as well as the cost due to real-time power imbalance, by simultaneously selecting: *1)* the amount of power to buy in the day-ahead market and *2)* the schedule for the controllable load. We propose a stochastic alternating direction method of multipliers (SADMM) to solve the resulting convex stochastic optimization problem and analyze its convergence. The effectiveness of the proposed approach is demonstrated via numerical experiments using real solar power data.

Index Terms— Smart grid, day-ahead power procurement, demand side management, stochastic programming, alternating direction method of multipliers

1. INTRODUCTION

Large-scale integration of renewable energy sources (RESs) to the power system is challenging. The intermittent nature of these sources, such as wind and solar powers, makes it difficult to achieve the desired balance between energy supply and demand, especially in cases where the demand in the grid is inelastic and random. Demand side management techniques [1], which make the demand elastic by judiciously managing controllable loads, have been proposed to mitigate the effects of the supply and demand fluctuations. For instance, multistage power procurement (e.g., day-ahead, hour-ahead etc), coupled with appropriate load scheduling, has been shown to result in a substantial cost saving for the smart grid system [2].

In this paper, we focus on the power procurement decisions at the day-ahead stage. These decisions are crucial since a significant amount of power (more than 65%) is allocated and traded through the day-ahead market [3]. Moreover, if such decisions can account for the presence of RESs, then larger amount of RESs may be incorporated in the grid. However, the power bidding and load scheduling decisions are challenging because the utilities only have limited knowledge about the amounts of RES and power demand advance. There have been some works that study the day-ahead power procurement and load scheduling problem. In [4], a joint day-ahead power procurement and real-time neighborhood-wise home energy management scheme was proposed using a Markov model for deferrable loads. In [5], a game theoretic formulation was proposed for day-ahead distributed energy and storage optimization. However, RESs were not considered in these works. The work in [6] considered simultaneous day-ahead power procurement and real-time load scheduling in the presence of RESs. In particular, a two-stage stochastic optimization problem was formulated and stochastic subgradient methods were used to solve the problem. Reference [7] considered a different scheme with a quality-of-usage constraint on the load scheduling optimization. However, the load models considered in [6, 7] do not fully describe the underlying problem. This in turn makes the numerical results derived from these models less reliable since they may not reflect the actual impact on the power grid. We note that in all the above mentioned works, the random demands due to uncontrollable loads were not considered.

In this paper, we study the joint day-ahead power procurement and load scheduling problem in the presence of RESs and (uncertain) random demands. We model the controllable loads as *deferrable loads* (such as manufacturing plants, PHEV, washing machine), whose operation schedules can be deferred by the controller under user-defined deadline constraints. We assume that the requesting times and operation deadlines of the controllable loads are known in the day ahead stage. The latter can be determined by letting the customers submit such information through their home energy management systems. We formulate the day-ahead joint power procurement and load scheduling problem as a stochastic optimization problem, whereby the goal is to minimize the total cost incurred by day-ahead power bidding as well as the expected cost due to real-time power imbalance. Solving the resulting stochastic optimization problem turns out to be challenging, due to the following reasons: *1)* there is no clear stochastic model for RESs and random demands, and *2)* the problem has a linear constraints coupling all the variables. In this paper we propose a *stochastic alternating direction method of multipliers* (SADMM) to efficiently solve the considered problem. We show that the proposed SADMM achieves an $O(1/\sqrt{n})$ rate of convergence, where n is the iteration index. Numerical results based on real solar power data show that the proposed approach yields promising performance.

2. LOAD MODEL AND PROBLEM FORMULATION

In this section, we briefly review the deferrable load model and present the joint power procurement and load scheduling problem.

2.1. Deferrable Load Model

We assume that the utility serves a neighborhood with N customers (e.g., factories and residential houses), and that each customer owns

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M deferrable loads whose operation schedule can be deferred by the utility, as long as the deadlines specified by the customers are met. We use the tuple (i, m) to denote the mth load of customer i. The load profiles of the deferrable loads are known to the utility, and once turned on, their operations cannot be interrupted. Examples of deferrable loads include manufacturing plants in factories as well as house appliances such as PHEV, dish washer and washing machine.

We adopt the signal model presented in [1], which describes the load request times and load operating times by an arrival process $a_{im}(t)$ and a departure process $d_{im}(t)$. Specifically, $a_{im}(t)$ counts the number of times that the load (i, m) has been requested at time t, and $d_{im}(t)$ counts the total number of times that the load has been launched to operate at time t. Thus, if the load (i, m) is requested to operate at time t then $a_{im}(t) - a_{im}(t-1) = 1$, otherwise the difference is zero. Similarly, $d_{im}(t) - d_{im}(t-1) = 1$ if the load is scheduled to launch at time t and zero otherwise. Let $\phi_{im}(t), t = 1, \ldots, L_{im}$ denote the profile of the load (i, m), where L_{im} is the length of operation. Then the total scheduled load for customer i at time t can be expressed as [8]

$$s_i(t) = \sum_{m=1}^{M} \sum_{k=1}^{\min\{t, L_{im}\}} [d_{im}(t-k+1) - d_{im}(t-k)]\phi_{im}(k). \quad (1)$$

Further assume that the load (i, m) has a maximum tolerable delay ξ_{im}. Then the controller has to schedule the load to operate no later than ξ_{im} time slots after it is being requested. Thus we have the following constraints on $d_{im}(t)$ [8]

$$a_{im}(t - \xi_{im}) \leq d_{im}(t), \quad (2a)$$

$$d_{im}(t-1) \leq d_{im}(t) \leq a_{im}(t), \; d_{im}(t) \in \mathbb{Z}_{++}, \quad (2b)$$

where $\mathbb{Z}_{++}$ denotes the positive integer set.

In addition to the deferrable loads, we assume that each customer owns some uncontrollable loads, denoted by $v_i(t)$. Moreover, we assume that the first N_r customers have installed either solar PVs or wind mills so that they also serve as RESs, or equivalently as negative loads. Specifically, for each $i \in \{1, \cdots, N_r\}$, $e_i(t)$ units of energy are generated at time slot t. In summary, the total load in the system at time slot t is given by $\psi(t) = \sum_{i=1}^{N}(s_i(t) + v_i(t)) - \sum_{i=1}^{N_r} e_i(t)$. Note that both $v_i(t)$ and $e_i(t)$ are random and unknown beforehand.

2.2. Joint Day-ahead Power Procurement and Load Scheduling

We consider a wholesale day-ahead market scenario where an utility purchases energy from the market to serve the customers in the neighborhood. Specifically, the utility determines the power bids $\boldsymbol{p} := [p(1), p(2), \ldots, p(24)]^T$ for the 24 hours in the next day. Moreover, it makes the scheduling decision for the deferable loads at the same time, by utilizing the the request times and the deadlines (i.e., $\{a_{im}(t)\}_{i,m}$ and $\{\xi_{im}\}_{i,m}$) made available by the customers.

In addition to paying for the day-ahead bids $\boldsymbol{p}$, the utility has to pay extra if, in real time, the power supply does not match the demand (i.e., power imbalance occurs). In particular, if the supply exceeds the demand, the utility has to pay for absorbing the excessive power in a real-time market. Similarly, if the supply is insufficient, then the utility is required to purchase additional amount of power in the real-time market to cover the shortfall. Therefore, it is desirable to minimize the cost incurred in the day-ahead market as well as the cost caused by real-time power imbalance.

We assume that each day is divided into 96 periods, each of 15 minutes duration. Consequently, the power supply at a given time period t can be expressed as $p(\lceil \frac{t}{4} \rceil)$, $t = 1, \ldots, 96$. Let $\pi_s(t)$ and $\pi_p(t)$ respectively denote the price for absorbing the excessive power and the price for purchasing additional power at time t, $t = 1, \ldots, 96$. Then the real-time cost due to power imbalance is given by

$$\sum_{t=1}^{96} \left[\pi_s(t) \Big(p(\lceil \frac{t}{4} \rceil) - \psi(t) \Big)^+ + \pi_p(t) \Big(\psi(t) - p(\lceil \frac{t}{4} \rceil) \Big)^+ \right], \quad (3)$$

where $(x)^+ := \max\{x, 0\}$. Unfortunately, since the uncontrollable loads $v_i(t)$'s and the RESs $e_i(t)$'s are random and unknown *a priori*, the utility is led to minimize the following *expected cost*

$$\mathbb{E}_{\boldsymbol{v},\boldsymbol{e}} \left\{ \sum_{t=1}^{96} \left[\pi_s(t) \Big(p(\lceil \frac{t}{4} \rceil) - \psi(t) \Big)^+ + \pi_p(t) \Big(\psi(t) - p(\lceil \frac{t}{4} \rceil) \Big)^+ \right] \right\},$$

where $\boldsymbol{v} := \sum_{i=1}^{N} [v_i(1), \ldots, v_i(96)]^T$ and $\boldsymbol{e} := \sum_{i=1}^{N_r} [e_i(1), \ldots, e_i(96)]^T$. Let $C_b(\boldsymbol{p})$ denote a convex cost function for power bids $\boldsymbol{p}$. The joint day-ahead power procurement and load scheduling problem is then formulated as

$$\min_{\substack{\boldsymbol{p} \succeq \boldsymbol{0}, \\ \boldsymbol{d}_{im} \succeq \boldsymbol{0}, \forall i,m}} \; C_b(\boldsymbol{p}) + \tau \, \mathbb{E}_{\boldsymbol{v},\boldsymbol{e}} \left\{ \sum_{t=1}^{96} \left[\pi_s(t) \Big(p(\lceil \frac{t}{4} \rceil) - \psi(t) \Big)^+ + \pi_p(t) \Big(\psi(t) - p(\lceil \frac{t}{4} \rceil) \Big)^+ \right] \right\} \quad (4a)$$

$$\text{s.t } d_{im}(t-1) \leq d_{im}(t), \; \forall i, m, \quad (4b)$$

$$a_{im}(t - \xi_{im}) \leq d_{im}(t) \leq a_{im}(t), \; \forall i, m, \quad (4c)$$

where $\tau > 0$ is a cost balance parameter, and the integer constraint in (2b) is relaxed to $d_{im}(t) \geq 0$ for the sake of tractability.

Note that although problem (4) is convex, finding its optimal solution is still challenging because the expected cost in (4a) is difficult to evaluate and has no closed-form expression. Moreover, its complicated objective function and constraint set make it difficult to directly apply the existing stochastic optimization methods [9, 10].

In the next section we will propose an efficient algorithm for problem (4). To facilitate the algorithm design, let us first rewrite (4a) in a more compact form. Let $\boldsymbol{d}_i := [\boldsymbol{d}_{i1}, \ldots, \boldsymbol{d}_{iM}]^T$ for $i = 1, \ldots, N$. Then the deferrable load due to customer i in (1) is linear in $\boldsymbol{d}_i$ and can be expressed as $\boldsymbol{s}_i := [s_i(1), \ldots, s_i(96)]^T = \boldsymbol{\phi}_i \boldsymbol{d}_i$, $\forall\, i$ [8], where $\boldsymbol{\phi}_i \in \mathbb{R}^{96 \times 96M}$ is composed of $\phi_{im}(k)$, $k = 1, \ldots, L_{im}$, $m = 1, \ldots, M$. The constraint set in (4b) and (4c) can also be expressed compactly as $\boldsymbol{D}\boldsymbol{d}_i \preceq \boldsymbol{0}$, $\boldsymbol{\ell}_i \preceq \boldsymbol{d} \preceq \boldsymbol{u}_i$, $i = 1, \ldots, N$, where $\boldsymbol{D} \in \mathbb{R}^{96M \times 96M}$ is a difference matrix; and the upper and lower bounds $\boldsymbol{u}_i, \boldsymbol{\ell}_i$ are due to (4c). Consequently one can rewrite problem (4) as

$$\min_{\substack{\boldsymbol{p} \succeq \boldsymbol{0}, \\ \boldsymbol{d}_i \succeq \boldsymbol{0}, \forall i}} \; C_b(\boldsymbol{p}) + \tau \, \mathbb{E}_{\boldsymbol{v},\boldsymbol{e}} \left\{ \boldsymbol{\pi}_s^T \big(\boldsymbol{p} \otimes \boldsymbol{1}_4 + \boldsymbol{e} - \boldsymbol{v} - \sum_{i=1}^{N} \boldsymbol{\phi}_i \boldsymbol{d}_i \big)^+ + \boldsymbol{\pi}_p^T \big(\sum_{i=1}^{N} \boldsymbol{\phi}_i \boldsymbol{d}_i + \boldsymbol{v} - \boldsymbol{e} - \boldsymbol{p} \otimes \boldsymbol{1}_4 \big)^+ \right\} \quad (5a)$$

$$\text{s.t } \boldsymbol{D}\boldsymbol{d}_i \preceq \boldsymbol{0}, \boldsymbol{\ell}_i \preceq \boldsymbol{d}_i \preceq \boldsymbol{u}_i, \; i = 1, \ldots, N, \quad (5b)$$

where $\boldsymbol{\pi}_s := [\pi_s(1), \ldots, \pi_s(96)]^T$, $\boldsymbol{\pi}_p := [\pi_p(1), \ldots, \pi_p(96)]^T$, $\otimes$ denotes the Kronecker product and $\boldsymbol{1}_4$ is the all-one vector in $\mathbb{R}^4$.

3. PROPOSED STOCHASTIC ADMM

In this section, we propose a new algorithm which can handle a general class of stochastic convex problems including problem (5) as a special case. At this point we will focus on describing the algorithm in a high-level, while leaving its implementation details for solving problem (5) to the next section.

Let us consider the following general stochastic convex optimization problem

$$\min_{\boldsymbol{x}\in\mathcal{X},\boldsymbol{y}\in\mathcal{Y}} \mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right] + g(\boldsymbol{y}) \quad \text{s.t. } \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} = \boldsymbol{b}, \tag{6}$$

where $\boldsymbol{\xi}$ is a random vector whose probability distribution has a support in a set $\Xi \subset \mathbb{R}^m$, $f(\boldsymbol{x},\boldsymbol{\xi}) : \mathcal{X} \times \Xi \mapsto \mathbb{R}$ is a stochastic function, and $\mathcal{X} \subset \mathbb{R}^{n_1}$ and $\mathcal{Y} \subset \mathbb{R}^{n_2}$ are both nonempty compact convex constraint sets. We assume that the expected function $\mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right]$ is continuous and convex on $\mathcal{X}$. For instance, if the function $f(\cdot,\boldsymbol{\xi})$ is convex on $\mathcal{X}$ for every $\boldsymbol{\xi} \in \mathbb{E}$, then clearly that $\mathbb{E}_{\xi}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right]$ is convex. The fundamental challenge in solving problem (6) lies in the fact that $\mathbb{E}_{\xi}\left[f(x,\xi)\right]$ is usually difficult to evaluate [9]. In order to make the problem tractable, we make the following two standard assumptions: (**A1**) It is possible to generate independent and identically distributed (i.i.d.) samples $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots$ of realizations of $\boldsymbol{\xi}$; (**A2**) For a given input point $(\boldsymbol{x},\boldsymbol{\xi}) \in \mathcal{X} \times \Xi$, one can easily obtain a stochastic subgradient, denoted $\hat{f}(\boldsymbol{x},\boldsymbol{\xi})$, such that $\mathbb{E}_{\boldsymbol{\xi}}[\hat{f}(\boldsymbol{x},\boldsymbol{\xi})] \in \partial\mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right]$ (a subgradient of $\mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right]$).

To proceed, we define the augmented Lagrangian function of problem (6) as

$$\mathcal{L}_\rho(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}) = \mathcal{L}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}) + \frac{\rho}{2}\|\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} - \boldsymbol{b}\|^2, \tag{7}$$

where

$$\mathcal{L}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}) = \mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right] + g(\boldsymbol{y}) - \langle \boldsymbol{\lambda}, \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{y} - \boldsymbol{b}\rangle \tag{8}$$

is the Lagrangian function of (6), $\rho > 0$ is a penalty parameter, and $\boldsymbol{\lambda}$ denotes the dual variable associated with the linear constraint in (6). The standard ADMM, when used to solve problem (6), consists of the following three steps at each iteration k [11–13]

$$\boldsymbol{x}^{k+1} = \operatorname*{argmin}_{\boldsymbol{x}\in\mathcal{X}} \mathcal{L}_\rho(\boldsymbol{x},\boldsymbol{y}^k,\boldsymbol{\lambda}^k), \tag{9}$$

$$\boldsymbol{y}^{k+1} = \operatorname*{argmin}_{\boldsymbol{y}\in\mathcal{Y}} \mathcal{L}_\rho(\boldsymbol{x}^{k+1},\boldsymbol{y},\boldsymbol{\lambda}^k), \tag{10}$$

$$\boldsymbol{\lambda}^{k+1} = \boldsymbol{\lambda}^k - \rho(\boldsymbol{A}\boldsymbol{x}^{k+1} + \boldsymbol{B}\boldsymbol{y}^{k+1} - \boldsymbol{b}). \tag{11}$$

Unfortunately, the term $\mathbb{E}_{\boldsymbol{\xi}}\left[f(\boldsymbol{x},\boldsymbol{\xi})\right]$ is difficult to evaluate, so the update in (9) is hard to implement. Therefore, we use the following simple stochastic subgradient projection step to replace (9)

$$\boldsymbol{x}^{k+1} = \mathcal{P}_{\mathcal{X}}\left\{\boldsymbol{x}^k - \gamma^k\left[\hat{f}(\boldsymbol{x}^k,\boldsymbol{\xi}^k) + \rho\boldsymbol{A}^T\left(\boldsymbol{A}\boldsymbol{x}^k + \boldsymbol{B}\boldsymbol{y}^k - \boldsymbol{b} - \frac{\boldsymbol{\lambda}^k}{\rho}\right)\right]\right\}, \tag{12}$$

where $\boldsymbol{\xi}^k$ is a realization of $\boldsymbol{\xi}$ at iteration k; $\mathcal{P}_{\mathcal{X}}(\cdot)$ denotes the projection operator on $\mathcal{X}$ and $\gamma^k > 0$ is the step size. Note that under (**A1**) and (**A2**), we are able to obtain the stochastic subgradient $\hat{f}(\boldsymbol{x}^k,\boldsymbol{\xi}^k)$.

The proposed SADMM algorithm is summarized below. We should emphasize here that the SADMM algorithm proposed above is different from existing stochastic ADMM methods such as the one in [14] and the online ADMM in [15], which do not use inexact sub-gradient projection scheme to handle the subproblem of $\boldsymbol{x}^{k+1}$. The authors of [16] and [17] are among the first to introduce the stochastic components into ADMM. They design the algorithms called D-LMS and D-RLS, which combine stochastic approximation with consensus based ADMM to solve in-network adaptive processing.

Algorithm 1: Proposed Stochastic ADMM

Initialize $\boldsymbol{x}^0, \boldsymbol{y}^0, \boldsymbol{\lambda}^0, \rho$ and γ^0. For $k = 0, 1, 2, \cdots$, do
- Given a new realization $\boldsymbol{\xi}^k$, compute $\boldsymbol{x}^{k+1}$ by (12),
- Compute $\boldsymbol{y}^{k+1}$ by (10),
- Compute $\boldsymbol{\lambda}^{k+1}$ by (11).

Next we present the convergence result of Algorithm 1. Since the primal-dual optimal solution pair $(\boldsymbol{x}^\star,\boldsymbol{y}^\star,\boldsymbol{\lambda}^\star)$ of (6) is also a saddle point of the Lagrangian function (8), we have

$$\mathcal{L}\left(\boldsymbol{x}^\star,\boldsymbol{y}^\star,\boldsymbol{\lambda}\right) \le \mathcal{L}\left(\boldsymbol{x}^\star,\boldsymbol{y}^\star,\boldsymbol{\lambda}^\star\right) \le \mathcal{L}\left(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}^\star\right), \ \forall(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}).$$

Thus, a primal-dual ϵ-optimal solution pair $(\tilde{\boldsymbol{x}},\tilde{\boldsymbol{y}},\tilde{\boldsymbol{\lambda}})$ to (6) would satisfy

$$\mathcal{L}\left(\tilde{\boldsymbol{x}},\tilde{\boldsymbol{y}},\boldsymbol{\lambda}\right) - \mathcal{L}\left(\boldsymbol{x},\boldsymbol{y},\tilde{\boldsymbol{\lambda}}\right) \le \epsilon, \ \forall(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda}). \tag{13}$$

Theorem 1 *Assume $\rho > 0$ and define $\gamma^k = \min\left\{\frac{1}{2\rho\|\boldsymbol{A}\|^2}, \frac{1}{\sqrt{k+1}}\right\}$. Let $\{(\boldsymbol{x}^k,\boldsymbol{y}^k,\boldsymbol{\lambda}^k)\}$ be the sequence generated by Algorithm 1, and $\hat{\boldsymbol{x}}^n = \frac{1}{n+1}\sum_{k=0}^n \boldsymbol{x}^k$, $\hat{\boldsymbol{y}}^n = \frac{1}{n+1}\sum_{k=0}^n \boldsymbol{y}^k$, $\hat{\boldsymbol{\lambda}}^n = \frac{1}{n+1}\sum_{k=0}^n \boldsymbol{\lambda}^k$. Then for any initial point $(\boldsymbol{x}^0,\boldsymbol{y}^0,\boldsymbol{\lambda}^0)$ and any feasible point $(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\lambda})$, we have*

$$\mathcal{L}(\hat{\boldsymbol{x}}^n,\hat{\boldsymbol{y}}^n,\boldsymbol{\lambda}) - \mathcal{L}(\boldsymbol{x},\boldsymbol{y},\hat{\boldsymbol{\lambda}}^n) \le \frac{1}{n+1}\left\{M_0 + M_1\sqrt{n+1}\right\},$$

where M_0 and M_1 are constants related to the initial points as well as the size of the feasibility sets $\mathcal{X}$ and $\mathcal{Y}$.

We omit the proof here due to space limitation. From Theorem 1 we can conclude that after n iterations the SADMM can find a solution of (6) that is within $O(1/\sqrt{n})$ to optimality.

3.1. Solving (5) by Proposed SADMM

In this section, we show how the proposed SADMM can be applied to the joint power procurement and load scheduling problem (5). Let $\boldsymbol{z} := \sum_{i=1}^N \boldsymbol{\phi}_i\boldsymbol{d}_i - \boldsymbol{p}\otimes\mathbf{1}_4$. Then (5) can be expressed as

$$\begin{aligned}\min_{\substack{\boldsymbol{p}\succeq\mathbf{0},\boldsymbol{z},\\ \boldsymbol{d}_i\in\mathcal{D}_i,\boldsymbol{y}_i\succeq\mathbf{0},\forall i}} \quad & C_b(\boldsymbol{p}) + \tau\,\mathbb{E}_{\boldsymbol{v},\boldsymbol{e}}\big\{\boldsymbol{\pi}_s^T\big(\boldsymbol{e}-\boldsymbol{v}-\boldsymbol{z}\big)^+ \\ & \qquad + \boldsymbol{\pi}_p^T\big(\boldsymbol{z}+\boldsymbol{v}-\boldsymbol{e}\big)^+\big\} \\ \text{s.t.} \quad & \boldsymbol{z} = \sum_{i=1}^N \boldsymbol{\phi}_i\boldsymbol{d}_i - \boldsymbol{p}\otimes\mathbf{1}_4, \boldsymbol{D}\boldsymbol{d}_i + \boldsymbol{y}_i = \mathbf{0}, \end{aligned} \tag{14a}$$

where $\mathcal{D}_i = \{\boldsymbol{d}_i |\ \boldsymbol{\ell}_i \preceq \boldsymbol{d}_i \preceq \boldsymbol{u}_i\}$ and $\boldsymbol{y}_i$'s are slack variables.

We consider the following correspondence between (6) and (14): $\boldsymbol{\xi} = (\boldsymbol{e},\boldsymbol{v})$, $\boldsymbol{x} = [\boldsymbol{d}_1^T,\ldots,\boldsymbol{d}_N^T,\boldsymbol{z}^T]^T$, $\boldsymbol{y} = [\boldsymbol{y}_1^T,\ldots,\boldsymbol{y}_N^T,\boldsymbol{p}^T]^T$, $f(\boldsymbol{x},\boldsymbol{\xi}) = \tau(\boldsymbol{\pi}_s^T\big(\boldsymbol{e}-\boldsymbol{v}-\boldsymbol{z}\big)^+ + \boldsymbol{\pi}_p^T\big(\boldsymbol{z}+\boldsymbol{v}-\boldsymbol{e}\big)^+)$ and $g(\boldsymbol{y}) = C_b(\boldsymbol{p})$, by which $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{b}$ in (6) can be obtained accordingly. Using Algorithm 1 and the above correspondence, the updates of $\boldsymbol{d}_i^{k+1}$ and $\boldsymbol{z}^{k+1}$ are given by

$$\boldsymbol{d}_i^{k+1} = \max\left\{\boldsymbol{\ell}_i, \min\left\{\boldsymbol{d}_i^k - \gamma^k\left[\rho\boldsymbol{\phi}_i^T\left(\sum_{i=1}^N \boldsymbol{\phi}_i\boldsymbol{d}_i^k - \boldsymbol{p}^k\otimes\mathbf{1}_4 - \boldsymbol{z}^k - \frac{\boldsymbol{\lambda}^k}{\rho}\right) + \sum_{i=1}^N \sigma_i\boldsymbol{D}^T\left(\boldsymbol{D}\boldsymbol{d}_i^k + \boldsymbol{y}_i^k - \frac{\boldsymbol{\mu}_i^k}{\sigma_i}\right)\right]\right\}, \boldsymbol{u}_i\right\}, \tag{15}$$

$$\boldsymbol{z}^{k+1} = \boldsymbol{z}^k - \gamma^k\left[\tau\boldsymbol{\pi}_p\odot\max\left\{\mathbf{0},\boldsymbol{q}^k\right\} - \tau\boldsymbol{\pi}_s\odot\min\left\{\mathbf{0},\boldsymbol{q}^k\right\} - \rho\left(\sum_{i=1}^N \boldsymbol{\phi}_i\boldsymbol{d}_i^k - \boldsymbol{p}^k\otimes\mathbf{1}_4 - \boldsymbol{z}^k - \frac{\boldsymbol{\lambda}^k}{\rho}\right)\right], \tag{16}$$

where $\boldsymbol{q}^k = \text{sign}\left(\boldsymbol{z}^k + \boldsymbol{v}^k - \boldsymbol{e}^k\right)$; $\odot$ denotes the componentwise product, e.g. $a \odot b = (a_1 b_1, \cdots, a_n b_n)^T$; $\boldsymbol{\lambda}$, $\{\boldsymbol{\mu}_i\}_{i=1}^N$ are Lagrangian multipliers; $\rho, \{\sigma_i\}_{i=1}^N$ are penalty parameters corresponding to the linear constraints respectively. Further, the update of $\boldsymbol{y}$ can be decomposed into the following independent steps

$$\boldsymbol{y}_i^{k+1} = \max\left\{\boldsymbol{0}, \frac{\boldsymbol{\mu}_i^k}{\sigma_i} - \boldsymbol{D}\boldsymbol{d}_i^{k+1}\right\}, i = 1, \cdots, N, \tag{17}$$

$$\boldsymbol{p}^{k+1} = \arg\min_{\boldsymbol{p} \succeq \boldsymbol{0}} C_b(\boldsymbol{p}) + \left\|\boldsymbol{p} \otimes \boldsymbol{1}_4 - \sum_{i=1}^N \boldsymbol{\phi}_i \boldsymbol{d}_i^{k+1} + \boldsymbol{z}^{k+1} + \frac{\boldsymbol{\lambda}^k}{\rho}\right\|^2, \tag{18}$$

where (18) usually has a closed-form solution. The update of multipliers $\boldsymbol{\lambda}$, $\{\boldsymbol{\mu}_i\}_{i=1}^N$ follows directly. As seen, the proposed SADMM for solving problem (5) involves simple steps and can be implemented efficiently.

4. NUMERICAL EXPERIMENTS AND DISCUSSIONS

We consider a scenario where there are 50 residential customers ($N = 50$) each of which owns 4 deferrable loads ($M = 4$) (washing machine, dish washer, tumble dryer and PHEV) and 14 uncontrollable loads. The detailed method to generate both the deferrable and uncontrollable load signals follows [4, 18]. We also assumed that the first N_r customers respectively own a PV solar panel which generate RES to the customer. The real data of the PV signals are obtained via NREL's PV Watts calculator[1]. A periodic autoregressive (PAR) model with a period of 24 hours [19] was built based on these real data and used to generate independent realizations of $\boldsymbol{e}$.

We consider a quadratic day-ahead power bidding cost $C_b(\boldsymbol{p}) = \sum_{t=1}^{24} \pi_{LMP}(t)p(t)^2$, where the local marginal prices was obtained online[2] on September 17, 2013. For comparison, we also consider a deterministic counterpart of problem (5) where the random parameters $\boldsymbol{e}$ and $\boldsymbol{v}$ are both replaced by their sample averages:

$$\min_{\substack{\boldsymbol{p} \succeq \boldsymbol{0}, \\ \boldsymbol{d}_i \succeq \boldsymbol{0}, \forall i}} C_b(\boldsymbol{p}) + \tau\Big\{\boldsymbol{\pi}_s^T\big(\boldsymbol{p} \otimes \boldsymbol{1}_4 - \sum_{i=1}^N \boldsymbol{\phi}_i \boldsymbol{d}_i + \frac{1}{L}\sum_{\ell=1}^L (\boldsymbol{e}^\ell - \boldsymbol{v}^\ell)\big)^+$$
$$+ \boldsymbol{\pi}_p^T\big(\sum_{i=1}^N \boldsymbol{\phi}_i \boldsymbol{d}_i - \boldsymbol{p} \otimes \boldsymbol{1}_4 + \frac{1}{L}\sum_{\ell=1}^L (\boldsymbol{v}^\ell - \boldsymbol{e}^\ell)\big)^+\Big\} \tag{19a}$$

$$\text{s.t } \boldsymbol{D}\boldsymbol{d}_i \preceq \boldsymbol{0}, \boldsymbol{\ell}_i \preceq \boldsymbol{d}_i \preceq \boldsymbol{u}_i, \; i = 1, \ldots, N, \tag{19b}$$

where $L = 10,000$ is the sample number. We refer problem (19) as a certainty equivalent control (CEC) formulation. Problem (19) can be solved by the standard convex solver `CVX` [20].

Let us first consider the case where 20 out of 50 customers (40%) own PV solar panels. The parameter τ is set to 10. Fig. 1 shows the signals of day-ahead power bidding $p(t)$, RES $e(t)$, total power supply $p(t) + e(t)$ and the scheduled load (including the deferrable and uncontrollable loads $\sum_{i=1}^{50}(s_i(t) + v_i(t))$) by the proposed SADMM. A total of 10000 realizations of real data $\boldsymbol{e}$ and $\boldsymbol{v}$ are used when implementing SADMM. From this figure, we observe that the proposed SADMM can provide promising power balance. We also observe from this figure that the day-ahead bidding is less around the noon since the solar PVs can generate more RES at noon. To understand how the penetration level of RESs to the grid can affect the day-ahead and power imbalance costs, respectively, we plot in Fig. 2 the optimal costs of problem (5) obtained by proposed SADMM versus the RES penetration level N_r/N. The costs due to the solutions of the CEC formulation (19) are also presented. Firstly, we observe that the proposed SADMM can yield lower costs than the CEC formulation. Secondly, it can be seen that the day-ahead costs can consistently decrease as there are more customers equipped with solar PVs. The power imbalance costs can decrease as there are less than 40% customers with solar powers, but the costs can increase if there is a high level of RES penetration. This is reasonable as more RESs induce more uncertainties and large deviation between the day-ahead prediction and real-time true signals.

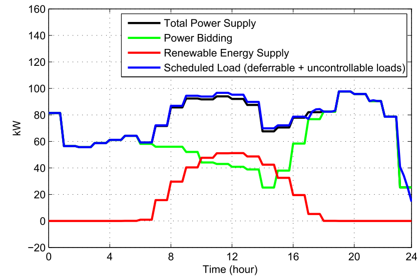

Fig. 1. Power supply and scheduled load by proposed SADMM

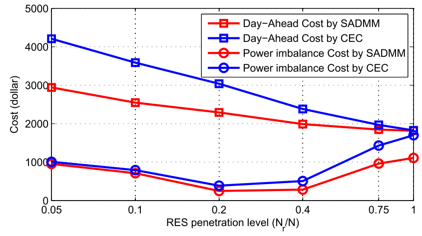

Fig. 2. Day-ahead and power imbalance costs versus RES penetration level (%). The parameter τ was set to 10.

Finally, in Table 1, we compare the proposed SADMM with the CEC formulation (19) for different values of the penalty parameters τ. Each value shown in the table represents the total cost of problem (5) by the solutions of the proposed SADMM and the CEC formulation (19). We observe that the proposed SADMM can consistently yield lower costs and the gap between the CEC formulation increases as the penalty parameter τ decreases.

Table 1. Comparison between Proposed SADMM and CEC

τ	30	10	1	0.1	0.01
CEC (19)	146.9	172.5	1486.3	2248.5	2961.5
SADMM	138.8	159.4	1220.6	1841.9	1912.5

In summary, we have formulated the joint day-ahead power procurement and load scheduling problem as a stochastic optimization problem (4) and proposed an SADMM algorithm that can effectively mitigate the effects of the uncertainties caused by RESs and uncontrollable random demands. The theoretical convergence of the proposed SADMM has been analyzed. The numerical results have shown that SADMM can yield promising performance for the considered problem. For future research, we will conduct more numerical tests with a larger number of customers. Moreover, we wish to extend the day-ahead framework to multi-timescale power procurement and load scheduling.

[1] `http://www.nrel.gov/rredc/pvwatts/`

[2] `http://www2.ameren.com/RetailEnergy/realtimeprices.aspx`

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