OPTIMAL MIMO BROADCASTING OVER TIME-VARYING WIRELESS CHANNELS FOR ENERGY HARVESTING TRANSMITTER WITH NON-IDEAL CIRCUIT POWER

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ABSTRACT

We develop a novel approach to optimal broadcast scheduling over time-varying channels for an energy harvesting transmitter with finite-capacity battery and non-ideal circuit power consumption. Relying on the convex optimization tools, a low-complexity algorithm is proposed to obtain the optimal transmission policy that maximizes the weighted sum-throughput for multi-input multi-output (MIMO) broadcast channels. Our approach provides the optimal benchmark to all the practical schemes for energy harvesting powered broadcasting with non-ideal circuit power.

Index Terms— Energy harvesting, broadcast channel, uplink-downlink duality, non-ideal circuit power, time-varying channel.

1. INTRODUCTION

Energy harvesting powered wireless communications have attracted growing interest in recent years. Different from the traditional communication systems, the intermittent nature of most energy harvesting sources causes bursty energy availability at the transmitter. Taking into account this new energy availability constraint, the optimal transmission policies for energy harvesting nodes were investigated for time-invariant and time-varying point-to-point channels [1–4], as well as time-invariant broadcast channels [5–7].

All the works [1–7] assumed an ideal (zero) circuit-power model. However, in practical short-range wireless transmissions, there exists non-negligible circuit power consumption; e.g., the AC/DC converter and RF amplifier can contribute to a significant portion of energy consumption when transmit-power P > 0. For many energy harvesting (e.g. sensor) applications, such a non-ideal circuit power consumption needs to be taken into account. This issue was only partly addressed for *point-to-point* transmissions in [8,9].

In this paper, we develop a novel approach to optimal *broadcast* scheduling over time-varying channels for energy harvesting transmitter with non-ideal circuit power consumption. Unlike the single-antenna broadcast models (where non-ideal circuit-power was not considered) in [5–7], we consider a more general multi-input multi-output (MIMO) model where both the transmitter and the receivers can have multiple antennas. Assuming full harvested energy and channel information, we develop the optimal policy that maximizes the system throughput. Using the uplink-downlink duality [10] and a "nested optimization" method [7,8], we convert the optimal user-power scheduling for a MIMO broadcast channel into an optimal sum-power allocation problem for an equivalent "point-topoint" link. Relying on convex optimization principles, an efficient algorithm is then proposed to obtain the optimal solution with a low computational complexity. With the optimal sum-power obtained, the optimal user-power scheduling strategy can be subsequently determined. Our approach provides the optimal benchmark to all the practical schemes for energy harvesting powered broadcasting.

The rest of the paper is organized as follows. Section II describes the broadcast channel, energy-harvesting, and circuit-power models. Section III presents the proposed approach to optimal MIMO broadcasting over time-varying channels. Section IV evaluates our scheme with numerical examples, followed by the conclusion.

2. MODELING PRELIMINARIES

Consider a MIMO broadcast channel (BC) where the transmitter has N_t antennas and each of the K users has N_r antennas. Let $H_k(t) \in \mathbb{C}^{N_r \times N_t}$ denote the channel coefficient matrix from the transmitter to the kth user, $k = 1, \ldots, K$, at time t. The received complex-baseband signal at user k is given by:

$$\boldsymbol{y}_k(t) = \boldsymbol{H}_k(t)\boldsymbol{x}(t) + \boldsymbol{z}_k(t), \qquad (1)$$

where $\boldsymbol{x}(t)$ is the transmitted vector signal, and $\boldsymbol{z}_k(t)$ is the additive random noise. We assume without loss of generality (w.l.o.g.) that $\boldsymbol{z}_k(t)$ is complex-Gaussian with zero mean and covariance matrix \boldsymbol{I} (the identity matrix of size N_r). The transmitted signal is the sum of the signal transmitted to individual users: $\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{x}_k(t)$. The overall transmit covariance matrix is then $\sum_{k=1}^{K} \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^{\dagger}] :=$ $\sum_{k=1}^{K} \Gamma_k$, where the positive semidefinite Γ_k (denoted by $\Gamma_k \succeq 0$) is the transmit covariance matrix for user k. The total transmit-power is given by $\sum_{k=1}^{K} \operatorname{tr}(\Gamma_k)$.

Assuming perfect channel state information at the transmitter, the capacity of the MIMO BC can be achieved by dirty paper coding (DPC). Let $\mathcal{H} := \{H_1, \ldots, H_K\}$. With a total transmit-power P, the capacity region achieved by the DPC for MIMO BC is given by:

$$\mathcal{C}_{\mathrm{BC}}(P;\mathcal{H}) = Co\left(\cup_{\pi}\mathcal{R}_{\pi}(P;\mathcal{H})\right),\tag{2}$$

where $Co(\cdot)$ denotes the convex hull, the union is over all permutation π of the user index set $\{1, 2, \ldots, K\}$, and

$$\mathcal{R}_{\pi}(P;\mathcal{H}) = \bigcup_{\{\boldsymbol{\Gamma}_k: \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{\Gamma}_k) \leq P\}} \{(r_1, \dots, r_K):$$
$$r_{\pi(u)} \leq \log \frac{|\boldsymbol{I} + \sum_{u=1}^{k} \boldsymbol{H}_{\pi(u)} \boldsymbol{\Gamma}_{\pi(u)} \boldsymbol{H}_{\pi(u)}^{\dagger}|}{|\boldsymbol{I} + \sum_{u=1}^{k-1} \boldsymbol{H}_{\pi(u)} \boldsymbol{\Gamma}_{\pi(u)} \boldsymbol{H}_{\pi(u)}^{\dagger}|}, \ \forall k\}.$$

Here $|\cdot|$ denotes the determinant operator.

2.1. Uplink-Downlink Duality

The BC capacity region can be alternatively characterized by the capacity regions of a set of "dual" multi-access channels (MACs). In the dual MAC, the received signal is:

$$\boldsymbol{y}(t) = \sum_{k=1}^{K} \boldsymbol{H}_{k}^{\dagger}(t) \boldsymbol{x}_{k}(t) + \boldsymbol{z}(t), \qquad (3)$$

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where $\boldsymbol{x}_k(t)$ is the transmitted signal by user k, and $\boldsymbol{z}(t)$ is additive complex-Gaussian with zero mean and covariance matrix I. Let $Q_k := \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^{\dagger}] \succeq 0$ denote the transmit covariance matrix for user k, and let $\boldsymbol{P} := [P_1, \dots, P_K]$ collect the transmit-power budgets for users. For a given P, the MAC capacity region is:

$$\begin{aligned} \mathcal{C}_{\text{MAC}}(\boldsymbol{P}; \mathcal{H}^{\mathsf{T}}) &= \cup_{\{\boldsymbol{Q}_k: \, \text{tr}(\boldsymbol{Q}_k) \leq P_k, \, \forall k\}} \{(r_1, \dots, r_K): \\ \sum_{k \in \mathcal{S}} r_k \leq \log |\boldsymbol{I} + \sum_{k \in \mathcal{S}} \boldsymbol{H}_k^{\dagger} \boldsymbol{Q}_k \boldsymbol{H}_k|, \, \forall \mathcal{S} \subseteq \{1, \dots, K\}\}. \end{aligned}$$

The uplink-downlink duality in [10] proves that the BC capacity region (2) equals the union of the above MAC capacity regions corresponding to all power vectors \boldsymbol{P} satisfying $\sum_{k=1}^{K} P_k \leq P$; i.e.,

$$\mathcal{C}_{\mathrm{BC}}(P;\mathcal{H}) = \bigcup_{\{\boldsymbol{P}: \sum_{k=1}^{K} P_k \leq P\}} \mathcal{C}_{\mathrm{MAC}}(\boldsymbol{P};\mathcal{H}^{\dagger}) := \mathcal{C}_{\mathrm{MAC}}(P;\mathcal{H}^{\dagger}).$$

2.2. Energy Harvesting Process

Suppose that the transmitter harvests renewable energy from the nature and then stores it in the battery for future use. The energy state changes when an energy arrival occurs. For a time-varying channel, the channel state \mathcal{H} can in general change between two energy arrivals. We assume that there are N + 1 channel or energy state changing instants $0 = t_0 < \cdots < t_N = T$ over the entire transmission interval [0, T]. We call the interval between two consecutive state changing instants an epoch, whose length is denoted by $L_i = t_i - t_{i-1}, i = 1, ..., N$, with $\sum_{i=1}^N L_i = T$. The energy arrival process is described by a set $\{(a_m, E_m)\}, m = 0, 1, ..., M$, where M denotes the number of energy arrivals, a_m denotes the epoch index of the *m*th energy arrival time, and E_m denotes the amount of energy arriving at time t_{a_m} . Let E_{\max} denote the capacity of the rechargeable battery. It is clear that $0 < E_i \leq E_{\text{max}}$, $i = 1, \ldots, M$; otherwise, the excess energy $E_i - E_{\text{max}}$ cannot be stored in the battery and we set w.l.o.g. $E_i = E_{\text{max}}$ in such cases.

2.3. Non-Ideal Circuit Power Consumption

In short-range wireless networks, circuit power consumption (e.g. RF amplifier) is non-negligible when transmit-power P > 0. On the other hand, the transmitter could turn off the power amplifier to avoid/reduce circuit power consumption [11] if no data transmission occurs. We refer the transmitter status with a transmit-power P > 0and that with P = 0 as the "on" and "off" modes. In practical systems, the circuit power in the "off" mode is usually much smaller than that in the "on" mode. Hence, we assume w.l.o.g. the circuitpower during the "on" and "off" modes to be $\alpha > 0$ Watts and 0 Watt, respectively. The total consumed power P_{total} is then [8]:

$$P_{total} = \begin{cases} P + \alpha, & P > 0, \\ 0, & P = 0. \end{cases}$$
(4)

3. BROADCASTING WITH NON-IDEAL CIRCUIT POWER

Let $\mathcal{H}_i := \{ \boldsymbol{H}_{i,1}, \dots, \boldsymbol{H}_{i,K} \}$ denote the channel per epoch i. To account for the non-ideal circuit power, the transmitter can be turned on for only a portion of an epoch [8]. Hence, we let $l_i \leq L_i$ denote the length of the "on" period, and $\Gamma_i = [\Gamma_{i,1}, \ldots, \Gamma_{i,K}]$ collect the transmit-covariance matrices during "on" period at the ith epoch. Define $\boldsymbol{l} := [l_1, \ldots, l_N]$ and $\boldsymbol{\Gamma} := [\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_N]$. With the covariance matrices in Γ_i , the sum transmit-power is then P_i := $\sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{\Gamma}_{i,k}). \text{ Let } r_k^B(\boldsymbol{\Gamma}_i) \text{ denote the achieved rate for user } k, \text{ and } r^B(\boldsymbol{\Gamma}_i) := [r_1^B(\boldsymbol{\Gamma}_i), \dots, r_K^B(\boldsymbol{\Gamma}_i)]. \text{ Define } E_m^a := \sum_{j=0}^{m-1} E_j, E_m^c := (\sum_{j=0}^m E_j - E_{\max})^+, \forall m, \text{ where } (x)^+ := \max\{x, 0\} \text{ [2].}$ Provided a priority weight vector $\boldsymbol{w} := [w_1, \ldots, w_K]$, we aim to maximize the weighted-sum of user throughput:

$$\max_{\mathbf{\Gamma}, \mathbf{l}} \sum_{k=1}^{K} [w_k \sum_{i=1}^{N} (r_k^B(\mathbf{\Gamma}_i) l_i)]$$
s. t. (C1):
$$\sum_{i=1}^{a_m} [(P_i + \alpha) l_i] \leq E_m^a, \quad m = 1, \dots, M,$$
(C2):
$$\sum_{i=1}^{a_m} [(P_i + \alpha) l_i] \geq E_m^c, \quad m = 1, \dots, M,$$

$$\boldsymbol{r}^B(\mathbf{\Gamma}_i) \in \mathcal{C}_{\mathsf{BC}}(P_i; \mathcal{H}_i), \quad 0 \leq l_i \leq L_i, \quad i = 1, \dots, N.$$
(5)

Here, (C1) are the causality constraints: the total energy consumed up to t_{a_m} cannot exceed the energy E_m^a that has been accumulatively harvested so far; whereas (C2) are the non-overflow constraints: the transmitter should at least consume the energy E_m^c to prevent energy overflow (i.e., waste) at t_{a_m} due to finite battery capacity E_{\max} .

Define $R(P_i; \mathcal{H}_i) := \max_{\mathbf{r}^B(\Gamma_i) \in \mathcal{C}_{\mathsf{BC}}(P_i; \mathcal{H}_i)} \sum_{k=1}^K w_k r_k^B(\Gamma_i).$ Relying on the uplink-downlink duality, we can show that:¹

Lemma 1 The strictly concave function $R(P_i, \mathcal{H}_i)$ can be alternatively obtained by the optimal value of the problem:

$$\max_{\mathbf{Q}_{i,k} \succeq 0} \sum_{k=1}^{K} (w_{\pi(k)} - w_{\pi(k+1)}) \log |\mathbf{I}| + \sum_{u=1}^{k} \mathbf{H}_{i,\pi(u)}^{\dagger} \mathbf{Q}_{i,\pi(u)} \mathbf{H}_{i,\pi(u)}$$

s. t. $\sum_{k=1}^{K} tr(\mathbf{Q}_{i,k}) = P_i$ (6)

where π is the permutation of user indices $\{1, \ldots, K\}$ such that $w_{\pi(1)} \ge w_{\pi(2)} \ge \cdots \ge w_{\pi(K)}$, and $w_{\pi(K+1)} = 0$.

Let $\mathbf{P} := [P_1, \ldots, P_N]$. Using $R(P_i)$, we can convert the optimal broadcasting problem (5) into an optimal sum-power allocation problem for an equivalent "point-to-point" link:

$$\max_{\substack{P, l \\ P, l}} \sum_{i=1}^{N} [R(P_i; \mathcal{H}_i)l_i] \\
\text{s. t.} \quad E_m^c \leq \sum_{i=1}^{a_m} [(P_i + \alpha)l_i] \leq E_m^a, \ m = 1, \dots, M, \quad (7) \\
P_i \geq 0, \ 0 \leq l_i \leq L_i, \ i = 1, \dots, N.$$

3.1. Convex Reformulation and Optimality Conditions

The non-convex problem (7) can be reformulated into a convex problem through a change of variables. Define $\Psi_i := (P_i + \alpha)l_i$. With $\Psi := [\Psi_1, \ldots, \Psi_N]$, we can rewrite (7) into:

$$\max_{\boldsymbol{\Psi}, l} \sum_{i=1}^{N} [R(\frac{\Psi_i}{l_i} - \alpha; \mathcal{H}_i)l_i]
s. t. \quad E_m^c \leq \sum_{i=1}^{a_m} \Psi_i \leq E_m^a, \quad m = 1, \dots, M, \quad (8)
\quad \Psi_i \geq \alpha l_i, \quad 0 \leq l_i \leq L_i, \quad i = 1, \dots, N.$$

Since $R(P_i; \mathcal{H}_i)$ is a concave function of P_i per Lemma 1, it can be shown that $R(\frac{\Psi_i}{l_i} - \alpha; \mathcal{H}_i)l_i$ is a jointly concave function of (Ψ_i, l_i) [12]. It then follows that (8) is a convex problem.

Let $\Lambda := \{\lambda_m, \mu_m, m = 1, \dots, M\}$ where λ_m and μ_m denote the Lagrange multipliers associated with the causality and nonoverflow constraints. The Lagrangian of (8) is given by:

$$\mathcal{L}(\boldsymbol{P},\boldsymbol{l},\boldsymbol{\Lambda}) = C(\boldsymbol{\Lambda}) + \sum_{i=1}^{N} \left[R(\frac{\Psi_i}{l_i} - \alpha; \mathcal{H}_i) l_i - \sum_{m=i}^{M} (\lambda_m - \mu_m) \Psi_i \right]$$

where $C(\mathbf{\Lambda}) := \sum_{m=1}^{M} (\lambda_m E_m^a) - \sum_{m=1}^{M} (\mu_m E_m^c)$. Let $(\mathbf{\Psi}^*, \mathbf{l}^*)$ denote the optimal solution for (8) and $\mathbf{\Lambda}^*$ the optimal Lagrange multiplier. Define $\theta_i := \sum_{m=i}^{M} \lambda_m^* - \sum_{m=i}^{M} \mu_m^*$. By the Karush-Kuhn-Tucker (KKT) conditions [12], we have: $\forall i$,

$$(\Psi_i^*, l_i^*) = \arg \max \left[R(\Psi_i/l_i - \alpha; \mathcal{H}_i) l_i - \theta_i \Psi_i \right]$$

s. t. $\Psi_i > \alpha l_i, \ 0 < l_i < L_i.$ (9)

¹Proof for lemmas and theorem can be found in our journal version [13].

In addition, the non-negative λ_m^* and μ_m^* satisfy the complementary slackness conditions: $\forall m$,

$$\begin{cases} \lambda_m^* = 0, & \text{if } \sum_{i=1}^{a_m} \Psi_i^* < E_m^a, \\ \sum_{i=1}^{a_m} \Psi_i^* = E_m^a, & \text{if } \lambda_m^* > 0; \end{cases}$$
(10)

$$\begin{cases} \mu_m^* = 0, \text{ if } \sum_{i=1}^{a_m} \Psi_i^* > E_m^c, \\ \sum_{i=1}^{a_m} \Psi_i^* = E_m^c, \text{ if } \mu_m^* > 0. \end{cases}$$
(11)

Let $P_i^* = \Psi_i^*/l_i^* - \alpha$ if $l_i^* > 0$, and P_i^* take any arbitrary non-negative value when $l_i^* = 0$, $\forall i$. It is clear that $(\mathbf{P}^*, \mathbf{l}^*)$ is the optimal solution of (7). From (9)–(11), the sufficient and necessary optimality conditions for (7) are: $\forall i, \forall m$,

$$(P_i^*, l_i^*) = \arg\max l_i [R(P_i; \mathcal{H}_i) - \theta_i (P_i + \alpha)]$$

s. t. $P_i > 0, \ 0 < l_i < L_i;$ (12)

$$\begin{pmatrix} \lambda_m^* = 0, & \text{if } \sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* < E_m^a, \\ \sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* = E_m^a, & \text{if } \lambda_m^* > 0; \end{cases}$$
(13)

$$\mu_m^* = 0, \text{ if } \sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* > E_m^c, \\
 \sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* = E_m^c, \text{ if } \mu_m^* > 0.$$
(14)

3.2. Energy Efficiency Maximizing Power

Before solving (7), we consider finding the bits-per-Joule energy efficiency (EE) maximizing powers: $\forall i$

$$P_{ee}(\mathcal{H}_i) = \arg\max_{P>0} R(P; \mathcal{H}_i)/(P+\alpha).$$
(15)

With an auxiliary variable b, we write the problem (15) into:

$$\max_{P \ge 0, b} b, \quad \text{s. t. } R(P; \mathcal{H}_i)/(P + \alpha) \ge b. \tag{16}$$

For a fixed *b*, consider the problem:

$$g(b) = \max_{P \ge 0} R(P; \mathcal{H}_i) - bP.$$
(17)

Define $f(\boldsymbol{Q}_i) = \sum_{k=1}^{K} (w_{\pi(k)} - w_{\pi(k+1)}) \log |\boldsymbol{I} + \sum_{u=1}^{k} \boldsymbol{H}_{i,\pi(u)}^{\dagger}$ $\boldsymbol{Q}_{i,\pi(u)} \boldsymbol{H}_{i,\pi(u)}|$. Then by Lemma 1, (17) is equivalent to:

$$g(b) = \max_{P \ge 0} \left[\max_{\sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{Q}_{i,k}) = P} f(\boldsymbol{Q}_i) - bP \right]$$

= $\max_{\boldsymbol{Q}_i \succeq 0} \left[f(\boldsymbol{Q}_i) - b \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{Q}_{i,k}) \right].$ (18)

The problem in (18) is a convex program, which can be efficiently solved by the Matlab CVX solver in polynomial time. Building on the solution for (18), a bisectional search can be implemented to solve (16) by finding the maximal *b* with $g(b) \ge \alpha b$; the solution $P_{ee}(\mathcal{H}_i)$ for (15) is in turn obtained.

3.3. Optimal Sum-Power Allocation

For any $l_i^* > 0$, it follows from (12) that

$$P_i^* = \arg \max_{P_i \ge 0} \left[R(P_i; \mathcal{H}_i) - \theta_i P_i \right].$$
(19)

Let $R'(P_i; \mathcal{H}_i)$ be the first derivative of $R(P_i; \mathcal{H}_i)$. We clearly have: $R'(P_i; \mathcal{H}_i) = \theta_i$, leading to : $P_i^* = R'^{-1}(\theta_i; \mathcal{H}_i)$, where R'^{-1} denotes the inverse function of R'. To obtain P_i^* , we need to solve: $\max_{P_i \ge 0} [R(P_i; \mathcal{H}_i) - \theta_i P_i]$. By Lemma 1, this is equivalent to solving the convex program (18) with $b = \theta_i$. Let $Q_i^*(\theta_i; \mathcal{H}_i)$ denote its optimal solution. Then

$$R'^{-1}(\theta_i; \mathcal{H}_i) = \sum_{k=1}^{K} \operatorname{tr}(\boldsymbol{Q}_{i,k}^*(\theta_i; \mathcal{H}_i)).$$
(20)

Note that for the single-antenna single-user case, the optimal power is given by the celebrated water-filling form: $P_i^* = (1/\theta_i - 1/|h_i|^2)^+$, where $1/\theta_i$ serves as a water-level. For this reason, we call $\omega_i := 1/\theta_i$ a generalized "water-level".

Substituting $R'(P_i^*; \mathcal{H}_i) = \theta_i$ into (12) implies:

$$l_{i}^{*} = \arg \max_{0 \le l_{i} \le L_{i}} l_{i} [R(P_{i}^{*}; \mathcal{H}_{i}) - R'(P_{i}^{*}; \mathcal{H}_{i})(P_{i}^{*} + \alpha)].$$
(21)

Relying on (21), we can show that:

Lemma 2 The optimal transmission policy for (7) can only adopt either one of the following three ("off", "on-off" and "on") strategies per epoch $i:(i) l_i^* = 0$, (ii) $P_i^* = P_{ee}(\mathcal{H}_i), l_i^* \leq L_i$, or (iii) $P_i^* > P_{ee}(\mathcal{H}_i), l_i^* = L_i$.

Lemma 2 states that the optimal sum-power allocation depends on the EE maximizing power $P_{ee}(\mathcal{H}_i)$; i.e. any transmit-power $P_i < P_{ee}(\mathcal{H}_i)$ per epoch *i* should not be adopted in the optimal policy. In fact, since $P_{ee}(\mathcal{H}_i)$ maximizes the bits-per-Joule EE, we can show that any transmission strategy with a $P_i < P_{ee}(\mathcal{H}_i)$ over an epoch can be dominated by an on-off transmission with $P_{ee}(\mathcal{H}_i)$, which yields a higher throughput reward with the same energy expenditure.

It is easy to see that $P_i^* = R'^{-1}(1/\omega_i; \mathcal{H}_i)$ increases as the water-level ω_i increases. Using this fact and the complementary slackness conditions (13)–(14), we can establish that:

Lemma 3 In the optimal policy, the powers for epoches i with $l_i > 0$ are given by a water-filling form: $P_i^* = R'^{-1}(1/\omega_i; \mathcal{H}_i)$, where the water-level ω_i increases after a t_{a_m} with $\sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* = E_m^a$, and it decreases after a t_{a_m} with $\sum_{i=1}^{a_m} (P_i^* + \alpha) l_i^* = E_m^c$.

The structures of the optimal policy revealed in Lemmas 2–3 imply that we should implement a water-level based "clipped string-tautening" approach to find the optimal solution for (7). To this end, let $\omega_{a_m}^+$ or $\omega_{a_m}^-$ denote the constant water-level to make the *m*th causality or non-overflow constraint become tight at t_{a_m} . Given an invariant water-level ω before t_{a_m} , the power per epoch *i* is given by $P_i^* = R'^{-1}(1/\omega; \mathcal{H}_i)$ if $l_i^* > 0$. On the other hand, it follows from Lemma 2 that we have $P_i^* \geq P_{ee}(\mathcal{H}_i)$ if $l_i^* > 0$. Define:

$$\omega_{ee}(\mathcal{H}_i) := 1/R'(P_{ee}(\mathcal{H}_i); \mathcal{H}_i), \quad \forall i.$$
(22)

This implies that we can have $l_i^* > 0$ only when $\omega \ge \omega_{ee}(\mathcal{H}_i)$. With the water-level ω , the optimal strategy per epoch *i* is then:

$$\begin{cases}
l_i^* = 0, & \text{if } \omega < \omega_{ee}(\mathcal{H}_i), \\
P_i^* = P_{ee}(\mathcal{H}_i), \ l_i^* \leq L_i, & \text{if } \omega = \omega_{ee}(\mathcal{H}_i), \\
P_i^* = R'^{-1}(1/\omega; \mathcal{H}_i), \ l_i^* = L_i, & \text{if } \omega > \omega_{ee}(\mathcal{H}_i).
\end{cases}$$
(23)

Let $\Psi_i(\omega; \mathcal{H}_i) = (P_i^* + \alpha)l_i^*$. By (23), we have: (i) $\Psi_i(\omega; \mathcal{H}_i) = 0$, if $\omega < \omega_{ee}(\mathcal{H}_i)$; (ii) $\Psi_i(\omega; \mathcal{H}_i) \in [0, (P_{ee}(\mathcal{H}_i) + \alpha)L_i]$, if $\omega = \omega_{ee}(\mathcal{H}_i)$; and (iii) $\Psi_i(\omega; \mathcal{H}_i) = (R'^{-1}(1/\omega; \mathcal{H}_i) + \alpha)L_i$, if $\omega < \omega_{ee}(\mathcal{H}_i)$. Thus, the values of $\omega_{a_m}^+$ and $\omega_{a_m}^-$, $m = 1, \ldots, M$, can be calculated by solving the equations: $\forall m$

$$\sum_{i=1}^{a_m} \Psi_i(\omega_{a_m}^+; \mathcal{H}_i) = E_m^a; \quad \sum_{i=1}^{a_m} \Psi_i(\omega_{a_m}^-; \mathcal{H}_i) = E_m^c.$$
(24)

Since $\sum_{i=1}^{a_m} \Psi_i(\omega; \mathcal{H}_i)$ is increasing in ω , the equations in (24) can be solved by a bisectional search.

With $\omega_{a_m}^-$ and $\omega_{a_m}^+$ obtained, we are ready for implementation of the proposed "clipped water-tautening" approach to solve (7). Let $\boldsymbol{\mathcal{E}} := \{E_0, E_1, \ldots, E_M\}, \, \boldsymbol{\mathcal{L}} := \{L_1, \ldots, L_N\}$ and $\boldsymbol{\mathcal{H}} := \{\mathcal{H}_1, \ldots, \mathcal{H}_N\}$. The optimal $\{\boldsymbol{P}^*, \boldsymbol{l}^*\}$ for (7) can be obtained by calling Procedure ScheduleW($\boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{L}}, \boldsymbol{\mathcal{H}}$) in Algorithm 1.

Algorithm 1 "Clipped Water-Tautening"

procedure SCHEDULEW(energy set \mathcal{E} , length set \mathcal{L} , channel set \mathcal{H}) $N_{\text{offset}} = 0$, and $P_i^* = 0$, $l_i^* = 0$, $\forall i$; 1: 2 3: while $N_{\text{offset}} < N' \mathbf{do}$ 4: $[\tau, \omega, E] = \text{FirstChangeW}(\mathcal{E}, \mathcal{L}, \mathcal{H});$ for i = 1 to τ do $P_{N_{\text{offset}}+i}^* = R'^{-1}(1/\omega; \mathcal{H}_i)$ given by (20); 5: 6: 7: if $\omega > \omega_{ee}(\mathcal{H}_i)$ then $l^*_{N_{\text{offset}}+i} = L_i$; end if 8: end for if there exists i_{ee} with $\omega_{ee}(\mathcal{H}_{i_{ee}}) = \omega$ then $l_{N_{\text{offset}}+i_{ee}} = \frac{E - \sum_{i=1}^{T}, i \neq i_{ee}}{P_{ee}(\mathcal{H}_{i_{ee}}) + \alpha};$ 9: 10: end if 11: 12: Noffset $N_{\text{offset}} + \tau$, and update $\boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{L}}, \boldsymbol{\mathcal{H}};$ 13. end while 14: end procedure 15: function $[\tau, \omega, E]$ =FIRSTCHANGEW($\mathcal{E}, \mathcal{L}, \mathcal{H}$) $\omega^-=0, \omega^+=\infty, \tau^-=\tau^+=0; M_e=|\boldsymbol{\mathcal{E}}|-1;$ for m=1 to M_e do 16: 17: $\begin{array}{l} m = 1 \text{ or } m_{e} \text{ to } \mathbf{u}^{-} \\ \text{calculate } \omega_{a_{m}}^{+} \text{ and } \omega_{a_{m}}^{-} \text{ by solving equations in (24);} \\ \text{if } \omega_{a_{m}}^{+} \leq \omega^{+} \text{ then } \tau^{+} = a_{m}, \omega^{+} = \omega_{a_{m}}^{+}, E^{+} = E_{m}^{a}; \text{ end if} \\ \text{if } \omega_{a_{m}}^{-} \geq \omega^{-} \text{ then } \tau^{-} = a_{m}, \omega^{-} = \omega_{a_{m}}^{-}, E^{-} = E_{m}^{c}; \text{ end if} \end{array}$ 18: 19: 20 $\begin{array}{l} \text{if } \omega^- > \omega^+ \, \& \, \tau^- < \tau^+ \, \text{then} \\ \text{return } \tau = \tau^-, \, \omega = \omega^-, \, E = E^-; \\ \text{else if } (\omega^- \ge \omega^+ \, \& \, \tau^- \ge \tau^+) \, \text{or} \, (\tau^+ = M_e) \, \text{then} \end{array}$ 21: 22: 23: 24: return $\tau = \tau^+, \omega = \omega^+, E = E^+;$ 25: end if 26: end for 27: end function

The key component in Algorithm 1 is Function FirstChangeW, which determines the first water-level changing time t_{τ} and the water-level ω used before it. The two candidate water-levels are updated as: $\omega^+ = \min_{a_m \leq i} \omega^+_{a_m}$ and $\omega^- = \max_{a_m \leq i} \omega^-_{a_m}$, which are in fact the maximum and minimum value for an invariant water-level to satisfy all the causality and non-overflow constraints before t_i , respectively. If we have $\omega^+ < \omega^-$ at a certain t_i , then the water-level can satisfy all the causality and non-overflow constraints so far. The first water-level changing time can be obtained by comparing τ^- and τ^+ to see which type of constraint first becomes tight. When the returned $t_i < T$, Function FirstChangeW can be reused for a new (\mathcal{E} , \mathcal{L} , \mathcal{H}) system over the remaining time to find the next water-level changing time and the next water-level.

The global optimality of the proposed Algorithm 1 is formally stated in the following theorem:

Theorem 1 Algorithm 1 computes the optimal transmission policy for (7) with a worst-case complexity $\mathcal{O}(M^2)$.

3.4. Optimal Broadcasting Solution

With the optimal sum-power P_i^* per epoch *i*, the optimal uplink matrices $Q_{i,k}(P_i^*)$ for (6) can be efficiently obtained in polynomial time. Then by uplink-downlink duality, we can obtain the optimal downlink matrices $\Gamma_{i,k}^*$ from $Q_{i,k}(P_i^*)$ as follows [10]. Define:

$$\begin{aligned} \boldsymbol{A}_{i,k} &= \boldsymbol{I} + \boldsymbol{H}_{i,\pi(k)} (\sum_{u=1}^{k-1} \Gamma^*_{i,\pi(u)}) \boldsymbol{H}^{\dagger}_{i,\pi(k)}, \\ \boldsymbol{B}_{i,k} &= \boldsymbol{I} + \sum_{u=k+1}^{K} (\boldsymbol{H}^{\dagger}_{i,\pi(u)} \boldsymbol{Q}^*_{i,\pi(u)} (P^*_i) \boldsymbol{H}_{i,\pi(u)}). \end{aligned}$$

Using $A_{i,k}$ and $B_{i,k}$, we have: $\forall k = 1, \ldots, K$,

$$\boldsymbol{\Gamma}_{i,\pi(k)}^{*} = \boldsymbol{B}_{i,k}^{-\frac{1}{2}} \boldsymbol{F}_{i,k} \boldsymbol{G}_{i,k}^{\dagger} \boldsymbol{A}_{i,k}^{\frac{1}{2}} \boldsymbol{Q}_{i,\pi(k)}^{*} (P_{i}^{*}) \boldsymbol{A}_{i,k}^{\frac{1}{2}} \boldsymbol{G}_{i,k} \boldsymbol{F}_{i,k}^{\dagger} \boldsymbol{B}_{i,k}^{-\frac{1}{2}}$$

where we obtain the matrices $F_{i,k}$ and $G_{i,k}$ by decomposing the effective channel using SVD: $B_{i,k}^{-\frac{1}{2}}H_{i,\pi(k)}^{\dagger}A_{i,k}^{-\frac{1}{2}} = F_{i,k}S_iG_{i,k}^{\dagger}$ with a square and diagonal S_i .



Fig. 1. Average throughput versus energy arrival rate λ_e .

Note that $\Gamma_{i,\pi(1)}^* = B_{i,1}^{-\frac{1}{2}} F_{i,1} G_1^{\dagger} Q_{i,\pi(1)}^* (P_i^*) G_{i,1} F_{i,1}^{\dagger} B_{i,1}^{-\frac{1}{2}}$, which only requires the knowledge of $Q_{i,k}(P_i^*)$. When calculating $\Gamma_{i,\pi(k)}, k > 1$, we need $A_{i,k}$ whose calculation requires the knowledge of previously calculated $\Gamma_{i,\pi(u)}, u = 1, \ldots, k - 1$. In such a sequential way, all $\Gamma_{i,k}^*$ can be determined.

Overall, the bisectional search to find $P_{ee}(\mathcal{H}_i)$ is geometrically fast. With P_i^* , transmit-covariance matrices $\Gamma_{i,k}^*$, $\forall k$, can be obtained through convex programming tools in polynomial time. It then readily follows that the optimal broadcasting schedule { Γ^* , l^* } for (5) can be obtained with a worst-case complexity $\mathcal{O}(KM^2)$.

4. NUMERICAL RESULTS

For a time-varying MIMO broadcast channel with K = 2 users, consider data transmission over T = 50 seconds. The weight vector w = [1, 1], and each element in channel matrix $H_{i,k}$, k = 1, 2, is a zero-mean complex Gaussian random variable with unit variance. The battery capacity of the transmitter is $E_{\text{max}} = 100$ Joules, and non-ideal circuit-power $\alpha = 3$ Watts. Assume a stochastic energy harvesting setup modeled by the compound Poisson process with mean λ_e [8]. The amount of energy in each arrival is assumed to be independent and uniformly distributed with mean 50 Joules. Fig. 1 shows the average throughput versus λ_e for a simulated wireless link when (N_t, N_r) is set to (2, 2) and (4, 2), respectively. Each result is obtained as the average of 10 randomly generated trial cases. The revealed structure of the optimal clipped water-tautening (CWT) policy also motivates us to develop an online CWT scheme which assume only causal knowledge of the harvested energy and channel realizations in [13]. In addition to the offline CWT algorithm, we include the performance of the online CWT policy and offline EE-SE policy (which we generalize to the MIMO BC case) in [8]. Without taking into account the finite battery-capacity, the EE-SE policy incurs throughput loss for all λ_e values. It is shown that the online CWT scheme achieves a reasonably good throughput for all λ_e values, and even outperforms the offline EE-SE policy for large λ_e . It is also clearly observed that the sum-rate is significantly improved for the MIMO BC as the number of transmit-antennas N_t doubles.

5. CONCLUSIONS

We proposed a novel approach to optimal transmission policy for energy-harvesting powered MIMO BC with non-ideal circuit power consumption over time-varying wireless channels. An efficient algorithm was developed to find the optimal solution with a low computational complexity.

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