OPTIMAL DECISION THRESHOLD FOR EIGENVALUE-BASED SPECTRUM SENSING TECHNIQUES

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ABSTRACT

This paper investigates optimization of the sensing threshold that minimizes the total error rate (i.e., the sum of the probabilities of false alarm and missed detection) of eigenvalue-based spectrum sensing techniques for multiple-antenna cognitive radio networks. Four techniques are investigated, which are maximum eigenvalue detection (MED), maximum minimum eigenvalue (MME) detection, energy with minimum eigenvalue (EME) detection, and the generalized likelihood ratio test (GLRT) detection. The contribution of this paper is of four parts. Firstly, we present the derivative of the matrix-variate confluent hypergeometric function, which is required for the MED case. Secondly, we derive the probabilities of false alarm for both cases MME and EME detection. Thirdly, we derive the probability of missed detection for the GLRT detector. Finally, we provide the exact expressions required to obtain the optimal sensing thresholds for all cases. The simulation results reveal that for all the investigated cases the chosen optimal sensing thresholds achieve the minimum total error rate.

Index Terms— Cognitive radio, eigenvalue-based spectrum sensing, optimal decision threshold, matrix-variate confluent hypergeometric function

1. INTRODUCTION

The problem of efficient utilization of the frequency spectrum has become a major issue within the research community. This is due to the well-known fact that the available spectrum is an overcrowded natural shared resource, and the fact that there is an overgrowing demand for communication services. However, under the current spectrum management schemes, unlicensed (secondary) users are prohibited to access the frequency bands which are allocated exclusively to licensed (primary) users [1]. An efficient solution to spectrum scarcity is cognitive radio which allows secondary (or cognitive) users (SUs) to detect and exploit spectrum holes whenever the primary users (PUs) are absent. However, during exploitation of such spectrum opportunities, SUs should not cause harmful interference to PUs. Therefore, spectrum sensing is an essential and crucial part for the implementation of the cognitive radio technology. An efficient spectrum sensing scheme should allow detection of PUs even under very low SNR conditions. For instance, the IEEE 802.22 standard for Wireless Regional Area Networks (WRANs) requires detection of Digital Television (DTV) signals using minimum probabilities of false alarm P_{fa} and missed detection P_{m} of 0.1.

Among many spectrum sensing techniques, eigenvalue-based detection is highly accurate and can provide minimal probability of error without much prior knowledge of the PU signals [2]. Four methods exist for eigenvalue-based detection which include MED [3], MME detector [4], EME detector [2] and GLRT detector [5]. The major objective of spectrum sensing is finding an optimal threshold that minimizes sensing errors.

Maintaining a minimal probability of missed detection serves the interests of PUs, as it protects them from secondary interference. On the other hand, maintaining a minimal probability of false alarm creates more chances for the SUs to access more channels. Since studying P_{m} is more difficult and complicated, the previous work [2–6] on eigenvalue-based detection have mainly focused on minimizing P_{fa} to benefit secondary users only. The optimization of energy detector (ED) was studied in [7], but the ED was sensitive to noise uncertainty which had no influences on eigenvalue-based detectors. Therefore, in this paper we focus on P_{m} and the total error rate P_{te} of eigenvalue-based detection to benefit both PUs and SUs simultaneously. This enables protection of PUs from secondary interference, while SUs can have higher probability of accessing the available frequency bands. Therefore, in this study we investigate the optimization of eigenvalue-based detection by finding the optimal decision threshold that minimizes P_{te}. Specifically, we derive the exact expressions of P_{te} for MED with arbitrary number of receive antennas and MME, EME and GLRT detectors with 2 receive antennas and find the optimal decision thresholds to minimize the P_{te}.

The remainder of this paper is organized as follows. Section II describes the system model, section III investigates optimization of the sensing threshold for the four methods, Section IV presents the simulation results, and finally Section V concludes the paper.

2. MULTIPLE-ANTENNA SPECTRUM SENSING MODEL

Let us consider a generic spectrum sensing scenario within a MIMO system that consists of p transmit antennas and m receive antennas. The transmitted signals are assumed to have a length of n samples, where $n > m$. Let $\mathcal{H}_0$ (PU is absent) and $\mathcal{H}_1$ (PU is present) denote the null and the alternate hypotheses respectively. During the sensing period, the received signal $\boldsymbol{Y}$ is given by

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{Y} = \mathbf{V}, \\ \mathcal{H}_1 &: \mathbf{Y} = \mathbf{H}\,\mathbf{X} + \mathbf{V}, \end{aligned} \tag{1}$$

where $\boldsymbol{Y} \in \mathbb{C}^{m\times n}$ is the matrix of the received signal samples at the secondary receivers, $\boldsymbol{H} \in \mathbb{C}^{m\times p}$ is a complex Gaussian channel matrix. The matrix $\boldsymbol{V} \in \mathbb{C}^{m\times n}$ represents an additive Gaussian white noise, which is independent of the channel, and the circularly symmetric complex Gaussian matrix of the transmitted signals $\boldsymbol{X} \in \mathbb{C}^{p\times n}$. Specifically, $\boldsymbol{V} \sim \mathcal{CN}(\mathbf{0}, \delta_v^2 \boldsymbol{I_m})$ and $\boldsymbol{X} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$. Therefore, the received signals by the cognitive antennas should be an $m \times n$ complex Gaussian matrix. Henceforth, the matrix $\boldsymbol{W} = \boldsymbol{Y} \times \boldsymbol{Y}'$ is a complex central Wishart matrix, i.e.,

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$\boldsymbol{W} \sim \mathcal{CW}_m(n, \boldsymbol{\Sigma_m})$, where the covariance matrix $\boldsymbol{\Sigma_m}$ is given by

$$\boldsymbol{\Sigma_m} = \begin{cases} \delta_v^2 \boldsymbol{I_m}, & \mathcal{H}_0, \\ \boldsymbol{H}\boldsymbol{\Sigma}\boldsymbol{H}^H + \delta_v^2 \boldsymbol{I_m}, & \mathcal{H}_1. \end{cases}$$

Let T denotes the test statistic used for eigenvalue-based spectrum sensing. The formulation of T can vary depending on the different types of eigenvalue-based detectors. Let $\lambda_{\max} = \lambda_1 > \cdots > \lambda_{\rm m} = \lambda_{\min}$ denote the eigenvalues of the matrix $\boldsymbol{W}$. Hence, the test statistics for MED, MME detection, EME detection and GLRT detection are given by: $T_{\rm MED} = \lambda_{\max}$, $T_{\rm MME} = \frac{\lambda_{\max}}{\lambda_{\min}}$, $T_{\rm EME} = \frac{\sum_{i=1}^m \lambda_i}{\lambda_{\min}}$ and $T_{\rm GLRT} = \frac{\lambda_{\max}}{\sum_{i=1}^m \lambda_i}$ respectively. The focus of this paper is to find the optimal sensing threshold $r_{\rm opt}$ that minimizes the total error rate $P_{\rm te}$ which is determined by

$$P_{\rm te} = P_{\rm fa} + P_{\rm m}, \tag{2}$$

where $P_{\rm fa}$ and $P_{\rm m}$ denote the probabilities of false alarm and missed detection respectively, and are given by

$$P_{\rm fa} = \text{Prob}\left[T > r | \mathcal{H}_0\right] = \int_r^{\infty} f_0(t) \mathrm{d}t, \tag{3}$$

$$P_{\rm m} = \text{Prob}\left[T < r | \mathcal{H}_1\right] = \int_{-\infty}^{r} f_1(t) \mathrm{d}t, \tag{4}$$

where $f_0(t)$ and $f_1(t)$ denote the probability distribution functions (PDFs) of the test statistic T under $\mathcal{H}_0$ and $\mathcal{H}_1$ respectively and r stands for the decision threshold.

An appropriate decision threshold r can achieve the desired value of probability of false alarm $P_{\rm fa}$, but the corresponding probability of missed detection $P_{\rm m}$ may not meet an acceptable value concurrently. Therefore, in practical applications, it is essential to find an optimal decision threshold $r_{\rm opt}$ that minimizes the total error rate. This can make both $P_{\rm fa}$ and $P_{\rm m}$ meet the acceptable values simultaneously.

3. OPTIMIZATION OF THE DECISION THRESHOLD

In this section, we investigate the optimal sensing thresholds for the cases of MED (with arbitrary number of receive antennas) and MME, EME and GLRT detectors (with 2 receive antennas), and assuming the case of one PU with a single antenna, i.e., $p = 1$. For each type of the eigenvalue-based detectors, and in order to find the optimal decision threshold $r_{\rm opt}$ and the total error rate $P_{\rm te}$, the exact cumulative distribution function (CDF) (or the PDF) of the corresponding test statistic is required for both cases of $\mathcal{H}_0$ and $\mathcal{H}_1$.

3.1. Optimal Threshold for the MED

In this subsection, we derive the exact expression of the total error rate $P_{\rm te}$ for MED assuming an arbitrary number of receive antennas. Also, the corresponding optimal decision threshold is analysed by using the derivative of the total error rate. However, in order to get the derivative of $P_{\rm te}$, we have to solve the issue of finding the derivative of a confluent hypergeometric function with a matrix argument. This case was not studied in the literature and only the derivative of the confluent hypergeometric function with a scalar argument is studied before.

Firstly, let us start with the case of $\mathcal{H}_0$, where in this case $\boldsymbol{W} \sim \mathcal{CW}_m(n, \sigma_v^2 \boldsymbol{I_m})$. Hence, by making use of the CDF of the maximum eigenvalues of an uncorrelated complex central Wishart matrix, the probability of false alarm is given by [8]

$$P_{\rm fa}(x) = 1 - \frac{C\Gamma_m(m)}{C\Gamma_m(n+m)} x^{mn} {}_1F_1(n; n+m; -x\sigma_v^2 \boldsymbol{I}), \tag{5}$$

where ${}_1F_1(.;.;.)$ is the matrix-variate confluent hypergeometric function and

$$C\Gamma_m(a) = \pi^{\frac{m(m-1)}{2}} \prod_{k=1}^{m} \Gamma(a - k + 1),$$

where $\Gamma(.)$ is the gamma function, $\Re(a) > (m-1) + k_1$, $k = k_1 + k_2 + \cdots + k_m$ and $k_1 \geq \cdots \geq k_m \geq 0$. On the other hand, considering the hypothesis $\mathcal{H}_1$, $\boldsymbol{W} \sim \mathcal{CW}_m(n, \boldsymbol{\Sigma_m})$. Therefore, the probability of missed detection is given by [8]

$$P_{\rm m}(x) = \frac{C\Gamma_m(m) x^{mn} {}_1F_1(n; n+m; -x{\boldsymbol{\Sigma_m}}^{-1})}{C\Gamma_m(n+m)(\det \boldsymbol{\Sigma_m})^n}. \tag{6}$$

Henceforth, the total error rate $P_{\rm te}$ for MED with an arbitrary number of receive antennas can be obtained by using (2). It can be seen that $P_{\rm te}(x)$ is a convex function, which indicates it has a global minimum value for x. Also, this implies that there exists one and only one value of x which minimizes $P_{\rm te}(x)$. Therefore, the optimal decision threshold $r_{\rm opt}$ is given by

$$r_{\rm opt} = \arg\min_x \; P_{\rm te}(x), \tag{7}$$

which can be achieved when the derivative of the total error rate is $\frac{\mathrm{d}P_{\rm te}(x)}{\mathrm{d}x} = 0$. The corresponding derivative is obtained as

$$\begin{aligned} \frac{\mathrm{d}P_{\rm te}(x)}{\mathrm{d}x} =& \frac{C\Gamma_m(m) {}_1F_1(n; n+m; -x{\boldsymbol{\Sigma_m}}^{-1})}{C\Gamma_m(n+m)(\det \boldsymbol{\Sigma_m})^n} \\ & \times \Big(mnx^{mn-1} + x^{mn}\Big(\mathrm{tr}(\boldsymbol{C}) - \mathrm{tr}(\boldsymbol{D})\Big)\Big) \\ & - \frac{C\Gamma_m(m) {}_1F_1(n; n+m; -x\sigma_v^2 \boldsymbol{I_m})}{C\Gamma_m(n+m)} \\ & \times \Big(mnx^{mn-1} + x^{mn}\Big(\mathrm{tr}(\boldsymbol{A}) - \mathrm{tr}(\boldsymbol{B})\Big)\Big), \end{aligned} \tag{8}$$

where the matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are given by

$$\begin{aligned} \boldsymbol{A} &= \Big((-x\sigma_v^2)^{k_j+m-j}\Big)^{-1}\Big(\sigma_v^2(-k_j - m + j)(-\sigma_v^2 x)^{k_j+m-j-1}\Big), \\ \boldsymbol{B} &= \Big((-x\sigma_v^2)^{m-j}\Big)^{-1}\Big(\sigma_v^2(j-m)(-\sigma_v^2 x)^{m-j-1}\Big), \\ \boldsymbol{C} &= \Big((x\beta_i)^{k_j+m-j}\Big)^{-1}\Big(\beta_i(k_j + m - j)(\beta_i x)^{k_j+m-j-1}\Big), \\ \boldsymbol{D} &= \Big((x\beta_i)^{m-j}\Big)^{-1}\Big(\beta_i(m-j)(\beta_i x)^{m-j-1}\Big), \end{aligned}$$

where $\beta_1, ..., \beta_m$ are the eigenvalues of the matrix $-{\boldsymbol{\Sigma_m}}^{-1}$. The solution to $\frac{\mathrm{d}P_{\rm te}(x)}{\mathrm{d}x} = 0$ can be evaluated numerically and represents the desired optimal decision threshold.

3.2. Optimal Threshold for the MME

In this subsection, we derive the exact expression of the total error rate for the MME detector assuming $m = 2$ and show the required steps to obtain the optimal decision threshold. Assuming $\mathcal{H}_0$, the PDF of the test statistic $T_{\rm MME}$ for the case of $m = 2$ is given by [4]

$$f_{\rm MME0}(x) = \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)}\Big(1 - \frac{1}{x}\Big)^2\Big(\frac{1}{x}\Big)^n\Big(1 + \frac{1}{x}\Big)^{-2n}. \tag{9}$$

By making use of the previous equation, (9), we derive the probability of false alarm as

$$\begin{aligned} P_{\rm fa}(x) =& \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)}\Big[\Delta_1(2n, n-1, x) - 2\Delta_1(2n, n, x) \\ & + \Delta_1(2n, n+1, x)\Big), \end{aligned} \tag{10}$$

$$f_{\mathrm{MME1}}(x) = \frac{(\delta_1\delta_2)^{1-n}}{(n-1)!(n-2)!(\delta_2-\delta_1)}\Big(\Delta_2(n-1,n-1,1/\delta_1,1/\delta_2,x) - \Delta_2(n-2,n,1/\delta_1,1/\delta_2,x) - \Delta_2(n-1,n-1,1/\delta_2,1/\delta_1,x) + \Delta_2(n-2,n,1/\delta_2,1/\delta_1,x)\Big), \quad x > 1 \tag{22}$$

$$S(y) = \Delta_3(n-1,n-1,1/\delta_1,1/\delta_2,y) - \Delta_3(n-2,n,1/\delta_1,1/\delta_2,y) - \Delta_3(n-1,n-1,1/\delta_2,1/\delta_1,y) + \Delta_3(n-2,n,1/\delta_2,1/\delta_1,y) \tag{23}$$

where $\Delta_1(a,b,y) = \frac{y^{-b}}{b}\,{}_2F_1(a,b;b+1,-y^{-1})$, and ${}_2F_1(.,.;.;.)$ is the Gaussian hypergeometric function. Assuming $\mathcal{H}_1$, $\boldsymbol{W} \sim \mathcal{CW}_2(n,\boldsymbol{\Sigma_2})$, and making use of the results in [8] yields the PDF of the test statistic T_{MME} associated with $\boldsymbol{W}$ as given by (22), where

$$\Delta_2(a,b,c,d,y) = -(b-1)!\sum_{k=0}^{b-1}\frac{(a+k)!y^{k-1}(kd-c(a+1)y)}{k!c^{b-k}(cy+d)^{a+k+2}},$$

and $\delta_1 > \delta_2$ are the non-zero ordered eigenvalues of $\boldsymbol{\Sigma_2}$. The probability of missed detection of the MME detector can then be obtained by

$$P_{\mathrm{m}}(x) = \frac{(\delta_1\delta_2)^{1-n}}{(n-1)!(n-2)!(\delta_2-\delta_1)}\Big(S(x)-S(1)\Big), \tag{11}$$

where $x > 1$, and $S(y)$ is given by (23) and Δ_3 is given by

$$\Delta_3(a,b,c,d,y) = (b-1)! \times\left(\frac{a!}{c^b d^{a+1}} - \sum_{k=0}^{b-1}\frac{(a+k)!y^k}{k!c^{b-k}(cy+d)^{a+k+1}}\right).$$

Thus, the exact expression of the total error rate P_{te} can be directly obtained from summing (10) and (11). In order to obtain the optimal decision threshold r_{opt}, the derivative $\frac{\mathrm{d}P_{\mathrm{te}}(x)}{\mathrm{d}x}$ is

$$\frac{\mathrm{d}P_{\mathrm{te}}(x)}{\mathrm{d}x} = \frac{\mathrm{d}(P_{\mathrm{fa}}(x)+P_{\mathrm{m}}(x))}{\mathrm{d}x} = f_{\mathrm{MME1}}(x) - f_{\mathrm{MME0}}(x).$$

The solution to $f_{\mathrm{MME1}}(x) - f_{\mathrm{MME0}}(x) = 0$ can be evaluated numerically and is the desired optimal decision threshold.

3.3. Optimal Threshold for the EME

In this subsection, the case of the EME detector is considered assuming 2 receive antennas. Since under $\mathcal{H}_0$, $\boldsymbol{W} \sim \mathcal{CW}_2(n,\sigma_v^2\boldsymbol{I_m})$ then the PDF of the test statistic T_{EME} is given by [9]

$$f_{\mathrm{EME0}}(x) = \frac{\Gamma(2n)x^{-2n}(x-1)^{n-2}(x-2)^2}{\Gamma(n)\Gamma(n-1)}, x \geq 2. \tag{12}$$

Hence, we derive the probability of false alarm P_{fa} as

$$P_{\mathrm{fa}}(x) = 1 - \frac{\Gamma(2n)}{\Gamma(n)\Gamma(n-1)}\sum_{k=0}^{n-2}\binom{n-2}{k}(-1)^{n-2-k} \times\Big(\frac{x^{k-2n+3}-2^{k-2n+3}}{k-2n+3} - 4\frac{x^{k-2n+2}-2^{k-2n+2}}{k-2n+2} + 4\frac{x^{k-2n+1}-2^{k-2n+1}}{k-2n+1}\Big). \tag{13}$$

Assuming $\mathcal{H}_1$, and using [10], the PDF of the threshold T_{EME}, that is associated with the dual correlated complex central Wishart matrix $\boldsymbol{W} \sim \mathcal{CW}_2(n,\boldsymbol{\Sigma_2})$ is given by

$$f_{\mathrm{EME1}}(x) = \frac{\Gamma(2n-1)(\delta_1\delta_2)^{1-n}(x-2)}{\Gamma(n)\Gamma(n-1)(\delta_1-\delta_2)}(x-1)^{n-2} \times\Big(\Delta_4(\delta_1,\delta_2,x)^{1-2n} - \Delta_4(\delta_2,\delta_1,x)^{1-2n}\Big), \tag{14}$$

where $x \geq 2$, $\Delta_4(\delta_1,\delta_2,x) = \frac{x-1}{\delta_1}+\frac{1}{\delta_2}$, $\Delta_4(\delta_2,\delta_1,x) = \frac{x-1}{\delta_2}+\frac{1}{\delta_1}$ and δ_1 and δ_2 are the non-zero ordered eigenvalues of $\boldsymbol{\Sigma_2}$, where $\delta_1 > \delta_2$. Hence, using the CDF in [10] , P_{m} is given by

$$P_{\mathrm{m}}(x) = \frac{\Gamma(2n-1)(\delta_1\delta_2)^{1-n}}{\Gamma(n)\Gamma(n-1)(\delta_1-\delta_2)} \times\left\{\Delta_5\left(\frac{\delta_1-\delta_2}{\delta_1},\frac{\delta_2}{\delta_1},\delta_2,x\right) - \Delta_5\left(\frac{\delta_2-\delta_1}{\delta_2},\frac{\delta_1}{\delta_2},\delta_1,x\right) - \Delta_5\left(\frac{\delta_1-\delta_2}{\delta_1},\frac{\delta_2}{\delta_1},\delta_2,2\right) + \Delta_5\left(\frac{\delta_2-\delta_1}{\delta_2},\frac{\delta_1}{\delta_2},\delta_1,2\right)\right\}, \tag{15}$$

where $x \geq 2$, and

$$\Delta_5(a,b,c,x) = \Big(\frac{c}{a}\Big)^{2n-1}\sum_{k=0}^{n-2}\binom{n-2}{k}(-1)^k \times\Big\{P(n-k-1,b,a,2n-1,x) - 2P(n-k-2,b,a,2n-1,x)\Big\}, \tag{16}$$

$$P(n,b,a,m,x) = \frac{x^{n+1}}{n+1}\,{}_2F_1\Big(m,n+1;n+2;-x\frac{b}{a}\Big). \tag{17}$$

Thus, the derivative of the total error rate P_{te} is $\frac{\mathrm{d}P_{\mathrm{te}}(x)}{\mathrm{d}x} = \frac{\mathrm{d}(P_{\mathrm{fa}}(x)+P_{\mathrm{m}}(x))}{\mathrm{d}x} = f_{\mathrm{EME1}}(x) - f_{\mathrm{EME0}}(x)$ and the solution to $f_{\mathrm{EME1}}(x) - f_{\mathrm{EME0}}(x) = 0$ is the desired optimal decision threshold.

3.4. Optimal Threshold for the GLRT

This subsection investigates the case of the optimal threshold for the GLRT detector with 2 receive antennas. Starting with the null hypothesis, and since in this case $\boldsymbol{W} \sim \mathcal{CW}_2(n,\sigma_v^2\boldsymbol{I_m})$, then using the PDF and CDF in [5], the PDF of the test statistic T_{GLRT} and the probability of false alarm are given by

$$f_{\mathrm{GLRT0}}(x) = \frac{\Gamma(2n)(2x-1)^2}{\Gamma(n)\Gamma(n-1)\left(x(1-x)\right)^{2-n}}, \tag{18}$$

$$P_{\mathrm{fa}}(x) = 1 - \frac{\Gamma(2n)\left(\Delta_6(n,x) - \Delta_6(n,\frac{1}{2})\right)}{\Gamma(n)\Gamma(n-1)}, \tag{19}$$

where $\frac{1}{2} \leq x \leq 1$ and

$$\Delta_6(n,y) = \sum_{k=0}^{n-2}\binom{n-2}{k}(-1)^{n-2-k} \times\Big\{\frac{y^{2n-k-3}}{2n-k-3} - 4\frac{y^{2n-k-2}}{2n-k-2} + 4\frac{y^{2n-k-1}}{2n-k-1}\Big\}. \tag{20}$$

Assuming $\mathcal{H}_1$, we have $\boldsymbol{W} \sim \mathcal{CW}_2(n,\boldsymbol{\Sigma_2})$, and therefore the PDF of T_{GLRT} is given by [5]

$$f_{\mathrm{GLRT1}}(x) = \frac{\left(x\delta_2+(1-x)\delta_1\right)^{1-2n} - \left(x\delta_1+(1-x)\delta_2\right)^{1-2n}}{\Gamma(n)\Gamma(n-1)(\delta_1-\delta_2)\left(x(1-x)\right)^{2-n}} \times\Gamma(2n-1)(\delta_1\delta_2)^n(2x-1), \quad \frac{1}{2} \leq x \leq 1, \tag{21}$$

$$
\begin{aligned}
P_{\mathrm{m}}(x) =& C_{\mathrm{g}} \sum_{k=0}^{n-2} \binom{n-2}{k} (-1)^k \Bigg[2\Bigg(\Delta_7\left(\delta_1, 2n-1, k+n, \delta_1-\delta_2, x\right) - \Delta_7\left(\delta_1, 2n-1, k+n, \delta_1-\delta_2, \frac{1}{2}\right)\Bigg) \\
& -2\Bigg(\Delta_7\left(\delta_2, 2n-1, k+n, \delta_2-\delta_1, x\right) - \Delta_7\left(\delta_2, 2n-1, k+n, \delta_2-\delta_1, \frac{1}{2}\right)\Bigg) \\
& -\Bigg(\Delta_7(\delta_1, 2n-1, n+k-1, \delta_1-\delta_2, x) - \Delta_7\left(\delta_1, 2n-1, n+k-1, \delta_1-\delta_2, \frac{1}{2}\right)\Bigg) \\
& +\Bigg(\Delta_7(\delta_2, 2n-1, n+k-1, \delta_2-\delta_1, x) - \Delta_7\left(\delta_2, 2n-1, n+k-1, \delta_2-\delta_1, \frac{1}{2}\right)\Bigg)\Bigg], \frac{1}{2} \le x \le 1. \quad (24)
\end{aligned}
$$

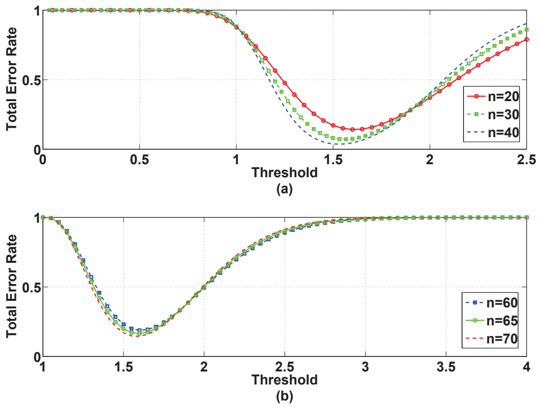

Fig. 1. The total error rate of MED (m=2, SNR=0 dB) (a) and MME detector (m=2, SNR=-5 dB) (b) vs decision threshold

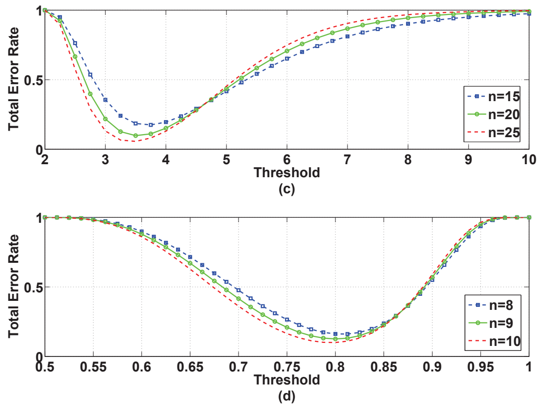

Fig. 2. The total error rate of EME (c) and GLRT (d) detector (m=2, SNR=0 dB) vs decision threshold

where $\delta_1 > \delta_2$ are the non-zero ordered eigenvalues of $\mathbf{\Sigma_2}$. By applying the binomial expansion and then integrating we derive the probability of missed detection as given by (24), where $C_{\mathrm{g}} = \frac{\Gamma(2n-1)(\delta_1\delta_2)^n}{\Gamma(n)\Gamma(n-1)(\delta_1-\delta_2)}$ and $\Delta_7(a,b,c,d,t) = \frac{a^{-b}t^c}{c}\ {}_2F_1(b,c;1+c;\frac{d}{a}t)$. Therefore, the derivative of the total error rate can be obtained as $\frac{\mathrm{d}P_{\mathrm{te}}(x)}{\mathrm{d}x} = \frac{\mathrm{d}(P_{\mathrm{fa}}(x)+P_{\mathrm{m}}(x))}{\mathrm{d}x} = f_{\mathrm{GLRT1}}(x) - f_{\mathrm{GLRT0}}(x)$ and the solution to $f_{\mathrm{GLRT1}}(x) - f_{\mathrm{GLRT0}}(x) = 0$ is the desired optimal decision threshold.

4. SIMULATION RESULTS

In this section, some simulation results are presented and discussed. For simplicity but without loss of generality, we assume a spectrum sensing scenario with 2 receive antennas and one transmit antenna. The calculation of matrix-variate confluent complex hypergeometric function utilizes the MATLAB codes provided by [11]. Starting with the case of MED, Figure (a) of Fig. 1 depicts the total error rate versus the decision threshold. The results are plotted for different cases of number of transmitted signal samples, $n = \{20, 30, 40\}$, and an SNR of 0dB using $\mathbf{\Sigma}_{\boldsymbol{m}} = \left(\begin{smallmatrix} 2 & 0 \\ 0 & 1.3 \end{smallmatrix}\right)$. The results show the corresponding optimal decision threshold is $r_{\mathrm{opt}} = \{1.60, 1.57, 1.53\}$ with a total error rate of $P_{\mathrm{te}} = \{0.14, 0.07, 0.04\}$.

Figure (b) of Fig. 1 shows the total error rate of MME detection versus the threshold. The simulation parameters are $n = \{60, 65, 70\}$, SNR $= -5$dB and $\mathbf{\Sigma}_{\boldsymbol{m}} = \left(\begin{smallmatrix} 1.7 & -0.3+0.2i \\ -0.3-0.2i & 1.2 \end{smallmatrix}\right)$. Here we can indicate the optimal decision threshold is $r_{\mathrm{opt}} = 1.6$, where $P_{\mathrm{te}} = \{0.19, 0.16, 0.15\}$. Figure (c) of Fig. 2 illustrates the total error rate of EME detection versus the threshold using $n = \{15, 20, 25\}$ when SNR $=$ 0dB and $\mathbf{\Sigma}_{\boldsymbol{m}} = \left(\begin{smallmatrix} 3.3 & -0.9+0.7i \\ -0.9-0.7i & 1.6 \end{smallmatrix}\right)$. Here we can find the optimal decision threshold is $r_{\mathrm{opt}} = 3.75$ for $n = 15$ and $r_{\mathrm{opt}} = 3.5$ for $n = \{20, 25\}$. The corresponding total error rates are $P_{\mathrm{te}} = \{0.17, 0.09, 0.06\}$. Finally, Figure (d) of Fig. 2 shows the total error rate of the GLRT detector versus the threshold, assuming $n = \{8, 9, 10\}$ when SNR $=$ 0dB and $\mathbf{\Sigma}_{\boldsymbol{m}} = \left(\begin{smallmatrix} 1.9 & 0.55+2.1i \\ 0.55-2.1i & 6.2 \end{smallmatrix}\right)$. Here it is found that the optimal decision threshold is $r_{\mathrm{opt}} = 0.82$ for $n = 8$ and $r_{\mathrm{opt}} = 0.80$ for $n = \{9, 10\}$, where the corresponding total error rates are $P_{\mathrm{te}} = \{0.17, 0.13, 0.10\}$. For all cases, it is clear that the proposed optimal decision thresholds minimize the total error rate which enables both P_{fa} and P_{m} achieving acceptable values simultaneously.

5. CONCLUSION

In this paper, the problem of finding the optimal decision threshold was considered for eigenvalue-based spectrum sensing. Specifically, we focused on the MED with arbitrary number of receive antennas, as well as the MME, EME and GLRT detectors with 2 receive antennas. The case of MED required finding the derivative of the matrix-variate confluent hypergeometric function. Also, for the cases of both the MME and EME detectors we presented accurate expressions for the probability of false alarm. Furthermore, for the case of the GLRT detector we derived an accurate expression for the probability of missed detection. For all cases, the exact total error rate was formulated and we presented the equations to numerically obtain the optimal decision thresholds which could minimize the total error rate.

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