

PERFORMANCE ANALYSIS OF MAX-SINR ALGORITHM UNDER CSI MISMATCH

S. Morteza Razavi and Tharmalingam Ratnarajah

Institute for Digital Communications, The King's Buildings
The University of Edinburgh, Edinburgh, EH9 3JL, UK
Email: Morteza.Razavi@ed.ac.uk

ABSTRACT

With interference alignment (IA), the achievable degrees of freedom (DoF) in wireless networks can be linearly scaled up with the number of users. However, to attain full DoF, the availability of perfect network channel state information (CSI) is mandatory, which impedes the practical deployment of IA since only partial network CSI may be accessible. In this paper, we investigate the effect of CSI mismatch on the performance of maximum signal-to-interference-plus-noise ratio (Max-SINR) algorithm. We show that while with perfect CSI, Max-SINR outperforms interference leakage minimization algorithms, with the presence of CSI mismatch, its comparative improvement becomes negligible. We then propose an adaptive Max-SINR which can notably improve the performance of original Max-SINR under CSI mismatch.

Index Terms— CSI mismatch, interference alignment, K -user constant MIMO interference channels, Max-SINR.

1. INTRODUCTION

Unlike orthogonal medium access techniques, say TDMA and FDMA, interference alignment (IA) is able to achieve significant throughput in wireless interference networks such that the total number of degrees of freedom (DoF) can be linearly scaled up with the number of users. In [1], it has been shown that in a K -user interference channel (IC) with a single antenna at each node, and with time-varying or frequency-selective channel coefficients, it is possible to achieve $\frac{K}{2}$ DoF by coding across sufficiently large symbol extension of the channel. This implies that the length of the symbol extension must tend to infinity which is not pragmatic. However, instead of aligning interfering signals in time and by deploying multiple antennae at transmit/receive nodes, it is possible to achieve IA without the need of symbol extension [2–9].

Nonetheless, for all IA techniques, the availability of perfect network channel state information (CSI) is necessary to achieve full DoF. However due to the realistic communication scenarios and also deployment challenges, only partial

network CSI may be available, which can adversely affect the achievable throughput and the total DoF in the network. It has been shown that for IA utilizing quantized feedback and for multi-tap single-input single-output (SISO) [10] or MIMO IC [11], the desired DoF may be achievable only if the number of feedback bits scales sufficiently fast with signal-to-noise ratio (SNR).

Since the performance of IA under quantized feedback has been widely investigated in the literature (see e.g., [10–14]), in this paper we consider a rather generalized imperfect CSI model where the variance of the measurement error is a function of SNR. We especially evaluate the performance of maximum signal-to-interference-plus-noise ratio (Max-SINR) algorithm described in [2] under CSI mismatch. Max-SINR is an interesting algorithm since it tries to maximize the SINR on a stream-by-stream basis instead of explicitly minimizing the leaked interference as being done by minimum weighted-leakage interference [2] and alternating minimization [7] algorithms, and thus achieves better performance. Because of its importance, some literature placed their focus on performance analysis of Max-SINR. For example, it has been established that Max-SINR is optimal within the class of linear beamformers at high SNRs [15], and it has been further shown that Max-SINR achieves better throughput than sum-rate gradient algorithms at low-to-intermediate SNRs [8]. Its convergence has been also addressed in [16].

However, performance analysis and improvement of Max-SINR under CSI mismatch has not been seriously considered so far. Therefore in this paper, we address this issue. First, it is shown that the comparative improvement of Max-SINR over interference leakage minimization algorithms becomes negligible subject to CSI mismatch. We then propose an adaptive Max-SINR algorithm which can notably improve the performance of original Max-SINR under CSI uncertainties.

2. SYSTEM MODEL

Consider a symmetric K -user MIMO interference channel consisting of $2K$ nodes, K of which are denoted as transmitters while the other K are receivers. Each transmitter is paired with a single receiver in a one-to-one mapping as denoted in

The project HiATUS acknowledges the financial support of the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission under FET-Open grant number: 265578.

Fig. 1. Specifically, each N -antenna transmitter communicates with its corresponding M -antenna receiver by sending d independent data streams. The channel output at receiver k is given by

$$\mathbf{y}_k = \mathbf{H}_{k,k}\mathbf{x}_k + \sum_{j=1, j \neq k}^K \mathbf{H}_{k,j}\mathbf{x}_j + \mathbf{z}_k \quad (1)$$

where $\mathbf{y}_k \in \mathbb{C}^{M \times 1}$ is the received signal, $\mathbf{x}_k \in \mathbb{C}^{N \times 1}$ is the transmitted signal from transmitter k and $\mathbf{x}_j \in \mathbb{C}^{N \times 1}$ is the interference received from transmitter j . $\mathbf{H}_{k,j} \in \mathbb{C}^{M \times N}$ describes the channel from transmitter j to receiver k . The magnitude of fading coefficients is assumed to be bounded away from zero and infinity. More specifically, the elements of channel matrices between each transmitter and receiver can be modeled by independent and identically distributed (*i.i.d.*) Gaussian random variables with zero mean and unit variance. $\mathbf{z}_k \in \mathbb{C}^{M \times 1}$ is the circularly symmetric additive white Gaussian noise with zero mean and variance σ_k^2 , i.e., $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_k^2 \mathbf{I})$. We also consider that each transmitted signal \mathbf{x}_k is equal to $\mathbf{V}_k \mathbf{s}_k$, where $\{\mathbf{V}_k\}_{k=1}^K \in \mathbb{C}^{N \times d}$ are truncated unitary transmit beamforming matrices (precoders), and $\mathbf{s}_k \in \mathbb{C}^{d \times 1}$ is the data stream intended for receiver k such that $\mathbb{E}\{\mathbf{s}_k \mathbf{s}_k^H\} = P_k \mathbf{I}$ where $\mathbb{E}\{\cdot\}$ represents the expectation of a random variable and $(\cdot)^H$ denotes the Hermitian transpose. Without loss of generality, we assume uniform power allocation across all users and DoF which is asymptotically optimal. In this case, $\rho = \frac{P}{\sigma^2}$ is defined as the nominal SNR.

Considering a zero-forcing (ZF) receiver, the conditions for perfect interference alignment can be described as [2, 17]

$$\mathbf{U}_k^H \mathbf{H}_{k,j} \mathbf{V}_j = \mathbf{0}_{d \times d}, \quad \forall j \neq k \quad (2)$$

$$\text{rank}(\mathbf{U}_k^H \mathbf{H}_{k,k} \mathbf{V}_k) = d \quad (3)$$

where $\{\mathbf{U}_j\}_{j=1}^K \in \mathbb{C}^{M \times d}$ are truncated unitary interference suppression matrices (combiners). Therefore by premultiplying the received signal at receiver k by \mathbf{U}_k^H we have

$$\begin{aligned} \mathbf{U}_k^H \mathbf{y}_k &= \mathbf{U}_k^H \mathbf{H}_{k,k} \mathbf{x}_k + \underbrace{\mathbf{U}_k^H \sum_{j=1, j \neq k}^K \mathbf{H}_{k,j} \mathbf{x}_j + \mathbf{U}_k^H \mathbf{z}_k}_{\bar{\mathbf{z}}_k} \\ &\Rightarrow \bar{\mathbf{y}}_k = \bar{\mathbf{H}}_{k,k} \mathbf{s}_k + \bar{\mathbf{z}}_k \end{aligned} \quad (4)$$

where $\bar{\mathbf{y}}_k = \mathbf{U}_k^H \mathbf{y}_k \in \mathbb{C}^{d \times 1}$, $\bar{\mathbf{H}}_{k,k} = \mathbf{U}_k^H \mathbf{H}_{k,k} \mathbf{V}_k \in \mathbb{C}^{d \times d}$, and following (2), at high enough SNRs we have $\bar{\mathbf{z}}_k = \mathbf{U}_k^H \mathbf{z}_k$. Therefore the transmitted symbol vector \mathbf{s}_k can be easily recovered by using, for example, a linear ZF equalizer, i.e., (pseudo-) inverse of $\bar{\mathbf{H}}_{k,k}$.

3. MAX-SINR ALGORITHM SUBJECT TO CSI MISMATCH

In this section, we propose an adaptive Max-SINR which can improve the performance of the original Max-SINR under

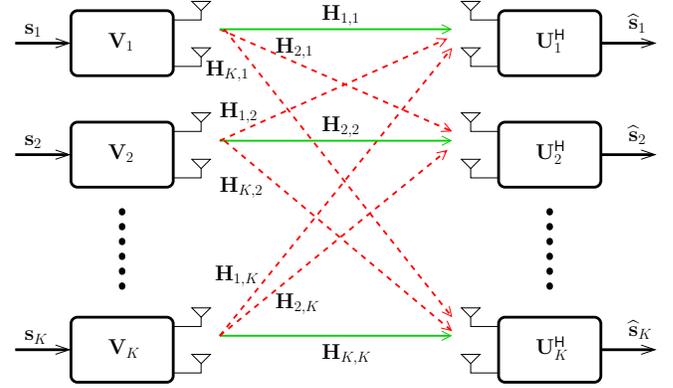


Fig. 1. K -user interference channel where green solid arrows denote direct links and red dash arrows represent interfering links. s_k designates the input data of the k th transmit node whereas \hat{s}_k indicates the recovered data at receive node k .

CSI mismatch. To do so, we first reintroduce a rather generalized CSI mismatch model in the following subsection.

3.1. Imperfect CSI Model

Following [18] and regardless of distributed or centralized processing, we assume that all precoders and combiners are constructed with the knowledge of unified CSI mismatch. We further model the imperfect CSI as

$$\hat{\mathbf{H}}_{k,j} = \tilde{\mathbf{H}}_{k,j} + \mathbf{H}_{k,j} \quad (5)$$

where the channel measurement error $\tilde{\mathbf{H}}_{k,j}$ is thought to be independent of actual channel matrix $\mathbf{H}_{k,j}$. Similar to [18], we consider $\tilde{\mathbf{H}}_{k,j}$ as a Gaussian matrix consisting of *i.i.d.* elements with mean zero and variance

$$\tau \triangleq \beta \rho^{-\alpha}, \quad \beta > 0, \quad \alpha \geq 0 \quad (6)$$

In this case, the error variance can depend on SNR ($\alpha \neq 0$) or be independent of that ($\alpha = 0$). Notice the variance model in (6) is versatile since it is potentially able to accommodate a variety of distinct scenarios, e.g., reciprocal channels ($\alpha = 1$) and CSI feedback ($\alpha = 0$). τ can be further interpreted as a parameter that captures the quality of the channel estimation which is possible to be known *a priori*, depending on the channel dynamics and channel estimation schemes (see e.g., [19] and references therein).

To facilitate the performance analysis of Max-SINR under CSI mismatch model in (5), it is more appropriate to have the statistical properties of $\mathbf{H}_{k,j}$ conditioned on $\hat{\mathbf{H}}_{k,j}$ by using following lemma:

Lemma 1 [20]: Conditioned on $\hat{\mathbf{H}}_{k,j}$, $\mathbf{H}_{k,j}$ has a Gaussian distribution with mean $\hat{\mathbf{H}}_{k,j} / (1 + \tau)$ and statistically independent elements of variance $\tau / (1 + \tau)$, i.e.,

$$\mathbf{H}_{k,j} = \frac{1}{1 + \tau} \hat{\mathbf{H}}_{k,j} + \check{\mathbf{H}}_{k,j} \quad (7)$$

$$\begin{aligned}
\mathbf{Q}_k^\ell &= \sum_{\substack{j=1 \\ j \neq k}}^K \sum_{n=1}^d P \mathbf{H}_{k,j} \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \mathbf{H}_{k,j}^H + \sum_{\substack{m=1 \\ m \neq \ell}}^d P \mathbf{H}_{k,k} \mathbf{V}_k^{*m} (\mathbf{V}_k^{*m})^H \mathbf{H}_{k,k}^H + \sigma^2 \mathbf{I} \\
&\stackrel{\textcircled{1}}{=} \sum_{\substack{j=1 \\ j \neq k}}^K \sum_{n=1}^d P \left(\frac{1}{1+\tau} \widehat{\mathbf{H}}_{k,j} + \check{\mathbf{H}}_{k,j} \right) \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \left(\frac{1}{1+\tau} \widehat{\mathbf{H}}_{k,j} + \check{\mathbf{H}}_{k,j} \right)^H \\
&+ \sum_{\substack{m=1 \\ m \neq \ell}}^d P \left(\frac{1}{1+\tau} \widehat{\mathbf{H}}_{k,k} + \check{\mathbf{H}}_{k,k} \right) \mathbf{V}_k^{*m} (\mathbf{V}_k^{*m})^H \left(\frac{1}{1+\tau} \widehat{\mathbf{H}}_{k,k} + \check{\mathbf{H}}_{k,k} \right)^H + \sigma^2 \mathbf{I}
\end{aligned} \tag{8}$$

where $\check{\mathbf{H}}_{k,j}$ is statistically independent of $\widehat{\mathbf{H}}_{k,j}$ with i.i.d. elements of mean zero and variance $\tau / (1 + \tau)$.

3.2. Adaptive Max-SINR Algorithm

Since Max-SINR maximizes the SINR on a stream-by-stream basis, we define $\mathbf{V}_k^{*\ell}$ and $\mathbf{U}_k^{*\ell}$ as the ℓ -th column of \mathbf{V}_k and \mathbf{U}_k , respectively, which are further to be considered as unit-norm vectors. Notice due to the coupled nature of the problem and regardless of what algorithm is being used, there are no closed form solutions for naive IA except for a very few particular cases, see e.g., [1, 21]. Consequently, finding the precoders and combiners requests an iterative procedure in general. Therefore, first we fix the precoders and seek the combiners, and then we fix the combiners and seek the precoders. Given randomly initialized precoders and with respect to the fact that only imperfect channel estimates $\widehat{\mathbf{H}}_{k,j}$ are available, the interference plus noise covariance matrix observed by the ℓ th stream of user k can be shown as (8) at the top of the page wherein $\textcircled{1}$ follows from equation (7).

To further proceed, we consider the following lemmas:

$$\begin{aligned}
&\text{Lemma 2: } \mathbb{E}_{\check{\mathbf{H}}|\widehat{\mathbf{H}}} \left\{ \widehat{\mathbf{H}}_{k,j} \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \check{\mathbf{H}}_{k,j}^H \right\} = \\
&\mathbb{E}_{\check{\mathbf{H}}|\widehat{\mathbf{H}}} \left\{ \check{\mathbf{H}}_{k,j} \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \widehat{\mathbf{H}}_{k,j}^H \right\} = \mathbf{0} \quad \forall k, j, n
\end{aligned}$$

Proof: All precoders and combiners are constructed upon channel estimates $\widehat{\mathbf{H}}_{k,j}$ which based on *Lemma 1* are independent of $\check{\mathbf{H}}_{k,j}$. ■

Lemma 3: If $\mathbf{A} \in \mathbb{C}^{M \times N}$ represents a Gaussian matrix with i.i.d. elements of mean zero and variance a , and $\mathbf{b} \in \mathbb{C}^{N \times 1}$ refers to a unit-norm vector independent of \mathbf{A} , then $\mathbb{E}_{\mathbf{A}} \{ \mathbf{A} \mathbf{b} \mathbf{b}^H \mathbf{A}^H \} = a \mathbf{I}$.

Proof: Since \mathbf{A} is a Gaussian matrix, it is bi-unitarily invariant¹, and consequently the joint distribution of its entries equals that of $\mathbf{A} \mathbf{b}$ for any unit-norm vector \mathbf{b} independent of \mathbf{A} [22]. Therefore $\mathbf{A} \mathbf{b}$ is a zero-mean Gaussian vector with covariance matrix $a \mathbf{I}$. ■

Following *Lemma 2* and *Lemma 3*, we substitute those parts of (8) including $\check{\mathbf{H}}_{j,k} \forall j, k$ with their expected values. This way, we can approximate \mathbf{Q}_k^ℓ in (8) with a simpler form,

¹A rectangular random matrix \mathbf{H} is called *bi-unitarily invariant* if the joint distribution of its entries equals that of $\mathbf{U}^H \mathbf{H} \mathbf{V}$ for any unitary matrices \mathbf{U} and \mathbf{V} independent of \mathbf{H} .

i.e., $\widehat{\mathbf{Q}}_k^\ell$, as follows:

$$\begin{aligned}
\widehat{\mathbf{Q}}_k^\ell &= \sum_{\substack{j=1 \\ j \neq k}}^K \sum_{n=1}^d \frac{P}{(1+\tau)^2} \widehat{\mathbf{H}}_{k,j} \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \widehat{\mathbf{H}}_{k,j}^H \\
&+ \sum_{\substack{m=1 \\ m \neq \ell}}^d \frac{P}{(1+\tau)^2} \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*m} (\mathbf{V}_k^{*m})^H \widehat{\mathbf{H}}_{k,k}^H \\
&+ \left(P(Kd - 1) \frac{\tau}{1+\tau} + \sigma^2 \right) \mathbf{I}
\end{aligned} \tag{9}$$

Therefore, with respect to $\widehat{\mathbf{Q}}_k^\ell$ in (9), the SINR maximizing receive filter is given by

$$\mathbf{U}_k^{*\ell} = \frac{\left(\widehat{\mathbf{Q}}_k^\ell \right)^{-1} \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*\ell}}{\left\| \left(\widehat{\mathbf{Q}}_k^\ell \right)^{-1} \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*\ell} \right\|} \tag{10}$$

which is the relative combiner of the ℓ th stream of user k , and $\| \cdot \|$ refers to the vector 2-norm.

As mentioned earlier, due to the coupled nature of the problem, finding precoders and combiners requests an iterative algorithm in general. Plus, with respect to the fact that only imperfect channel estimates $\widehat{\mathbf{H}}_{k,j}$ are available, and with the knowledge of error variance τ in advance (as discussed in Section 3.1), the proposed algorithm can be concisely presented as follows:

Adaptive Max-SINR

1. Set $\mu := \rho^{-1} (1 + \tau)^2 + \tau (\tau + 1) (Kd - 1)$
2. Initialize random unit-norm vectors $\mathbf{V}_k^{*\ell} \forall \ell, k$
- 3.

$$\begin{aligned}
\mathbf{T}_k^\ell &= \sum_{\substack{j=1 \\ j \neq k}}^K \sum_{n=1}^d \widehat{\mathbf{H}}_{k,j} \mathbf{V}_j^{*n} (\mathbf{V}_j^{*n})^H \widehat{\mathbf{H}}_{k,j}^H \\
&+ \sum_{\substack{m=1 \\ m \neq \ell}}^d \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*m} (\mathbf{V}_k^{*m})^H \widehat{\mathbf{H}}_{k,k}^H + \mu \mathbf{I}
\end{aligned}$$

4. $\mathbf{U}_k^{*\ell} = \frac{\left(\mathbf{T}_k^\ell \right)^{-1} \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*\ell}}{\left\| \left(\mathbf{T}_k^\ell \right)^{-1} \widehat{\mathbf{H}}_{k,k} \mathbf{V}_k^{*\ell} \right\|}$ for all k and ℓ

5.

$$\mathbf{R}_k^\ell = \sum_{\substack{j=1 \\ j \neq k}}^K \sum_{n=1}^d \hat{\mathbf{H}}_{j,k}^H \mathbf{U}_j^{*n} (\mathbf{U}_j^{*n})^H \hat{\mathbf{H}}_{j,k} \\ + \sum_{\substack{m=1 \\ m \neq \ell}}^d \hat{\mathbf{H}}_{k,k}^H \mathbf{U}_k^{*m} (\mathbf{U}_k^{*m})^H \hat{\mathbf{H}}_{k,k} + \mu \mathbf{I}$$

6. $\mathbf{V}_k^{*\ell} = \frac{(\mathbf{R}_k^\ell)^{-1} \hat{\mathbf{H}}_{k,k}^H \mathbf{U}_k^{*\ell}}{\|(\mathbf{R}_k^\ell)^{-1} \hat{\mathbf{H}}_{k,k}^H \mathbf{U}_k^{*\ell}\|}$ for all k and ℓ

7. Go to Step 3

Remark: Note that similar to the original Max-SINR, the precoders and combiners calculated by adaptive Max-SINR are not unitary (although they have unit-norm columns) which may result in rank-deficient precoders and/or combiners for at least one user and hence at high SNRs, they may not be able to achieve full multiplexing gain and thus degrade the performance. However, to preserve full DoF, one solution to make precoders and combiners unitary is inserting orthogonalization steps for \mathbf{U}_k and \mathbf{V}_k within each iteration, which ensure no performance degradation at high SNR ranges.

4. NUMERICAL RESULTS

In this section and by using numerical results, we corroborate the improved performance achieved by adaptive Max-SINR (with orthogonalization) compared to the original Max-SINR (with orthogonalization) defined in [2]. We also consider the performance of alternating minimization (Alt-Min) algorithm described in [7]. Although the promised improvement can be gleaned for various values of α , we focus on two representative cases: $\alpha = 0$ (which mimics the CSI feedback scenario), and $\alpha = 1$ (which mimics the reciprocal channels). Without loss of generality, we consider a symmetric constant MIMO IA with $K = 3$ and $d = 4$. To meet the sufficient conditions of feasibility for IA, we set $M = N = 8$ [23].

By considering i.i.d. Gaussian input signaling and uniform power allocation, we evaluate the achievable sum rates as [18]

$$R = \sum_{k=1}^K \log_2 \det \left(\mathbf{I} + \left(\rho^{-1} \mathbf{I} + \sum_{j=1, j \neq k}^K \Phi_{k,j} \right)^{-1} \Phi_{k,k} \right) \quad (11)$$

where

$$\Phi_{k,j} = \mathbf{U}_k^H \mathbf{H}_{k,j} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{k,j}^H \mathbf{U}_k \quad (12)$$

and in the case of imperfect CSI, all precoders and combiners are constructed based on erroneous channel estimations in (5).

Figs. 2 and 3, respectively, depict the average sum rate and symbol-error rate (SER) for $\beta = 0.1, \alpha = 0$ and $\beta = 10, \alpha = 1$. As observed, with the presence of perfect CSI, Max-SINR outperforms Alt-Min algorithm. However, subject to CSI mismatch, while the expected improvement by Max-SINR over Alt-Min becomes negligible, adaptive Max-SINR

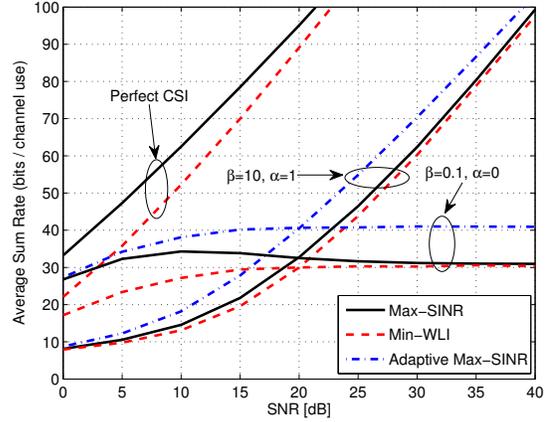


Fig. 2. Average sum rates for $K = 3, d = 4, M = N = 8$ and for the cases $\beta = 0.1, \alpha = 0$, and $\beta = 10, \alpha = 1$.

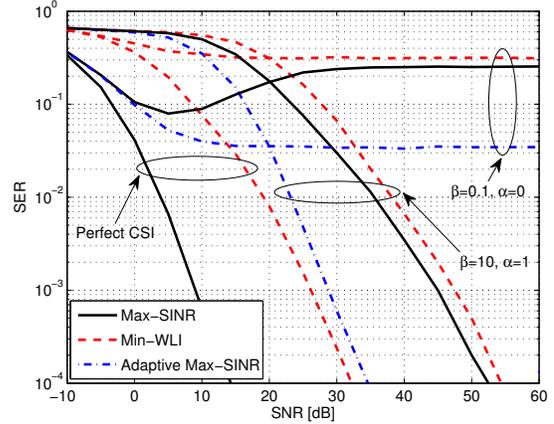


Fig. 3. SER for $K = 3, d = 4, M = N = 8$ and for the cases $\beta = 0.1, \alpha = 0$, and $\beta = 10, \alpha = 1$. Each transmitted block consists of 100 QPSK symbols.

achieves notably better performance. For example, adaptive Max-SINR achieves at least 18 dB gain compared to Max-SINR to achieve the same SER as of 10^{-4} for $\alpha = 1$. Also at high SNRs, adaptive Max-SINR achieves 10 bits per channel use gain in sum rate compared to Max-SINR for $\alpha = 0$.

5. CONCLUSION

With the presence of perfect CSI, Max-SINR outperforms interference leakage minimization algorithms since it maximizes the SINR of each stream of each user. However, under imperfect CSI, its performance becomes significantly degraded and the achieved improvement compared to leakage minimization algorithms becomes negligible, especially at high SNRs. In this paper, we proposed an adaptive Max-SINR algorithm which can notably improve the performance of the original Max-SINR under CSI mismatch.

6. REFERENCES

- [1] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K -user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
- [2] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.
- [3] H. Yu and Y. Sung, "Least squares approach to joint beam design for interference alignment in multiuser multi-input multi-output interference channels," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4960–4966, Sep. 2010.
- [4] D. S. Papailiopoulos and A. G. Dimakis, "Interference alignment as a rank constrained rank minimization," *IEEE Trans. Signal Process.*, vol. 60, no. 8, pp. 4278–4288, Aug. 2012.
- [5] H. Du, T. Ratnarajah, M. Sellathurai, and C. B. Papadias, "Reweighted nuclear norm approach for interference alignment," *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3754–3765, Sep. 2013.
- [6] S. M. Razavi and T. Ratnarajah, "Interference alignment in K -user MIMO interference channels with partially coordinated receivers," *IET Commun.*, vol. 8, no. 1, pp. 50–57, Jan. 2014.
- [7] S. W. Peters and R. W. Heath, "Interference alignment via alternating minimization," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, 2009, pp. 2445–2448.
- [8] I. Santamaria, O. Gonzalez, R. W. Heath, and S. W. Peters, "Maximum sum-rate interference alignment algorithms for MIMO channels," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, 2010.
- [9] D. A. Schmidt, C. Shi, R. A. Berry, M. L. Honig, and W. Utschick, "Minimum mean squared error interference alignment," in *Proc. Asilomar Conf. Signal Syst. Comput. (ACSSC)*, 2009, pp. 1106–1110.
- [10] J. Thukral and H. Bolcskei, "Interference alignment with limited feedback," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, 2009, pp. 1759–1763.
- [11] R. T. Krishnamachari and M. K. Varanasi, "Interference alignment under limited feedback for MIMO interference channels," *IEEE Trans. Signal Process.*, vol. 61, no. 15, pp. 3908–3917, Aug. 2013.
- [12] J.-S. Kim, S.-H. Moon, S.-R. Lee, and I. Lee, "A new channel quantization strategy for MIMO interference alignment with limited feedback," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 358–366, Jan. 2012.
- [13] X. Rao, L. Ruan, and V. K. N. Lau, "CSI feedback reduction for MIMO interference alignment," *IEEE Trans. Signal Process.*, vol. 61, no. 18, pp. 4428–4437, Sep. 2013.
- [14] S. Cho *et al.*, "Feedback-topology designs for interference alignment in MIMO interference channels," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6561–6575, Dec. 2012.
- [15] J. Park, Y. Sung, and H. Vincent Poor, "On beamformer design for multiuser MIMO interference channels." [Online]. Available: <http://arxiv.org/abs/1011.6121v1>
- [16] C. Wilson and V. V. Veeravalli, "A convergent version of the Max SINR algorithm for the MIMO interference channel," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2952–2961, Jun. 2013.
- [17] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, "On feasibility of interference alignment in MIMO interference networks," *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771–4782, Sep. 2010.
- [18] S. M. Razavi and T. Ratnarajah, "Performance analysis of interference alignment under CSI mismatch," *IEEE Trans. Veh. Technol.*
- [19] T. Yoo and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 2203–2214, May 2006.
- [20] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. New Jersey: Prentice-Hall, 1993.
- [21] R. Tresch, M. Guillaud, and E. Riegler, "On the achievability of interference alignment in the K -user constant MIMO interference channel," in *Proc. IEEE/SP Workshop Stat. Signal Process.*, 2009, pp. 277–280.
- [22] A. M. Tulino and S. Verdú, "Random matrix theory and wireless communications," *Foundations and Trends in Communications and Information Theory*, vol. 1, no. 1, pp. 1–182, 2004.
- [23] M. Razaviyayn, G. Lyubeznik, and Z.-Q. Luo, "On the degrees of freedom achievable through interference alignment in a MIMO interference channel," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 812–821, Feb. 2012.