2014 IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP)

MULTIUSER DOWNLINK BEAMFORMING WITH INTERFERENCE CANCELLATION USING A SDP-BASED BRANCH-AND-BOUND ALGORITHM

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ABSTRACT

We consider in this paper multiuser downlink beamforming with interference cancellation (BFIC). In our BFIC problem, the total transmitted power of the base station (BS) is minimized under signal-to-interference-plus-noise ratio (SINR) requirements of the mobile stations (MSs) and single-stage interference cancellation (SSIC) is adopted at the MSs. The challenge of the problem lies in its combinatorial and non-convex nature. We propose a semidefinite programming (SDP) based branch-and-bound (BnB) algorithm to (optimally) solve the BFIC problem. The SDP-based BnB algorithm employs SDP and sequential second-order cone programming. We further develop a fast heuristic algorithm for large-scale applications. Simulations show that employing SSIC achieves significant reductions in total transmitted BS power. The proposed SDP-based BnB algorithm optimally solves all considered instances of the BFIC problem, and the heuristic algorithm yields near-optimal solutions.

Index Terms— Downlink Beamforming, Interference Cancellation, SDP-based Branch-and-Bound, Fast Heuristic Algorithm

1. INTRODUCTION

Multiuser downlink beamforming represents one of the key enabling technologies for current and future cellular networks and has already been intensively investigated in the literature (see, e.g., [1–14]). In the conventional downlink beamforming problem, the total transmitted power of the base station (BS) is minimized while guaranteeing the received signal-to-interference-plus-noise ratio (SINR) requirements of the mobile stations (MSs). Both, efficient convex optimization techniques (see, e.g., [6–9]) and specialized iterative algorithms (see, e.g., [10–14]) have been proposed to optimally solve the SINR-constrained multiuser downlink beamforming problem. In most of the existing works, it is assumed that the co-channel interference received at the MSs is treated as noise.

While treating co-channel interference as noise simplifies the mobile receivers it overlooks the potential of exploiting the co-channel interference. For instance, it is well-known that interference cancellation (IC), in which the MSs decode and subtract the strong interference, achieves better performance in terms of, e.g., increase in link capacity and reduction in transmitted BS power [13–17]. In this paper, we consider multiuser downlink beamforming with interference cancellation (BFIC). To limit the decoding complexity at the MSs, we focus on single-stage interference cancellation (SSIC). That is, each MS at most decodes and cancels one interfering signal. Depending on the strength of the interfering signal at a MS, it may directly decode its own signal by treating other signals as noise, or decode and cancel one interfering signal and then decode its desired signal (treating the remaining interference as noise). Multiuser downlink beamforming combined with dirty-paper coding was studied in [13–15]. However, the optimization of the decoding order was not considered in [13–15] and the complexity of dirty-paper coding may be prohibitive in practice.

Similar to existing work on IC [13–17], to simplify the system model and the presentation, the overhead of IC is not explicitly taken into account in this paper. Hence, the results presented in this paper serve as performance bounds for practical systems.

Since the decisions about the decoding orders at the MSs are coupled in the downlink SINR constraints through co-channel interference when employing SSIC, the BFIC problem of interest represents a combinatorial nonlinear program. We formulate the BFIC problem as a non-convex quadratically constrained quadratic program (QCQP) with on-off constraints [18–24], where the on-off constraints [24] model the decisions about the decoding orders at the MSs.

We develop a semidefinite programming (SDP) based branch-and-bound (BnB) algorithm to (optimally) solve the BFIC problem. In the BnB algorithm, a binary BnB search tree is constructed, with each node on the tree representing a non-convex QCQP. The non-convex QCQP at a node represents a subproblem of the BFIC problem and arises from fixing the decoding orders at one or more of the MSs. We adopt the celebrated semidefinite relaxation (SDR) approach for the non-convex QCQP at each node, relaxing the non-convex QCQP into a semidefinite program (SDP) by dropping the non-convex rank-one constraints [19–23]. Since the SDR may not be exact, we further include the sequential second-order cone (SSOC) algorithm to compute a (possibly local) solution of the non-convex QCQP at the *leaf* nodes when the SDR approach does not yield rank-one solutions. We further propose a customized branching rule to speed up the BnB solution process. Moreover, a low-complexity heuristic algorithm is developed to approximately solve the BFIC problem for applications in large-scale systems.

The simulations show that significant reductions in the total transmitted BS power can be achieved by employing SSIC. The proposed SDP-based BnB algorithm optimally solves the considered instances of the BFIC problem, and the fast heuristic algorithm yields total transmitted BS power that is close to the solution obtained by the proposed SDP-based BnB method.

2. RELATION TO PRIOR WORK

Downlink beamforming has been intensively studied in [6–14, 20–22], but without considering IC. IC in the context of max-min power control and admission control has been studied in [16, 17], but without beamforming. The authors of [13–15] consider IC and

This work was supported by the LOEWE Priority Program Cocoon (http://www.cocoon.tu-darmstadt.de).

978-1-4799-2893-4/14/$31.00 ©2014 IEEE

beamforming assuming nonlinear dirty-paper coding schemes, but do not present any practical algorithm. In contrast to the existing approaches [6–13, 13–17, 20–22], we consider in this paper the joint optimization of multiuser downlink beamforming and interference cancellation (BFIC). A SDP-based BnB algorithm, which is able to solve the BFIC problem for all considered instances, and a practical heuristic algorithm yielding close-to-optimal solutions, are developed.

3. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a cellular downlink system consisting of one BS, equipped with $L > 1$ antennas, that serves $K > 1$ single-antenna MS. Denote $\mathbf{h}_k \in \mathbb{C}^L$ and $\mathbf{w}_k \in \mathbb{C}^L$ as the frequency flat channel vector and the beamforming vector of the kth MS, $k \in \mathcal{K}$, with the MS indices set $\mathcal{K} \triangleq \{1, \ldots, K\}$. The received signal y_k at the kth MS can then be written as (see, e.g., [6–14, 20–22]):

$$y_k = \mathbf{h}_k^H \mathbf{w}_k x_k + \sum_{j=1, j\neq k}^{K} \mathbf{h}_k^H \mathbf{w}_j x_j + n_k, \quad \forall k \in \mathcal{K}, \tag{1}$$

where $x_k \in \mathbb{C}$ represents the unit-power data symbol, designated for the kth MS, and $n_k \in \mathbb{C}$ denotes the additive noise at the kth MS, with zero mean and variance $\sigma_k^2, \ \forall k \in \mathcal{K}$. As in [6–14, 20–22], it is assumed that the data symbols for different MSs are mutually statistically independent and also independent from the receiver noise.

In our BFIC problem, the kth MS may either directly decode its own signal (treating the co-channel interference as noise), or decode and cancel/subtract one interfering signal and then decode its own signal (treating the residual co-channel interference as noise) [17]. Following the SINR-constrained design as presented in [6–14], to be able to directly decode its own signal, for the kth MS the received SINR at the kth MS, denoted by SINR_k, shall exceed a given threshold γ_k, i.e.,

$$\text{SINR}_k \triangleq \frac{\mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k}{\sum_{j=1, j\neq k}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2} \geq \gamma_k. \tag{2}$$

Accordingly, if the kth MS decodes and cancels the signal of the lth MS and then decodes its own signal, it is required that [16, 17]:

$$\text{SINR}_k^{(l)} \triangleq \frac{\mathbf{w}_l^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_l}{\sum_{j=1, j\neq l}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2} \geq \gamma_l \tag{3a}$$

$$\text{SINR}_k^{(-l)} \triangleq \frac{\mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k}{\sum_{j=1, j\neq k,l}^{K} \mathbf{w}_j^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \tag{3b}$$

where $\text{SINR}_k^{(l)}$ denotes the SINR associated with the signal of the lth MS seen at the kth MS, and $\text{SINR}_k^{(-l)}$ represents the SINR of the kth MS after cancelling the interfering signal of the lth MS.

We introduce binary variables $a_{k,l} \in \{0, 1\}, k, l \in \mathcal{K}$ modelling the IC procedure at the MSs. For a MS $k \in \mathcal{K}$, we set $a_{k,k} = 1$ if and only if MS k directly decodes its own signal, and $a_{k,l} = 1$ if and only if MS k decodes and cancels MS lth signal first. With SSIC, for each MS k exactly one of the binary variables $\{a_{k,l}, l \in \mathcal{K}\}$ is equal to one. Let $\mathbf{A} = (a_{k,l}) \in \{0, 1\}^{K\times K}$, be the binary matrix, summarizing the decoding order of all MS, i.e., the kth row of the matrix $\mathbf{A}$ stores the decoding order of the kth MS.

It is further assumed in this paper, that the channel vectors $\{\mathbf{h}_k, k \in \mathcal{K}\}$ are known at the BS. In the BFIC problem, the BS determines the decoding order for all MSs and jointly computes the beamformers. Similar to the existing works [16, 17] that consider power control and SSIC (without beamforming), the BFIC problem, in which the total transmitted BS power is minimized while ensuring the SINR targets of the MSs, can be stated as

$$\begin{aligned}
\min_{\{\mathbf{w}_k, \mathbf{A}\}} \quad & \sum_{k=1}^{K} \|\mathbf{w}_k\|_2^2 && \text{(4a)}\\
\text{s.t.} \quad & \text{SINR}_k \geq \gamma_k \qquad \text{if } a_{k,k} = 1 \ k \in \mathcal{K} && \text{(4b)}\\
& \text{SINR}_k^{(l)} \geq \gamma_l \qquad \text{if } a_{k,l} = 1, l, k \in \mathcal{K}, l \neq k && \text{(4c)}\\
& \text{SINR}_k^{(-l)} \geq \gamma_k \qquad \text{if } a_{k,l} = 1, l, k \in \mathcal{K}, l \neq k && \text{(4d)}\\
& \sum_{l=1}^{K} a_{k,l} = 1, \ k \in \mathcal{K} && \text{(4e)}\\
& \mathbf{w}_k \in \mathbb{C}^L, \ k \in \mathcal{K}, \ \mathbf{A} \in \{0, 1\}^{K\times K} && \text{(4f)}
\end{aligned}$$

where the SINR constraint in (4b) are present for the MSs that do not adopt IC, and the SINR constraints in (4c) and (4d) correspond to the MSs that employ SSIC [16, 17].

The formulated BFIC problem (4) represents a non-convex QCQP with on/off constraints [24] and belongs to the class of mixed integer nonlinear programs (MINLP). MINLPs are not easy to solve, due to the combinatorial and nonlinear nature of the problem. As the numerical results will show, even the state-of-the-art MINLP solver Baron [25] is not able to compute the optimal solutions of the BFIC problem (4) in reasonable time. Thus, we develop a customized SDP-based BnB algorithm for solving the BFIC problem (4), making use of the SDR technique [19–23] and the SSOC algorithm.

4. SDP-BASED BRANCH-AND-BOUND ALGORITHM

We now present our SDP-based BnB algorithm for solving the BFIC problem (4) that is a modification of a standard BnB algorithm [26, 27]. We deal with the combinatorial nature of the BFIC problem resulting from the IC decisions at the MSs within the BnB algorithm. In the proposed SDP-based BnB algorithm Alg.1 for solving (4), at every node of the BnB-tree a subproblem of the BFIC problem (4) is considered that is obtained by fixing the decoding order for a subset of the MSs, omitting the constraints (4e). The subproblem is solved or at least a lower bound is determined using SDR techniques. Similar to general BnB algorithms [26, 27], depending on the generated lower and upper bounds for (4), subtrees of the BnB-tree can be pruned or we have to branch deeper into the BnB-tree by fixing the decoding order of more MSs. In the end, if we have searched the whole BnB-tree, we have either found a global solution for the BFIC problem (4), or, since we are using SDR techniques, a feasible solution for (4). In the latter case we can state the (possible) gap between the global solution of (4) and the best solution found by our algorithm. The details of the proposed SDP-based BnB algorithm are presented next.

The problem at a node in the SDP-based BnB algorithm is a subproblem of (4) (omitting (4e)), and depends on the (partial) decoding order fixation $\mathbf{A} = (a_{k,l}) \in \{0, 1\}^{K\times K}$. In oder to derive good lower bounds at the nodes in the BnB-tree, we additionally add the SNR constraint

$$\text{SNR}_k \triangleq \mathbf{w}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{w}_k \geq \gamma_k \sigma_k^2 \tag{5}$$

to the problem of the node, if the decoding order for the MS k is not fixed yet, i.e., $\sum_{l=1}^{K} a_{k,l} = 0$. Note that we can add the SNR constraint (5) to the BFIC problem (4) without changing the optimal solution since the SNR defined in (5) is always bigger than the SINR in (4b) and (4d). The problem at a node in the SDP-based BnB algorithm is given by:

$$\min_{\{\mathbf{w}_k\}} \sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{w}_k \text{ s.t. (4b), (4c), (4d), (4f),} \\ \text{(5), if } \sum_{l=1}^{K} a_{k,l} = 0. \tag{6}$$

For many (non-convex) QCQPs the consideration of the corresponding SDR has turned out to be very useful [20–23]. Thus, in our SDP-based BnB algorithm, we do not solve the non-convex QCQP of a node (6) directly but consider its SDR [20–23] that is obtained by introducing matrix variables $\mathbf{W}_k$ for $\mathbf{w}_k\mathbf{w}_k^H$ and dropping the non-convex constraints $\text{rank}(\mathbf{W}_k) = 1, \forall k \in \mathcal{K}$. The resulting SDP is given by:

$$\min_{\{\mathbf{W}_k\}} \quad \sum_{k=1}^{K} \text{tr}(\mathbf{W}_k) \tag{7}$$

$$\begin{aligned}
\text{s.t.} \quad & \text{tr}(\mathbf{H}_k\mathbf{W}_k) \geq \gamma_k \sum_{j=1, j\neq k}^{K} \text{tr}(\mathbf{H}_k\mathbf{W}_k) + \gamma_k\sigma_k^2, \text{if } a_{k,k} = 1 \\
& \text{tr}(\mathbf{H}_k\mathbf{W}_l) \geq \gamma_l \sum_{j=1, j\neq l}^{K} \text{tr}(\mathbf{H}_k\mathbf{W}_j) + \gamma_l\sigma_k^2, \text{ if } a_{k,l} = 1 \\
& \text{tr}(\mathbf{H}_k\mathbf{W}_k) \geq \gamma_k \sum_{j=1, j\neq l,k}^{K} \text{tr}(\mathbf{H}_k\mathbf{W}_j) + \gamma_k\sigma_k^2, \text{ if } a_{k,l} = 1 \\
& \text{tr}(\mathbf{H}_k\mathbf{W}_k) \geq \gamma_k\sigma_k^2, \text{ if } \sum_{l=1}^{K} a_{k,l} = 0 \\
& \mathbf{W}_k \succeq 0, \forall k \in \mathcal{K}.
\end{aligned}$$

Let $\overline{\mathbf{W}}_k, k \in \mathcal{K}$, be the solution of the SDP (7). Similar to [20–23], we call an optimal solution of a SDP "rank-one" if each matrix $\overline{\mathbf{W}}_k, k \in \mathcal{K}$ is of rank one. If the solution $\overline{\mathbf{W}}_k, k \in \mathcal{K}$, is rank-one, i.e., $\overline{\mathbf{W}}_k = \overline{\mathbf{w}}_k(\overline{\mathbf{w}}_k)^H, \forall k \in \mathcal{K}$, then $\overline{\mathbf{w}}_k, k \in \mathcal{K}$, is an optimal solution of the non-convex QCQP (6) with the same decoding order fixation $\mathbf{A}$. Otherwise, we only get a lower bound for the non-convex QCQP (6). Due to the results in [21, 22], we know in advance, that some instances of (7) have a rank-one solution, namely: If at most two MSs perform SSIC, then we can construct a rank-one solution in polynomial time, given the solution of the SDP (7), since the number of constraints is less or equal the number of separable blocks plus two.

Regardless of whether the solution of the SDP at a node (7), where the decoding order is only partly fixed, is rank-one or not we further branch on the decoding order. At leaf nodes of the BnB-tree the decoding order for all MS has been fixed and satisfies (4e). If the solution of a leaf node SDP (7) is rank-one, we have found a feasible solution (upper bound) to the BFIC problem (4). If the solution is better than the best solution found so far, we store the new solution as the current best solution. At leaf nodes, where the SDP does not have a rank-one solution, but the objective of the SDP is better than the current best solution, we apply the SSOC algorithm Alg.2, which is described in Sec. 5. With the SSOC algorithm Alg.2 we can find a (local) solution of the non-convex QCQP (6) of the leaf node, if the SDP (7) does not have a rank-one solution. We further store the decoding order $\mathbf{A}^{sdp}$ and the objective of the SDP P^{sdp} of all leaf nodes, which do not have a rank-one solution in the list of *crucial* nodes $\mathcal{C}$. However, if the SDP solution of a leaf node is worse than the best solution found so far, we can cut off the node regardless of the possible gap between the solution of the SDP (7) and QCQP (6).

At the end of our SDP-based BnB algorithm, we compare the objective value of the best solution found P^{best} with the minimum of the SDP solutions stored in the list of crucial nodes $\mathcal{C}$, denoted by P^{lb}. If $P^b \leq P^{lb}$, we have solved the BFIC problem to optimality. Otherwise, we have found a feasible solution for the BFIC problem and can state the relative gap $(P^b - P^{lb})/P^{lb}$.

In BnB algorithms the development of a suitable branching rule is important to speed up the solution process. As mentioned above, in the SDP-based BnB algorithm Alg.1 we branch on the decoding order of the MSs. In our SDP-based BnB algorithm the order in which we fix the decoding order of the MSs depends on the channel gain $\|\mathbf{h}_k\|_2$ for all MS $k \in \mathcal{K}$. We fix the decoding order for MSs with less channel gain first. The branching order is stored in $\mathbf{f} \in \{1, \ldots, K\}^K$, i.e., $\mathbf{f}_1$ is the index of the MS with lowest channel gain, $\mathbf{f}_2$ the index of the MS with second lowest channel gain, and so on. In depth one of the BnB tree, the decoding order of MS $\mathbf{f}_1$ is fixed, in depth two in addition the one of the MS $\mathbf{f}_2$, and so on. Moreover, in depth m, we first consider subproblems of (4), corresponding to decoding order fixation, in which signals of MSs with less channel gain are decoded first at the MS $\mathbf{f}_m$. We further perform depth first search [26, 27]. Whenever we have to branch, we update the set of unsolved nodes $\mathcal{L}$, adding the K children nodes of the current node to the list of unsolved nodes according to the proposed branching rule stored in $\mathbf{f}$.

In the preprocessing step of Alg.1, we apply the low- channel-gain heuristic Alg.3, presented in Sec.6, to generate a close-to-optimal solution of the BFIC problem. With this initial solution we can prune a lot of nodes in the SDP-based BnB algorithm.

In Alg.1, we denote a solution by the tuple $S = ((\mathbf{w_k}), \mathbf{A}, P)$, indicating with the superscript, $b, heu, sdp, ssoc$ the best solution of the BFIC problem (4) found so far, the solution of Alg.3, the solution of the SDP (7), and the solution of Alg.2, respectively.

Algorithm 1: SDP-based BnB algorithm for BFIC

```
Preprocessing: Apply heuristic Alg.3 from Sec.6.
Initialization: Determine branching rule f, set S^b = S^heu.
Initialize set of unsolved nodes L, set C = ∅.
while L ≠ ∅ do
    Select problem A^node from L, set L = L \ {A^node}.
    Solve the SDP (7) corresponding to A^node.
    if SDP (7) is infeasible or P^sdp > P^b then
        prune subtree.
    if P^sdp < P^best and (4e) is not satisfied then
        update L according to branching rule f.
    if P^sdp < P^b and (4e) is satisfied then
        if (W_k)^sdp is rank-one then
            S^b = S^sdp, where
            w_k^sdp (w_k^sdp)^H = W_k^sdp, ∀k ∈ K.
        else
            store (A^sdp, P^sdp) in C, apply Alg.2 from
            Sec.5.
            if P^ssoc < P^b then
                S^b = S^ssoc.
Determine relative gap using C and P^b.
Output: S^b, relative gap.
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As the numerical results will show, for the considered instances we can solve the BFIC problem to optimality, i.e., we have a gap of 0%. Note that in contrast to standard MINLP solver, such as Baron [25], we do not need spatial branching to achieve global optimality.

5. SEQUENTIAL SECOND-ORDER CONE ALGORITHM

In our SDP-based BnB algorithm, at every leaf node we solve the SDP relaxation (7) of the non-convex QCQP (6). If the SDP solution is not rank-one, we apply the SSOC algorithm Alg.2, which is a special case of the sequential convex programming algorithm [28]. The convergence of the SSOC algorithm has been proven in [28]. For easier presentation, we consider the slightly more general real-valued optimization problem:

$$\min_{\{\mathbf{z}\in\mathbb{R}^n\}} \mathbf{z}^T\mathbf{z} \quad \text{s.t. } \|((\mathbf{C}_m\mathbf{z})^T,1)\|_2 - \|\mathbf{B}_m\mathbf{z}\|_2 \le 0,\ m\in\mathcal{M} \tag{8}$$

where $\mathbf{C}_m \in \mathbb{R}^{k_m\times n},\ \mathbf{B}_m \in \mathbb{R}^{l_m\times n}, m\in\mathcal{M}$. Casting the QCQP subproblems (6) of the BFIC problem (4) into this form is straightforward. In particular, there exist mappings

$$[\mathbf{w}_1,\dots,\mathbf{w}_K]\in\mathbb{C}^{L\times K} \mapsto \mathbf{z}\in\mathbb{R}^{2LK\times 1} \tag{9}$$

$$[\mathbf{h}_1,\dots,\mathbf{h}_K]\in\mathbb{C}^{L\times K} \mapsto \mathbf{B}_m\in\mathbb{R}^{2\times 2LK}, m\in\mathcal{M} \tag{10}$$

$$[\mathbf{h}_1,\dots,\mathbf{h}_K]\in\mathbb{C}^{L\times K} \mapsto \mathbf{C}_m\in\mathbb{R}^{2K\times 2LK}, m\in\mathcal{M}. \tag{11}$$

The SSOC algorithm consists of successive linear approximation of the non-convex part of the constraint set in (8) [28–31]. Given an iterate $\mathbf{z}^{(j)}$, the convex optimization problem solved in the jth iteration of the SSOC algorithm, is given by:

$$\min_{\{\mathbf{z}\in\mathbb{R}^n\}} \mathbf{z}^T\mathbf{z} \tag{12}$$

$$\text{s.t. } \|((\mathbf{C}_m(\mathbf{z}))^T,1)\|_2 \le \frac{\sum_{i=1}^{l_m}(\mathbf{B}_m)_{i\cdot}\mathbf{z}^{(j)}\cdot(\mathbf{B}_m)_{i\cdot}}{\|\mathbf{B}_m\mathbf{z}^{(j)}\|_2}\,\mathbf{z},\ m\in\mathcal{M}$$

where $(\mathbf{B}_m)_{i\cdot}$ denotes the ith row of the matrix $\mathbf{B}_m$. When applying Alg.2, in our SDP-based BnB Alg.1 the required starting point is generated as follows: Given the solution $\overline{\mathbf{W}}_k, k\in\mathcal{K}$, of the SDP (7) we set $\mathbf{w}_k = (\lambda_{\max}(\overline{\mathbf{W}}_k))^{1/2}\,\mathbf{v}_k,\ k\in\mathcal{K}$, where $\lambda_{\max}(\overline{\mathbf{W}}_k)$ denotes the maximal eigenvalue of the matrix $\overline{\mathbf{W}}_k$ with corresponding normalized eigenvector $\mathbf{v}_k$. The vector $\mathbf{z}^{(1)}$ is obtained from $\{\mathbf{v}_k\}_{k=1}^K$ according to the mapping in (9).

Algorithm 2: Sequential Second-Order Cone Algorithm

Initialization: Choose starting point $\mathbf{z}^{(1)}$. Set $j=1$.
Step 1: Given $\mathbf{z}^{(j)}$, solve (12), denote the solution by $\mathbf{z}^{(j+1)}$.
Step 2: If $\|\mathbf{z}^{(j)}-\mathbf{z}^{(j+1)}\|\le\varepsilon$, Stop, Return $\mathbf{z}^{(j)}$.
Otherwise, increase j by 1, and go back to Step 1.

6. A LOW- COMPLEXITY HEURISTIC FOR BFIC

Although Alg.1 is able to globally solve the BFIC problem to optimality for all considered instances, computational low-complexity suboptimal solutions for the BFIC problem (7) are important for large-scale applications. We propose the following low-channel-gain procedure. Besides its entitlement as a stand-alone heuristic, the low-channel-gain procedure is used in the preprocessing step of Alg.1. In the low-channel-gain procedure, summarized in Alg. 3, we first fix the decoding order for all MSs, denoted by $\mathbf{A}$. In the second step, the corresponding problem (4) is approximately solved, via solving the corresponding SDP (7) and if needed using the SSOC algorithm Alg.2 from Sec. 5 to generate a feasible solution for the BFIC problem (4). We compare the objective value of the generated solution to that of the downlink beamforming problem without IC ($\mathbf{A}^{db}=(a_{l,k}), a_{k,k}=1,\forall k\in\mathcal{K}$) and take the best of the two. For the low-channel-gain procedure two SDPs have to be solved, and Alg. 2 has to be applied at most once.

Algorithm 3: Low-channel-gain procedure for BFIC problem

Initialization: Determine $\|\mathbf{h}_k\|, \forall k\in\mathcal{K}$. We assume without loss of generality $\|\mathbf{h}_1\|\le\|\mathbf{h}_2\|\le\cdots\le\|\mathbf{h}_K\|$.
Choose the $\lfloor K/2\rfloor$ MSs with lowest channel gain, and store indices in the set $\mathcal{I}:=\{1,2,\dots,\lfloor K/2\rfloor\}$.
Calculate $D_{k,l}:=|\mathbf{h}_k^H\mathbf{h}_l|/(\|\mathbf{h}_k\|\|\mathbf{h}_l\|), \forall k,l\in\mathcal{K}$.
Step 1: Fix the decoding order for each MS $\mathbf{A}=(a_{k,l})$:
for $k\in\mathcal{K}$ **do**
  $\bar{l}_k = \text{argmax}_{l\in\mathcal{I}}\ D_{k,l}$.
  if $\max_{l\in\mathcal{I}}\ D_{k,l}\ge 0.3$ set $a_{k,\bar{l}_k}=1$ **else** $a_{k,k}=1$.
Step 2: (Locally) solve problem (4) for $\mathbf{A}=(a_{k,l})$ fixed.
Step 3: Choose best out of $\mathbf{A}$ and $\mathbf{A}^{db}$.

7. NUMERICAL RESULTS

We simulate a network comprising one BS equipped with 4 antennas and K MS, where $K\in\{3,4,5,6\}$. The SINR threshold $\gamma=\gamma_k,\ k\in\mathcal{K}$, varies from -4 dB to 6 dB with a step size of 2. The channel model and parameters are chosen as in [32, 33]. Results are averaged over 200 Monte Carlo runs. The merit of SSIC in terms of power reduction compared to downlink beamforming without IC is summarized in Fig.1. The curves from top to bottom correspond respectively to the optimal solution computed by our SDP-based BnB algorithm, the solution of the heuristic, the solution of the downlink beamforming problem, and, as a benchmark, the best solution found by Baron, using a Big-M formulation of (4), with a runtime limit of 100s and the downlink beamforming solution as an initial solution. The time limit is chosen such that a fair comparison to our proposed BnB algorithm can be made which requires a maximal running time of 52.9s (on average 6.9s) for $K=5$ MSs. In contrast to Baron, we were able to globally solve the BFIC (4) for all considered instances using the proposed Alg.1. Fig. 1, for example shows that for $\gamma=2$dB a power reduction of 70%, 59% can be achieved with Alg.1, Alg.3, respectively. In addition to the achievable power reduction, with SSIC more MS can be served compared to the case of downlink beamforming without IC. For instance, for $K=6$ and $\gamma_k\ge 4$dB the downlink beamforming problem was always infeasible whereas the BFIC problem was feasible and could be solved using Alg.1. The average number of visited nodes, i.e., the number of solved SDPs in Alg.1, to solve the BFIC problem (4), is displayed in the table below. In the second row, we added the number of possible decoding orders. The SSOC algorithm Alg.2 was applied in 1%, 9% of the considered instances for $K=5,6$, respectively.

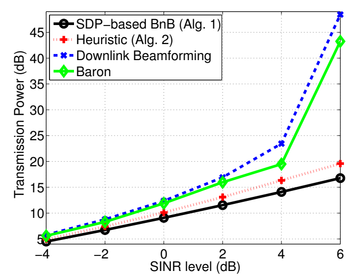

Fig. 1. Power reduction due to SSIC for K=5.

K (number of MSs)	3	4	5	6
nodes in Alg.1	9.5	21	73	430
exhaustive search	27	256	3126	46656

8. REFERENCES

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