SINR BALANCING FOR NON-REGENERATIVE TWO-WAY RELAY NETWORKS WITH INTERFERENCE NEUTRALIZATION

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ABSTRACT

In this paper we consider a multi-pair two-way relaying network with two types of relays, namely, smart multi-antenna amplify and forward relays and dumb repeaters. The smart relays are able to perform adaptive linear precoding while the dumb repeaters are only able to forward the received signals. Utilizing an interference neutralization scheme, a closed-form transmit strategy can be computed for our scenario. We derive necessary and sufficient conditions for the feasibility of interference neutralization. This provides interesting insights how to choose system parameters like the number of antennas and the number of relays. When the SINR balancing problem is considered, simulation results show that the interference neutralization solution provides a balance between the computational complexity and the performance when compared to optimal transmit strategies.

Index Terms— interference cancellation, wireless relays, MIMO, mathematical programming

1. INTRODUCTION AND CONTRIBUTIONS

Relays can be deployed to extend the coverage of wireless networks. Considering the complexity of signal processing at the relays, they can be categorized into two types: smart relays and dumb repeaters. The smart relays are able to gather channel state information and perform linear precoding while the dumb repeaters are only able to serve as amplifiers. Since the repeaters are oblivious to the source and destination nodes and the system is interference limited, it can only benefit a little from the repeaters [1]. Therefore, additional smart relays are suggested to be deployed to further improve the system performance in [1]. Multi-pair relay networks with both types of relays are in general difficult to handle due to the existence of different types of interference [2], [3]. On the other hand, the interference neutralization technique, which tunes the interfering signals such that they neutralize each other at the destination node [4], is proven to be a powerful tool to handle interference in a multi-pair one-way relaying (OWR) network with both smart relays and repeaters [2], in deterministic channels [4, 5] and two-hop relay channels [6]. Nevertheless, a multi-pair two-way relaying (TWR) network with both types of relays has not been considered in the literature.

Thus, in this paper we study a multi-pair TWR network with repeaters and multiple smart multi-antenna amplify and forward (AF) relays, where the relay amplification matrices at each relay are designed. We first apply the interference neutralization technique, which nulls the interference in the network. Afterwards, we obtain a system which consists of multiple independent point-to-point two-way relaying subsystems, and thus the system design is simplified. The necessary and sufficient condition for the feasibility of interference neutralization is characterized and a closed-form solution is obtained. When the SINR balancing problem is considered, we propose an iterative algorithm which is based on the Dinkelbach type II method [7]. The proposed method guarantees a superlinear convergence speed to the optimal solution. When compared to optimal transmit strategies, the interference neutralization solution provides a balance between the computational complexity and the performance.

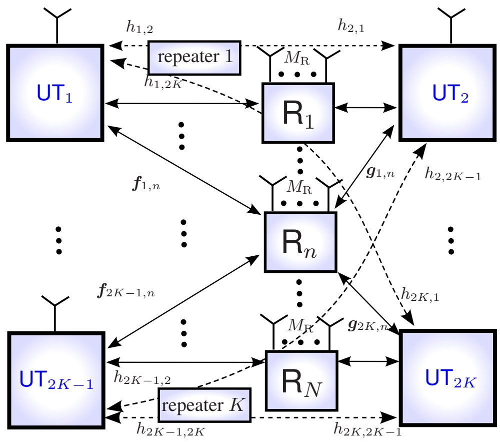

Fig. 1. Multi-pair two-way relaying with multiple amplify and forward relays where each relay has M_R antennas.

2. SYSTEM MODEL

The scenario under investigation is shown in Fig. 1, where K pairs of single-antenna user terminals (UTs) would like to communicate with each other via the help of N *smart* relays and K *dumb* repeaters. Each smart relay has M_R antennas. We assume the channel is i.i.d. frequency flat and quasi-static block fading. The channel vector from the $(2k-1)$-th UT to the n-th relay is denoted as $\boldsymbol{f}_{2k-1,n}$ ($n \in \{1, \cdots, N\}$) and the cascaded channel vector of the $(2k-1)$-th UT to all the relays is $\boldsymbol{f}_{2k-1} = [\boldsymbol{f}_{2k-1,1}^\mathrm{T}, \boldsymbol{f}_{2k-1,2}^\mathrm{T}, \ldots, \boldsymbol{f}_{2k-1,N}^\mathrm{T}]^\mathrm{T} \in \mathbb{C}^{NM_\mathrm{R}}$. Meanwhile, the channel from the $(2k)$-th user to the n-th relay is denoted as $\boldsymbol{g}_{2k,n}$ and the cascaded channel vector of the $(2k)$-th UT to all the relays is $\boldsymbol{g}_{2k} = [\boldsymbol{g}_{2k,1}^\mathrm{T}, \boldsymbol{g}_{2k,2}^\mathrm{T}, \ldots, \boldsymbol{g}_{2k,N}^\mathrm{T}]^\mathrm{T} \in \mathbb{C}^{NM_\mathrm{R}}$, for $k \in \{1, 2, \cdots, K\}$. The repeaters in the network do not cooperate with each other and amplify only their received signals [2]. Therefore, the equivalent channel from the i-th UT to the j-th UT via the network of repeaters is modeled as a single variable, which is denoted as $h_{i,j}$ ($\{i, j\} \in \{1, \cdots, 2K\}$). We assume that the reciprocity holds for the smart relay channel as well as for the

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repeaters' channel such that $h_{i,j} = h_{j,i}$. This is valid in an ideal reciprocal time-division duplex (TDD) system. The signals passing through the repeaters and the smart relays are assumed to arrive at the destination at the same time (symbol-synchronous). The transmission takes two time slots. In the first time slot, all the UTs transmit to the relays and the repeaters. The signal received at the n-th relay can be combined in a vector as

$$\boldsymbol{r}_n = \sum_{k=1}^{K} (\boldsymbol{f}_{2k-1,n} s_{2k-1} + \boldsymbol{g}_{2k,n} s_{2k}) + \boldsymbol{n}_{\mathrm{R},n} \in \mathbb{C}^{M_\mathrm{R}} \quad (1)$$

where s_m ($m \in \{1, 2, 3, \ldots, 2K\}$) is i.i.d. with zero mean and unit variance and $\boldsymbol{n}_{\mathrm{R},n}$ represents the zero-mean circularly symmetric complex Gaussian (ZMCSCG) noise with covariance matrix $\mathbb{E}\{\boldsymbol{n}_{\mathrm{R},n}\boldsymbol{n}_{\mathrm{R},n}^{\mathrm{H}}\} = \sigma_\mathrm{R}^2 \boldsymbol{I}_{M_\mathrm{R}}, \forall n$. In the second time slot, the repeaters simply amplify and forward the received signal while the n-th smart relay transmits $\bar{\boldsymbol{r}}_n = \boldsymbol{W}_n \boldsymbol{r}_n$, where $\boldsymbol{W}_n \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{R}}$ is the relay amplification matrix and the relay transmit power constraint has to be fulfilled such that $\sum_{n=1}^{N} \mathbb{E}\{\|\bar{\boldsymbol{r}}_n\|^2\} \le P_{\mathrm{R,max}}$, if a total sum relay power constraint is considered as in [8]. Finally, the received signal at the $(2k-1)$-th user can be written as

$$\begin{aligned} y_{2k-1} = & \underbrace{\left(h_{2k-1,2k} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{g}_{2k}\right) s_{2k}}_{\text{desired signal}} \\ & + \underbrace{\left(h_{2k-1,2k-1} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{f}_{2k-1}\right) s_{2k-1}}_{\text{self-interference}} \\ & + \underbrace{\sum_{\substack{\ell \neq k \\ \ell=1}}^{K} (h_{2k-1,2\ell-1} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{f}_{2\ell-1}) s_{2\ell-1}}_{\text{inter-pair interference}} \\ & + \underbrace{\sum_{\substack{\ell \neq k \\ \ell=1}}^{K} (h_{2k-1,2\ell} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{g}_{2\ell}) s_{2\ell}}_{\text{inter-pair interference}} + \underbrace{\boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \bar{\boldsymbol{n}}_\mathrm{R} + n_{2k-1}}_{\text{effective noise}} \end{aligned}$$

where $\tilde{\boldsymbol{W}} \in \mathbb{C}^{NM_\mathrm{R} \times NM_\mathrm{R}}$ is defined as $\tilde{\boldsymbol{W}} = \mathrm{blkdiag}\{\boldsymbol{W}_n\}$ and $\mathrm{blkdiag}\{\cdot\}$ stands for the block diagonal operation. The ZMCSCG noise n_{2k-1} has variance $\sigma_U^2, \forall k$. If the channel is known at the receiver, the self-interference term can be subtracted and thus we get

$$\hat{y}_{2k-1} = y_{2k-1} - \left(h_{2k-1,2k-1} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{f}_{2k-1}\right) s_{2k-1}.$$

Let $\mathrm{vec}\{\cdot\}$ stand for the operation which stacks the columns of a matrix into a vector and the $\mathrm{unvec}_{M \times N}\{\cdot\}$ operator stand for the inverse function of $\mathrm{vec}\{\cdot\}$. Let $\otimes$ denote the Kronecker product. Using the fact that $\mathrm{vec}\{\boldsymbol{\Gamma X \zeta}\} = (\boldsymbol{\zeta}^{\mathrm{T}} \otimes \boldsymbol{\Gamma})\mathrm{vec}\{\boldsymbol{X}\}$, we can show that

$$\boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{g}_{2k} = \boldsymbol{h}_{2k-1,2k}^{\mathrm{T}} \tilde{\boldsymbol{w}} = \left(\mathrm{vec}\{\tilde{\boldsymbol{G}}_{2k} \diamond \tilde{\boldsymbol{F}}_{2k-1}\}\right)^{\mathrm{T}} \tilde{\boldsymbol{w}}$$

where $\diamond$ denotes the Khatri-Rao product, which is defined as the column-wise Kronecker product [9], and where $\tilde{\boldsymbol{w}}$ is defined as

$$\tilde{\boldsymbol{w}} = \begin{bmatrix} \mathrm{vec}\{\boldsymbol{W}_1\}^{\mathrm{T}} & \cdots & \mathrm{vec}\{\boldsymbol{W}_N\}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{NM_\mathrm{R}^2}, \quad (2)$$

where the matrices $\tilde{\boldsymbol{F}}_{2k-1} = \mathrm{unvec}_{M_\mathrm{R} \times N}\{\boldsymbol{f}_{2k-1}\}$ and $\tilde{\boldsymbol{G}}_{2k} = \mathrm{unvec}_{M_\mathrm{R} \times N}\{\boldsymbol{g}_{2k}\}$. Moreover, we introduce an auxiliary variable $\alpha \in \mathbb{C}$ where $|\alpha|^2 = 1$. Utilizing the Charnes-Cooper transform [10], we let $\boldsymbol{\eta_1}/\alpha = \tilde{\boldsymbol{w}}$ and $\boldsymbol{w} = \begin{bmatrix} \boldsymbol{\eta_1}^{\mathrm{T}} & \alpha \end{bmatrix}^{\mathrm{T}} \in \mathbb{C}^{NM_\mathrm{R}^2+1}$. Then it is possible to derive the signal-to-noise-plus-interference ratio (SINR) of the $(2k-1)$-th UT as a function of $\boldsymbol{w}$, yielding

$$\gamma_{2k-1} = \frac{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{E}_{2k-1}^{(\mathrm{g})} \boldsymbol{w}}{\boldsymbol{w}^{\mathrm{H}} \boldsymbol{F}_{2k-1}^{(\mathrm{g})} \boldsymbol{w}} \quad (3)$$

where $\boldsymbol{E}_{2k-1} = \bar{\boldsymbol{h}}_{2k-1,2k}^{*} \bar{\boldsymbol{h}}_{2k-1,2k}^{\mathrm{T}}$ and

$$\begin{aligned} \boldsymbol{F}_{2k-1} = & \sum_{\substack{\ell \neq k \\ \ell=1}}^{K} \bar{\boldsymbol{h}}_{2k-1,2\ell-1}^{*} \bar{\boldsymbol{h}}_{2k-1,2\ell-1}^{\mathrm{T}} + \bar{\boldsymbol{h}}_{2k-1,2\ell}^{*} \bar{\boldsymbol{h}}_{2k-1,2\ell}^{\mathrm{T}} \\ & + \mathrm{blkdiag}\{\sigma_\mathrm{R}^2 \tilde{\boldsymbol{H}}_{2k-1}^{*} \tilde{\boldsymbol{H}}_{2k-1}^{\mathrm{T}}, \sigma_\mathrm{U}^2\} \end{aligned}$$

with $\bar{\boldsymbol{h}}_{i,j} = \begin{bmatrix} \boldsymbol{h}_{i,j}^{\mathrm{T}} & h_{i,j} \end{bmatrix}^{\mathrm{T}}$ and $\tilde{\boldsymbol{H}}_{2k-1} = \mathrm{blkdiag}\{\boldsymbol{I}_{M_\mathrm{R}} \otimes \boldsymbol{f}_{2k-1,n}\}, \forall n$.

Similarly, the total transmit power constraint of the relays in the network can be expressed as

$$\sum_{n=1}^{N} \mathbb{E}\{\|\bar{\boldsymbol{r}}_n\|^2\} = \tilde{\boldsymbol{w}}^{\mathrm{H}} \tilde{\boldsymbol{C}} \tilde{\boldsymbol{w}} \le P_{\mathrm{R,max}} \Leftrightarrow \boldsymbol{wCw} \le 0 \quad (4)$$

where $\boldsymbol{C} = \mathrm{blkdiag}\{\tilde{\boldsymbol{C}}, -P_{\mathrm{R,max}}\}$, $\tilde{\boldsymbol{C}} = \mathrm{blkdiag}\Big\{(\bar{\boldsymbol{F}}_n^{*} \bar{\boldsymbol{F}}_n^{\mathrm{T}} + \bar{\boldsymbol{G}}_n^{*} \bar{\boldsymbol{G}}_n^{\mathrm{T}} + \sigma_\mathrm{R}^2 \boldsymbol{I}_{M_\mathrm{R}}) \otimes \boldsymbol{I}_{M_\mathrm{R}}\Big\}$, and where we have the following identities $\bar{\boldsymbol{F}}_n = \begin{bmatrix} \boldsymbol{f}_{1,n} & \cdots & \boldsymbol{f}_{2K-1,n} \end{bmatrix} \in \mathbb{C}^{M_\mathrm{R} \times K}$ and $\bar{\boldsymbol{G}}_n = \begin{bmatrix} \boldsymbol{g}_{2,n} & \cdots & \boldsymbol{g}_{2K,n} \end{bmatrix} \in \mathbb{C}^{M_\mathrm{R} \times K}$.

For a UT with even index, e.g., γ_{2k}, the SINR expression can be obtained by replacing $2k$ and $(2k-1)$ by $(2k-1)$ and $2k$, correspondingly. *Note that all the derivations and the proofs in Sections 3 and 4 are omitted due to space limitations.*

3. FEASIBILITY OF INTERFERENCE NEUTRALIZATION

In this section we show how the relay forwarding strategy can be chosen to neutralize all interference and which conditions are necessary and sufficient to achieve this. To this end, the following equalities must be satisfied at the same time. For all $\ell, k \in \{1, \cdots, K\}, \ell \neq k$,

$$h_{2k-1,2\ell-1} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{f}_{2\ell-1} = 0 \quad (5a)$$

$$h_{2k-1,2\ell} + \boldsymbol{f}_{2k-1}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{g}_{2\ell} = 0 \quad (5b)$$

$$h_{2k,2\ell-1} + \boldsymbol{g}_{2k}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{f}_{2\ell-1} = 0 \quad (5c)$$

$$h_{2k,2\ell} + \boldsymbol{g}_{2k}^{\mathrm{T}} \tilde{\boldsymbol{W}} \boldsymbol{g}_{2\ell} = 0. \quad (5d)$$

Equation (5a) describes the interference from any odd-indexed UT to another odd-indexed UT. Similarly, (5b), (5c), (5d) describe the interference from any even-indexed UT to any odd-indexed UT, from any odd-indexed UT to any even-indexed UT and from any even-indexed UT to even-indexed UT, respectively. The feasibility conditions in (5) can be quantified by four parameters: the number of relay nodes N, the number of antennas at each relay node M_R, the number of UT pairs K, and the maximum available power at the relay $P_{\mathrm{R,max}}$. The conditions are summarized in the following main result:

Theorem 1. *Assume that we have a two-way relay channel with $2K$ UTs, N relay nodes each with M_R antennas, and sufficient power*

$P_{\mathrm{R,max}}$ *at the relay. The interference neutralization requirements, given in* (5)*, can be satisfied if and only if both of the following criteria are satisfied:*

1. *The total available number of antennas in the network should satisfy*
$$2K(K-1) \leq \frac{1}{2} N M_{\mathrm{R}}(M_{\mathrm{R}}+1). \quad (6)$$
2. *Given the interference neutralization solution as* $\boldsymbol{w}^{(\mathrm{I})}$*, the available relay power* $P_{\mathrm{R,max}}$ *should satisfy*
$$P_{\mathrm{R,max}} \geq \boldsymbol{w}^{(\mathrm{I})^{\mathrm{H}}} \tilde{\boldsymbol{C}} \boldsymbol{w}^{(\mathrm{I})}. \quad (7)$$

Corollary 1. *Assume that interference neutralization is feasible, i.e., inequalities (6) and (7) are satisfied. Let* $\boldsymbol{K}_{M_{\mathrm{R}}^2}$ *be a commutation matrix as defined in [11]. Define the SVD of* $\bar{\boldsymbol{K}} = \boldsymbol{I}_N \otimes (\boldsymbol{I}_{M_{\mathrm{R}}^2} - \boldsymbol{K}_{M_{\mathrm{R}}^2})$ *as* $\bar{\boldsymbol{K}} = \boldsymbol{U}\boldsymbol{\Sigma}[\boldsymbol{V}_{\mathrm{s}} \quad \boldsymbol{V}_{\mathrm{n}}]^{\mathrm{H}}$ *where* $\boldsymbol{V}_{\mathrm{n}} \in \mathbb{C}^{NM_{\mathrm{R}}^2 \times (NM_{\mathrm{R}}^2 - r_2)}$ *spans the null space of* $\bar{\boldsymbol{K}}$ *and* r_2 *is the rank of* $\bar{\boldsymbol{K}}$*. The interference neutralization solution of* $\boldsymbol{w}^{(\mathrm{I})}$ *is computed as*

$$\boldsymbol{w}^{(\mathrm{I})} = \boldsymbol{V}_{\mathrm{n}}((\bar{\boldsymbol{A}}\boldsymbol{V}_{\mathrm{n}})^{+}\boldsymbol{b} + (\boldsymbol{I}_{NM_{\mathrm{R}}^2 - r_2} - (\bar{\boldsymbol{A}}\boldsymbol{V}_{\mathrm{n}})^{+}\bar{\boldsymbol{A}}\boldsymbol{V}_{\mathrm{n}})\tilde{\boldsymbol{w}}_{\mathrm{n}}) \quad (8)$$

where $\{\cdot\}^{+}$ *denotes the Moore-Penrose pseudoinverse and the vector* $\boldsymbol{w}^{(\mathrm{I})} \in \mathbb{C}^{NM_{\mathrm{R}}^2 - r_2}$ *contains the degrees of freedom that can be used for further system improvements. Define* $\bar{i} \in \{1, \cdots K\}$ *and* $\bar{j} \in \{\bar{i}+1, \cdots K\}$*. The column-vector* $\boldsymbol{b}$ *is generated by*

$$\boldsymbol{b} = -\begin{bmatrix} h_{2\bar{i}-1,2\bar{j}-1} & h_{2\bar{i}-1,2\bar{j}} & h_{2\bar{i},2\bar{j}-1} & h_{2\bar{i},2\bar{j}} \end{bmatrix}^{\mathrm{T}}, \forall \bar{i}, \bar{j}$$

and the corresponding $\bar{\boldsymbol{A}}$ *is generated via*

$$\bar{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{h}_{2\bar{i}-1,2\bar{j}-1} & \boldsymbol{h}_{2\bar{i}-1,2\bar{j}} & \boldsymbol{h}_{2\bar{i},2\bar{j}-1} & \boldsymbol{h}_{2\bar{i},2\bar{j}} \end{bmatrix}^{\mathrm{T}}, \forall \bar{i}, \bar{j}.$$

The interference neutralization solution modifies the original system model. Nevertheless, one can easily prove that the SINR and the power constraint can be written in the same form as (3) and (4), correspondingly.

Remark 1. Antenna layout design is a relevant problem in getting better network planning and resource management in a relay-assisted wireless network. Given the total number of antennas in the network, in one extreme it is possible to group all antennas in one "mega relay" which is powerful and manages all network resources and traffic. In the other extreme, one can distribute the antennas uniformly in the geometric space, such as in sensor networks. Or, a compromise between both schemes: bundles of antennas are distributed at various locations in the network. Our feasibility study on interference neutralization provides an interesting result on *how the total number of antennas in the network, which are used for interference management / neutralization, can be decreased when clusters of relays can be formed*. For example, when single antenna relays cannot cooperate with each other, condition (6) implies that we need $2K(K-1)$ relays (number of antennas) in total. However, if we allow 3 single-antenna relays to form a cluster - a multi-antenna relay, according to (6), the number of antennas required in the network decreases by a factor of two: $NM_R = K(K-1)$.

Remark 2. The interference neutralization solution obtained by (8) is a closed-form solution if $\tilde{\boldsymbol{w}}_{\mathrm{n}}$ is chosen randomly but fulfils the total relay transmit power constraint. Moreover, since $\tilde{\boldsymbol{w}}_{\mathrm{n}}$ has a lower dimension compared to $\tilde{\boldsymbol{w}}$, the computational complexity of the corresponding optimization problem will be lower.

4. SINR BALANCING VIA DINKELBACH-TYPE METHOD

SINR balancing aims at maximizing the minimum SINR of the UTs in the network subject to the transmit power constraint at the relay. In the following we discuss the SINR balancing solution with and without interference neutralization.

Since the two cases possess the same expressions for the SINR and the power constraint, we take the general system model in Section 2 as an example. Our optimization problem with a sum power constraint is formulated as

$$\begin{aligned} \max_{\boldsymbol{w}} \min_{m} \quad & \gamma_m \\ \text{s.t.} \quad & \boldsymbol{w}^{\mathrm{H}}\boldsymbol{C}\boldsymbol{w} \leq 0 \\ & \boldsymbol{w}^{\mathrm{H}}\boldsymbol{C}_1\boldsymbol{w} = 1 \end{aligned} \quad (9)$$

where $\boldsymbol{C}_1 = \mathrm{blkdiag}\{\boldsymbol{0}_{NM_{\mathrm{R}}^2}, 1\}$ and the second constraint comes from the fact that $|\alpha|^2 = 1$.

Problem (9) is non-convex. Hence, it may not be solvable in polynomial time. But its approximate solution can be obtained by using the semidefinite relaxation (SDR) techniques in [12]. We introduce a new variable $\boldsymbol{X} = \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$ and rewrite problems (9) as

$$\begin{aligned} \lambda_{\mathrm{opt}} = \max_{\boldsymbol{X}} \min_{m} \quad & \frac{\mathrm{Tr}\{\boldsymbol{E}_m\boldsymbol{X}\}}{\mathrm{Tr}\{\boldsymbol{F}_m\boldsymbol{X}\}} \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq 0 \\ & \mathrm{Tr}\{\boldsymbol{C}_1\boldsymbol{X}\} = 1 \end{aligned} \quad (10)$$

With an additional non-convex constraint $\mathrm{rank}\{\boldsymbol{X}\} = 1$, the newly formulated problems (10) are equivalent to the original problem. Therefore, if the optimal solution $\boldsymbol{X}_{\mathrm{opt}}$ to problems (10) is a rank-1 matrix, it is also the optimal solution to the original problem (9). Otherwise, rank-1 extraction/approximation techniques should be applied [12]. Due to the minimum over m, our problem (10) contains more than three constraints and thus a rank-one solution is not guaranteed according to [13, Theorem 3.2 & Corollary 3.4]. Hence, the randomization technique in [12], which is a rank-1 approximation technique, will be used to get an approximate solution after we solve (10). In the following we will propose an algorithm to solve (10), which is based on the Dinkelbach-type II (DT-II) algorithm.

We start from introducing a parametric programming formulation for our problem. As the SDR technique will be eventually applied, we take a short-cut and start with (10). A parametric programming formulation of (10) is given by

$$\begin{aligned} f(\lambda) = \max_{\boldsymbol{X}} \min_{m} \quad & \mathrm{Tr}\{\boldsymbol{E}_m\boldsymbol{X}\} - \lambda \mathrm{Tr}\{\boldsymbol{F}_m\boldsymbol{X}\} \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq 0 \\ & \mathrm{Tr}\{\boldsymbol{C}_1\boldsymbol{X}\} = 1 \end{aligned} \quad (11)$$

where parametric here implies that we consider the solution of this optimization problem for varying values of λ. This formulation is especially useful if $f(\lambda)$ is a convex function with respect to $\boldsymbol{X}$. Because it is easier to solve a convex problem (11) than a non-convex problem (10). Problem (11) is equivalent to (10) if there exists λ such that $f(\lambda) = 0$ [7]. Thus, this gives rise to finding the root of the equation $f(\lambda) = 0$. One of the iterative methods derived under this concept is the so called Dinkelbach algorithm, where the original version refers to the case $m = 1$. When $m > 1$, the Dinkelbach type II (DT-II) algorithm is suggested in [7]. One prerequisite for finding the optimal solution using the Dinkelbach type algorithms is that the objective value of the original problem, i.e., (9), needs to be

Table 1. The DT-II algorithm for solving (10)

Input: initial value $\lambda^{(1)}$, a threshold value ϵ, $\boldsymbol{C}$, $\boldsymbol{C}_1$, $\boldsymbol{E}_m$, $\boldsymbol{F}_m$, $\forall m$, and maximum number of iterations $N_{\max}$.
Output: an optimal solution $\boldsymbol{X}$ with arbitrary rank.

Main step:

for $p = 1$ to $N_{\max}$ **do**

Obtain $(\boldsymbol{X}^{(p)}, t_1^{(p)})$ by solving

$$\begin{aligned} \max_{\boldsymbol{X}, t_1} \quad & t_1 \\ \text{s.t.} \quad & \mathrm{Tr}\{\boldsymbol{C}\boldsymbol{X}\} \leq 0, \\ & \mathrm{Tr}\{\boldsymbol{C}_1\boldsymbol{X}\} = 1 \qquad (12) \\ & \frac{\mathrm{Tr}\{\boldsymbol{E}_m\boldsymbol{X}\} - \lambda^{(p)}\mathrm{Tr}\{\boldsymbol{F}_m\boldsymbol{X}\}}{\mathrm{Tr}\{\boldsymbol{F}_m\boldsymbol{X}^{(p-1)}\}} \geq t_1, \forall m. \end{aligned}$$

calculate $\lambda^{(p+1)}$ using

$$\lambda^{(p+1)} = \min_m \frac{\mathrm{Tr}\{\boldsymbol{E}_m\boldsymbol{X}^{(p)}\}}{\mathrm{Tr}\{\boldsymbol{F}_m\boldsymbol{X}^{(p)}\}}.$$

if $|t_1^{(p)}| \leq \epsilon$ or $|\lambda^{(p+1)} - \lambda^{(p)}| \leq \epsilon$ **then**

return $\boldsymbol{X}^{(p)}$

end if

end for

finite. This is the case for our problem. The SINR expressions in (3) are Rayleigh quotients, which are bounded between the smallest and largest eigenvalue[1]. Thus, the objective value is finite and it is possible to extend the DT-II algorithm for our problem. The proposed iterative algorithm based on the DT-II method is summarized in Table 1. Based on the results in [7], we can show the following corollary on the convergence rate of the proposed DT-II algorithm:

Corollary 2 ([7]). *The DT-II algorithm converges at least superlinearly to the optimal solution of (10).*

5. SIMULATION RESULTS

In this section, the performance of the SINR balancing problem with and without interference neutralization are evaluated via Monte-Carlo simulations. For this purpose, we consider a system with $K = 2$ pairs of UTs. The simulated flat fading channels are spatially uncorrelated Rayleigh fading channels. The total transmit power at the relay $P_{\mathrm{R,max}}$ is fixed to unity and the noise variance is identical at all nodes, i.e., $\sigma_{\mathrm{R}}^2 = \sigma_{\mathrm{U}}^2 = \sigma_{\mathrm{n}}^2$. Thus, the SNR is defined as $1/\sigma_{\mathrm{n}}^2$. All the simulation results are obtained by averaging over 1000 channel realizations. "DT-II" denotes the case without interference neutralization and "DT-II INL" denotes the case with interference neutralization.

Fig. 2 demonstrates the convergence speed for the proposed DT-II algorithm with and without interference when $N = 4$ and $M_{\mathrm{R}} = 2$. When interference neutralization is applied, the convergence speed becomes much faster. Moreover, when interference neutralization is not used, we observe from numerical simulations that the obtained $\boldsymbol{X}_{\mathrm{opt}}$ is almost always rank-one.

In Fig. 3 we compare the achievable minimum SINR of various algorithms under two different system settings, i.e., $N = 2$, $M_{\mathrm{R}} = 4$ and $N = 4$, $M_{\mathrm{R}} = 2$. Other than the proposed algorithms "DT-II" and "DT-II INL", the following two algorithms have also been compared. The first one is denoted as "Non-smart", which refers to the scheme where smart relays are not deployed. The second one is denoted as "TDMA". This scheme refers to an orthogonal resource access where each pair of the UTs utilize the relays and repeaters in the network in a time-division multiple access (TDMA) fashion. Thus, for a fair comparison, peak power constraints are used in the simulation and the simulation results obtained using the "TDMA" scheme are additionally divided by K. Clearly, when there are no smart relays in the network, the presence of interferences will significantly affect the system performance. On the other hand, the orthogonal resource access scheme "TDMA" has its benefits especially in the low SNR regime. Among the two non-orthogonal resource access schemes, the interference neutralization scheme "DT-II INL" provides a balance between the computational complexity and the performance. Moreover, when the total number of antennas is limited in a network, to have a better system performance, it is more reasonable to have a few relays but many antennas at each relay.

[1]The parameter λ is bounded between $\min_m \mathcal{P}_{\min}\{\boldsymbol{F}_m^{-1}\boldsymbol{E}_m\}$ and $\min_m \mathcal{P}_{\max}\{\boldsymbol{F}_m^{-1}\boldsymbol{E}_m\}$, where $\mathcal{P}\{\cdot\}$ denotes the eigenvalue.

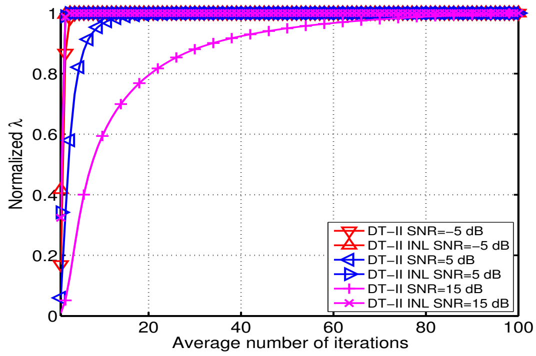

Fig. 2. Convergence speed of the proposed DT-II algorithm.

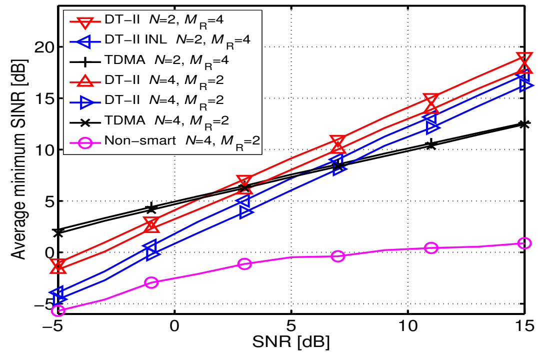

Fig. 3. A comparison of the achievable minimum SINR with and without interference neutralization.

6. CONCLUSION

Multi-pair TWR networks with both repeaters and smart relays suffer from various types of interference. When the interference neutralization condition (5) is satisfied, a closed-form solution can be obtained to neutralize the interference in the network. When SINR balancing is chosen as the system design criterion, we have compared the proposed non-orthogonal resource access schemes with or without interference neutralization to a TDMA scheme, which can be seen as an orthogonal resource access scheme. Simulation results show that the proposed non-orthogonal resource access schemes have larger degrees of freedom, i.e., a better SINR slope, than the TDMA scheme in the high SNR regime.

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