

# FILTER-AND-FORWARD RELAY BEAMFORMING USING OUTPUT POWER MINIMIZATION

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## ABSTRACT

In this paper, we consider designing the Filter-and-forward (FF) relay beamforming in frequency-selective channels using a new approach. The proposed approach aims to minimize the output power at the destination side while keeping the response to the desired signal at a constant level, and the problem is subject to both total and individual relay transmit power constraints. It is shown that the proposed beamforming design scheme is equivalent to the SINR maximization formulation, in terms of achieving the same output SINR. Despite the equivalence in performance, the proposed approach requires significantly lower computational load for solving the problem.

**Index Terms**— Relay beamforming, filter-and-forward, output power minimization.

## 1. INTRODUCTION

Distributed relay beamforming [1–5], which is realized by the cooperating of distributed single-antenna nodes in a relay network to form a virtual multi-antenna communication system, has recently attracted considerable interests since cooperative diversity gain can be obtained from employing it.

Due to its simplicity, relay beamforming based on amplify-and-forward (AF) relay scheme is most widely studied. However, the AF scheme which aims at working on frequency-flat channels cannot be directly adopted for frequency-selective channels. In order to deal with frequency-selective channels, Chen *et al.* have proposed a filter-and-forward (FF) relaying scheme for relay beamforming [6, 7]. In this scheme, relay nodes process the received signals with finite impulse response (FIR) filters and then re-transmit the filtered signals towards the destination. Since FF distributed relay beamforming functions as distributed channel equalization, Liang *et al.* have introduced an additional decision feedback equalizer (DFE) at the destination side [8, 9]. Following the FF relaying scheme, a relay beamforming approach is proposed in [10] to address the problem of efficiently selecting the decision delay at the destination, which is shown to significantly affect the system performance.

In this paper, the FF relay beamforming design problem is considered based on a new approach of output power minimization. Specifically, the proposed approach aims to minimize the output power while keeping the response to the desired signal at a constant level, subject to both total and individual relay transmit power constraints. We also demonstrate that, in terms of output SINR performance, the proposed approach is equivalent to the SINR maximization formulation of [6]. However, in spite of the equivalence in SINR performance, the proposed approach requires significantly lower computational loads for solving the problem.

## 1.1. Relation to Prior Work

In the existing literature dealing with the relay beamforming design problems, the criterion of output SNR optimization [1–5], or output SINR optimization (when interference is taken into account) [6–9] is most widely used. Also in [10], the relay beamforming design problem is discussed based on maximizing the received SINR at the destination subject to a distortionless constraint. However, the proposed beamforming approach amounts to minimize the destination output power, with introducing a controllable scalar amplifier. Hence the response to the desired signal can be directly adjusted, thus further facilitating the subsequent symbol decision process.

The SINR maximization scheme proposed in [6] leads to a second-order cone programming (SOCP) along with a bisection search. Although we show that the proposed approach renders the same output SINR performance as the SINR maximization, our approach solves an SOCP which has a computational load comparable to merely a single iteration of the bisection search employed by [6]. That is, we propose an alternative approach to achieve the optimal SINR with significantly reduced computational complexity. In addition, we present an FF relay network signal model with different definition, with is more straightforward and concise than that used in [6, 7].

In some of the aforementioned literature on relay beamforming designs [2, 5–7], criteria based on optimizing transmit power at the source or relay side are also frequently used. By employing these criteria, the major concern lies in the power control aspects, such as the battery life of terminal equipments and interference control in a multi-user environment. But our proposed approach minimizes the output power at the destination side, and focuses on suppressing interference and noise for each destination receiver.

## 2. SIGNAL MODEL

As depicted in Figure 1, in a frequency-selective wireless channel environment, we consider a relay network with one source node, one destination node and  $R$  relay nodes, and all nodes are each equipped with a single antenna. We also assume that the source-destination direct link does not exist, and the relay nodes work in a time-division duplexing (TDD) mode. That is, a signal transmission from the source to the destination consists of two phases. In the first phase, the source broadcasts its signal to all relays, and in the second phase, the signal received at each relay is filtered and re-transmitted to the destination.

The impulse responses of the frequency-selective backward channel and forward channel that are corresponding to the  $i$ th relay are denoted by  $\mathbf{f}_i = [f_i(0), \dots, f_i(L_f - 1)]^T$  and  $\mathbf{g}_i = [g_i(0), \dots, g_i(L_g - 1)]^T$ , respectively. As also assumed in [6–10], all the instantaneous channel state information are perfectly known.

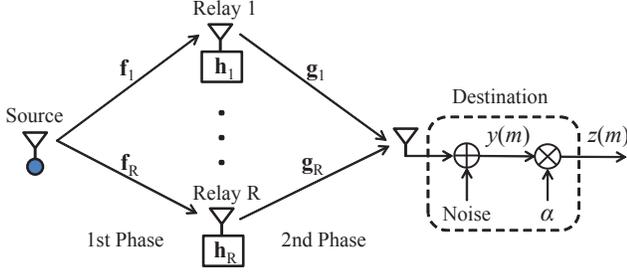


Fig. 1. Network signal model.

Hence, the signal received at the  $i$ th relay is

$$r_i(m) = s(m) * f_i(m) + n_i(m), \quad (1)$$

where  $s(m)$  is the source transmit signal with the power of  $P_S = E\{|s(m)|^2\}$ ,  $n_i(m)$  denotes the additive white Gaussian noise with power  $\sigma_n^2 = E\{|n_i(m)|^2\}$ , and  $*$  denotes the convolution sum between sequences. The relay received signal  $r_i(m)$  is processed by the relay filter with impulse response of  $\mathbf{h}_i = [h_i(0), \dots, h_i(L_h - 1)]^T$ , and then it is forwarded to the destination. Thus the signal received at the destination is

$$y(m) = \sum_{i=1}^R r_i(m) * h_i(m) * g_i(m) + v(m), \quad (2)$$

where  $v(m)$  is the additive white Gaussian noise with power  $\sigma_v^2 = E\{|v(m)|^2\}$ . Furthermore,  $y(m)$  is amplified by a controllable scalar  $\alpha \in \mathbb{C}$ , and the output signal is expressed as

$$z(m) = \alpha \cdot y(m) = \alpha \cdot s(m) * h_{\text{eqv}}(m) + \alpha \cdot n_{\text{pro}}(m) + \alpha \cdot v(m), \quad (3)$$

where  $h_{\text{eqv}}(m) = \sum_{i=1}^R f_i(m) * h_i(m) * g_i(m)$  denotes the impulse response of the equivalent channel from the source to the input of the destination, and  $n_{\text{pro}}(m) = \sum_{i=1}^R n_i(m) * h_i(m) * g_i(m)$  is the noise propagated from the relay nodes.

For better illustration and to facilitate the subsequent problem formulation, we next rewrite the overall signal model (3) in matrix form. To start with, by representing the result of the convolution  $f_i(m) * g_i(m)$  as a column vector  $\mathbf{b}_i = \mathbf{f}_i * \mathbf{g}_i = [b_{i,1}, \dots, b_{i,L_b}]^T$ , where  $L_b = (L_f + L_g - 1)$ . Then the equivalent channel  $h_{\text{eqv}}(m)$  can be rewritten in matrix form as

$$\mathbf{h}_{\text{eqv}} = \sum_{i=1}^R \Theta_i \mathbf{h}_i = \Psi \mathbf{w}^*, \quad (4)$$

where  $\Psi = [\Theta_1, \dots, \Theta_R]$ ,  $\mathbf{w} = [\mathbf{h}_1^H, \dots, \mathbf{h}_R^H]^T$ , and

$$\Theta_i = \begin{bmatrix} b_{i,1} & 0 & \dots & 0 \\ \vdots & b_{i,1} & \ddots & \vdots \\ b_{i,L_b} & \vdots & \vdots & \vdots \\ 0 & b_{i,L_b} & \ddots & 0 \\ \vdots & 0 & \vdots & b_{i,1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \vdots & b_{i,L_b} \end{bmatrix} \quad (5)$$

is a column-circulant matrix of dimension  $L_c \times L_h$ ,  $L_c = L_f + L_g + L_h - 2$ . On the other hand, the propagation noise  $n_{\text{pro}}(m)$  can be expressed in matrix form as

$$n_{\text{pro}}(m) = \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m), \quad (6)$$

where  $\mathbf{n}_i = [n_i(m), n_i(m-1), \dots, n_i(m-L_g-L_h+2)]^T$ , and  $\mathbf{G}_i$  is an  $(L_g + L_h - 1) \times L_h$  column-circulant matrix with  $[g_i(0), \dots, g_i(L_g - 1), \mathbf{0}_{1 \times (L_h - 1)}]^T$  as the first column, which has a similar structure to  $\Theta_i$  defined in (5). Therefore, from (4) and (6), we can rewrite the overall signal model (3) in matrix form as

$$z(m) = \alpha \cdot \mathbf{w}^H \Psi^T \mathbf{s}(m) + \alpha \cdot \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m) + \alpha \cdot v(m), \quad (7)$$

where  $\mathbf{s}(m) = [s(m), s(m-1), \dots, s(m-L_c+1)]^T$ .

Moreover, the intersymbol interference (ISI) in  $z(m)$  can also be represented. Suppose that at the time instance  $(m)$ , we want to estimate the transmitted symbol  $s(m-\tau)$  from  $z(m)$ , and  $\tau$  denotes the decision delay. By separating the  $(\tau+1)$ th column from the matrix  $\Psi^T$ , we can further decompose (7) as:

$$z(m) = \underbrace{\alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T s(m-\tau)}_{\text{Desired signal}} + \underbrace{\alpha \cdot \mathbf{w}^H \bar{\Psi}_\tau^T \bar{\mathbf{s}}_\tau(m)}_{\text{ISI}} + \underbrace{\alpha \cdot \sum_{i=1}^R (\mathbf{G}_i \mathbf{h}_i)^T \mathbf{n}_i(m)}_{\text{Noise}} + \alpha \cdot v(m), \quad (8)$$

where  $\vec{\psi}_\tau$  is the  $(\tau+1)$ th row of  $\Psi$ , and hence  $\bar{\Psi}_\tau$  is the submatrix of  $\Psi$  with removing the  $(\tau+1)$ th row, and  $\bar{\mathbf{s}}_\tau(m) = [s(m), \dots, s(m-\tau+1), s(m-\tau-1), \dots, s(m-L_c+1)]^T$ . Therefore, the desired symbol  $s(m)$  is separated from the ISI introduced by neighboring symbols.

### 3. FF BEAMFORMING USING OUTPUT POWER MINIMIZATION

In this section, we discuss the proposed relay beamforming design scheme of output power minimization, and its equivalence in performance to the output SINR minimization scheme is also shown.

#### 3.1. Problem Formulation

We consider an FF relay beamforming design problem which amounts to minimize the destination output power subject to both the individual relay transmit power constraint and the total relay transmit power constraint, while also keeping the response to the desired signal at a constant level:

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & P_{\text{out}} \\ \text{s.t.} \quad & \alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T = \gamma \\ & P_{\text{tot}} \leq P_0 \\ & P_i \leq P_{0,i}, \quad i = 1, \dots, R \end{aligned} \quad (9)$$

where  $P_{\text{out}}$  is the destination output power,  $P_{\text{tot}}$  is the total relay transmit power with  $P_0$  as its budget, and  $P_i$  is the individual transmit power of the  $i$ th relay with  $P_{0,i}$  as its budget. Also note that the

linear constraint is based on the "Desired Signal" term of (8), where  $\gamma \in \mathbb{R}$  is a pre-defined system parameter that indicates the constant response level to the desired signal.

First, we derive the expressions for the output power  $P_{\text{out}}$ . According to the expression for output signal decomposition (8), the output power at the destination side is given by

$$E|z(m)|^2 = |\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} + |\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_i^{(\tau)} \mathbf{w} + |\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_n \mathbf{w} + |\alpha|^2 \sigma_v^2, \quad (10)$$

where

$$\begin{aligned} \mathbf{Q}_s^{(\tau)} &= P_s \cdot \vec{\psi}_\tau^T \vec{\psi}_\tau^*, \\ \mathbf{Q}_i^{(\tau)} &= P_s \cdot \vec{\Psi}_\tau^T \vec{\Psi}_\tau^*, \\ \mathbf{Q}_n &= \sigma_n^2 \cdot \text{blkdiag}\{\mathbf{G}_1^T \mathbf{G}_1^*, \dots, \mathbf{G}_R^T \mathbf{G}_R^*\}. \end{aligned} \quad (11)$$

Next, we derive the expressions for the transmit power at the relay side. According to the network signal model, the transmit signal from the  $i$ th relay is expressed as

$$t_i(m) = s(m) * f_i(m) * h_i(m) + n_i(m) * h_i(m). \quad (12)$$

It can be further expressed in matrix form as:

$$t_i(m) = (\mathbf{F}_i \mathbf{h}_i)^T \tilde{\mathbf{s}}(m) + \mathbf{h}_i^T \tilde{\mathbf{n}}_i(m), \quad (13)$$

where  $\tilde{\mathbf{s}}(m) = [s(m), s(m-1), \dots, s(m-L_f-L_h+2)]^T$ ,  $\tilde{\mathbf{n}}_i(m) = [n_i(m), n_i(m-1), \dots, n_i(m-L_h+1)]^T$ , and  $\mathbf{F}_i$  is an  $(L_f + L_h - 1) \times L_h$  column-circulant matrix with  $[f_i(0), \dots, f_i(L_g-1), \mathbf{0}_{1 \times (L_h-1)}]^T$  as the first column, which has a similar structure to matrix  $\Theta_i$  defined in (5). According to (13), we can obtain the individual relay transmit power as

$$P_i = E|t_i(m)|^2 = \mathbf{h}_i^T (P_s \cdot \mathbf{F}_i^T \mathbf{F}_i^* + \sigma_n^2 \cdot \mathbf{I}_{L_h}) \mathbf{h}_i^*. \quad (14)$$

Define a notation  $\mathbf{D}_i = \text{diag}\{\mathbf{m}_i\} \otimes (P_s \cdot \mathbf{F}_i^T \mathbf{F}_i^* + \sigma_n^2 \cdot \mathbf{I}_{L_h})$ , where  $\otimes$  denotes the Kronecker product, and  $\mathbf{m}_i = [\mathbf{0}_{1 \times (i-1)}, 1, \mathbf{0}_{1 \times (R-i)}]$  is a  $1 \times R$  vector with the  $i$ th element being the only non-zero element, then (14) can be further rewritten as

$$P_i = \mathbf{w}^H \mathbf{D}_i \mathbf{w}. \quad (15)$$

Consequently, the total relay transmit power is obtained as

$$P_{\text{tot}} = \sum_{i=1}^R P_i = \mathbf{w}^H \mathbf{D} \mathbf{w}, \quad (16)$$

where  $\mathbf{D} = \sum_{i=1}^R \mathbf{D}_i$ .

Therefore, using (10), (15) and (16), the problem formulation (9) is rewritten as

$$\begin{aligned} \min_{\mathbf{w}, \alpha} \quad & |\alpha|^2 \cdot \mathbf{w}^H (\mathbf{Q}_s^{(\tau)} + \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + |\alpha|^2 \sigma_v^2 \\ \text{s.t.} \quad & \alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T = \gamma \\ & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0 \\ & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R. \end{aligned} \quad (17)$$

### 3.2. Relation to SINR Maximization Approach

Now we show that the proposed minimizing output power scheme is in fact equivalent to the scheme of "SINR maximization under total/individual relay power constraint" in [6], in terms of achieving the optimal output SINR performance.

According to the linear constraint in (17), we have  $\alpha = \gamma / (\mathbf{w}^H \vec{\psi}_\tau^T)$ . Substituting this expression for  $\alpha$  into the objective function of (17) and also noting  $\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = P_s \cdot |\mathbf{w}^H \vec{\psi}_\tau^T|^2$  given by (11), the optimization variable  $\alpha$  in (17) is eliminated, and we obtain a problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \gamma^2 P_s + \frac{\gamma^2 P_s \cdot (\mathbf{w}^H (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2)}{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0 \\ & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R. \end{aligned} \quad (18)$$

Note that  $\gamma^2 P_s$  is a constant term in the objective function and can be omitted without affecting the optimal solution for variable  $\mathbf{w}$ . Thus (18) can be further recast to a maximization problem:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \frac{\mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w}}{\mathbf{w}^H (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \mathbf{w} + \sigma_v^2} \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{D} \mathbf{w} \leq P_0 \\ & \mathbf{w}^H \mathbf{D}_i \mathbf{w} \leq P_{0,i}, \quad i = 1, \dots, R. \end{aligned} \quad (19)$$

However, this is just the "SINR maximization under total/individual relay power constraint" formulation in [6] (although the expression of  $\mathbf{w}$ ,  $\mathbf{Q}_s^{(\tau)}$ ,  $\mathbf{Q}_i^{(\tau)}$  and  $\mathbf{Q}_n$  are totally different). Here we also note that the optimal solution for  $\mathbf{w}$  is irrelevant of parameter  $\gamma$ . That is, the parameter  $\gamma$  does not affect the optimal SINR, but it does control the output power.

Despite the equivalence in SINR performance between the two formulations, we will subsequently show that our proposed formulation requires a lower computational load for solving the problem.

### 3.3. Problem Solution

To adopt efficient numerical methods, we transform the original problem (17) to an SOCP. To begin with, using the linear constraint  $\alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T = \gamma$  and the definition of  $\mathbf{Q}_s$  in (11), we can obtain

$$|\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w} = P_s \cdot |\alpha \cdot \mathbf{w}^H \vec{\psi}_\tau^T|^2 = \gamma^2 P_s. \quad (20)$$

Thus the term  $(|\alpha|^2 \cdot \mathbf{w}^H \mathbf{Q}_s^{(\tau)} \mathbf{w})$  in the objective of (17) can be omitted without affecting the optimal solution. By further doing variable substitution  $\hat{\mathbf{w}} = \alpha \cdot \mathbf{w}$ , problem (17) can be recast as

$$\begin{aligned} \min_{\hat{\mathbf{w}}, \alpha} \quad & \hat{\mathbf{w}}^H (\mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n) \hat{\mathbf{w}} + |\alpha|^2 \sigma_v^2 \\ \text{s.t.} \quad & \hat{\mathbf{w}}^H \vec{\psi}_\tau^T = \gamma \\ & \|\hat{\mathbf{w}}^H \mathbf{D}^{1/2}\|_2 \leq |\alpha| \sqrt{P_0} \\ & \|\hat{\mathbf{w}}^H \mathbf{D}_i^{1/2}\|_2 \leq |\alpha| \sqrt{P_{0,i}}, \quad i = 1, \dots, R. \end{aligned} \quad (21)$$

where  $\mathbf{D}^{1/2}$  and  $\mathbf{D}_i^{1/2}$  are the principal square roots of matrices  $\mathbf{D}$  and  $\mathbf{D}_i$ , respectively. Observing the power constraints of (21), we note that an arbitrary phase rotation of  $\alpha$  does not affect the objective

function. Hence, without loss of generality, we can assume  $\alpha$  is real valued. Moreover, by defining the following notations:

$$\begin{aligned} \mathbf{b}^{(\tau)} &= [\vec{\psi}_\tau, 0]^T, \quad \mathbf{u} = [\hat{\mathbf{w}}^T, \alpha]^T, \\ \tilde{\mathbf{Q}}^{(\tau)} &= \begin{bmatrix} \mathbf{Q}_i^{(\tau)} + \mathbf{Q}_n & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & \sigma_v^2 \end{bmatrix}, \\ \mathbf{d} &= [\mathbf{0}_{1 \times RL_h}, \sqrt{P_0}]^T, \quad \tilde{\mathbf{D}} = \begin{bmatrix} \mathbf{D}^{1/2} & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & 0 \end{bmatrix}, \\ \mathbf{d}_i &= [\mathbf{0}_{1 \times RL_h}, \sqrt{P_{0,i}}]^T, \quad \tilde{\mathbf{D}}_i = \begin{bmatrix} \mathbf{D}_i^{1/2} & \mathbf{0}_{RL_h \times 1} \\ \mathbf{0}_{1 \times RL_h} & 0 \end{bmatrix}, \end{aligned}$$

problem (21) can be solved via the following optimization:

$$\begin{aligned} \min_{\mathbf{u}} \quad & \|\mathbf{u}^H \tilde{\mathbf{Q}}^{(\tau)}\|_2 \\ \text{s.t.} \quad & \mathbf{u}^H \mathbf{b}^{(\tau)} = \gamma \\ & \text{Im}\{\mathbf{u}_{\text{end}}\} = 0 \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}\|_2 \leq \mathbf{u}^H \mathbf{d} \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}_i\|_2 \leq \mathbf{u}^H \mathbf{d}_i, \quad i = 1, \dots, R, \end{aligned} \quad (22)$$

where  $\mathbf{u}_{\text{end}}$  denotes the last element of vector  $\mathbf{u}$ , *i.e.*,  $\alpha$ . By introducing an auxiliary variable  $t$ , problem (22) can be further rewritten as a standard SOCP:

$$\begin{aligned} \min_{\mathbf{u}, t} \quad & t \\ \text{s.t.} \quad & \|\mathbf{u}^H \tilde{\mathbf{Q}}^{(\tau)}\|_2 \leq t \\ & \mathbf{u}^H \mathbf{b}^{(\tau)} = \gamma \\ & \text{Im}\{\mathbf{u}_{\text{end}}\} = 0 \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}\|_2 \leq \mathbf{u}^H \mathbf{d} \\ & \|\mathbf{u}^H \tilde{\mathbf{D}}_i\|_2 \leq \mathbf{u}^H \mathbf{d}_i, \quad i = 1, \dots, R. \end{aligned} \quad (23)$$

which can be efficiently solved using interior point methods [11, 12].

We note that in [6], the scheme of ‘‘SINR maximization subject to individual relay power constraints’’ is solved as a feasibility problem using the bisection search technique. That is, the SOCP feasibility problem has to be solved multiple times for finding a solution, and the number of times of solving SOCP depends on the initial value and the termination condition (error tolerance). However, the proposed approach solves an SOCP (23) only once, while it is also shown to be equivalent to the SINR maximization in terms of output SINR performance. The worst-case complexity of solving (23) using interior point method is comparable to that of solving one iteration of SINR maximization, since in terms of problem dimension problem (23) has only one more variable than the SINR maximization problem of [6]. Hence, the proposed approach consumes lower computational resources to achieve the optimal SINR.

#### 4. SIMULATION RESULTS

We consider a network where the number of relays is  $R = 10$ . The relay noise power and destination noise power are assumed to be  $\sigma_n^2 = \sigma_v^2 = 1$ , and the source power  $P_s$  is 10 dB higher than the noise power. The coefficients of channel impulse responses are modeled as independent quasi-static Rayleigh fading, and are hence generated as zero-mean complex Gaussian

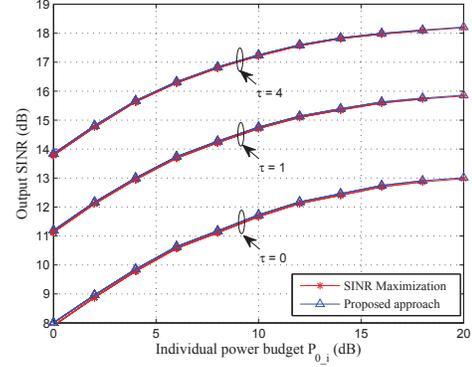


Fig. 2. SINR performance versus individual relay transmit power budget  $P_{0,i}$  for different decision delays  $\tau$ .

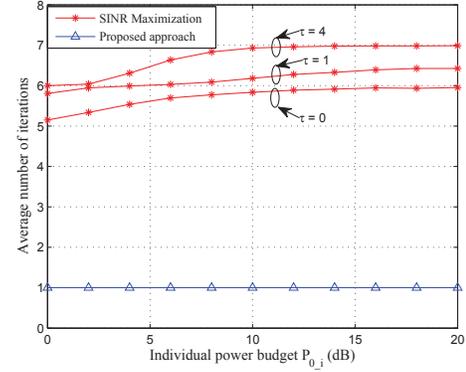


Fig. 3. Average number of iterations versus individual relay transmit power budget  $P_{0,i}$  for different decision delays  $\tau$ .

random variables with the exponential power delay profile [13]  $p(m) = (1/\sigma_t) \cdot \sum_{l=0}^{L_x-1} e^{-m/\sigma_t} \delta(m-l)$ , where  $\sigma_t$  is the delay spread factor and here  $\sigma_t = 2$  is assumed, and  $L_x = L_f$  and  $L_g$  for the length of the backward channels and forward channels, respectively. Assume  $L_f = L_g = 5$ , and  $L_h = 5$ .

In Fig. 2, we show the output SINR performances of the proposed approach and SINR maximization of [6], with respect to the individual power budget. We assume all the relays have the same power budget  $P_{0,i}$ , and the total power budget is  $P_0 = 8P_{0,i}$ . The error tolerance for the bisection search is set to be  $\epsilon = 0.1$ . Therefore, we can see that if the error tolerance is not small enough, the SINR maximization approach which needs bisection search shows inferior performance. In Fig. 3, we show the average number of bisection iterations, corresponding to result of Fig. 2. We note that for the SINR maximization approach, the iteration number increases with the increase of output SINR. While the proposed approach is not iteratively solved, and thus it always requires one iteration only.

#### 5. CONCLUSION

We have proposed an alternative FF relay beamforming approach, which is based on output power minimization criterion. We demonstrated that the proposed approach is equivalent to the SINR maximization scheme and have lower computational load.

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