Multi-antenna Relay Network Beamforming Design for Multiuser Peer-to-Peer Communications

Jiangwei Chen and Min Dong
Department of Electrical, Computer and Software Engineering
University of Ontario Institute of Technology, Ontario, Canada
Email: {jiangwei.chen,min.dong}@uoit.ca

***Abstract*—We consider a multi-user peer-to-peer relay network with multiple multi-antenna relays for amplify-and-forward relaying. Assuming distributed relay beamforming strategy, we investigate the design of each relay processing matrix to minimize the per-antenna relay power usage for given users' SNR targets. As the problem is NP-hard, we developed an approximate solution through the Lagrange dual domain. Through a sequence of transformations, we obtain a semi-closed form solution which can be determined by solving an efficient semi-definite programming problem. We also considered the semi-definite relaxation (SDR) approach. Compared with the SDR approach, the proposed solution has significantly lower computational complexity. The benefit of such solution is apparent when the optimal solution can be obtained by both approaches. When the solution is suboptimal, simulations show that the SDR approach has better performance. Thus, we proposed a combined method of the two approaches to trade-off performance and complexity. Simulations showed the effectiveness of such combined method.**

1. Introduction

Cooperative relaying is one of the key techniques to support dynamic ad-hoc networking for the next generation wireless systems. In such a network, there are typically multiple communicating pairs as well as available relays. Thus, efficient physical layer design of cooperative relaying to support such simultaneous transmissions is important. We study the design of amplify-and-forward (AF) multi-antenna relaying in such multiuser peer-to-peer (MUP2P) relay networks, where multiple source-destination pairs communicate through the assistance of multiple relays. We consider using distributed relay beamforming technique among relays, each equipped with multiple antennas, to maximize the transmission power gain for data forwarding. The major goal for this problem is to develop efficient algorithms to determine each relay processing matrix so that good performance at each user can be achieved. To devise a practical design, we consider each relay antenna has its own individual power budget. This constraint corresponds to the reality of individual RF front-end power amplifier at each antenna. Such per-antenna power control makes the design problem significantly more challenging than the sum-power constraint consideration in most of the existing works.

Many existing works focus on the relay processing design in a single source-destination pair setting, either with a multi-antenna relay [1]–[5], or with multiple single-antenna relays forming distributed beamforming [6]–[9]. For MUP2P relay networks, distributed relay beamforming with multiple single-antenna relays were studied in [10]–[12] for total power minimization among relays or among all network nodes. Due to the inherent complexity of the problem, numerical methods were proposed to obtain approximate solutions. Most existing designs focus on total power constraint, either among relay antennas, or across relays, which lead to more analytically tractable problems. However, these results or techniques cannot be applied to the problem where the per-antenna/per-node power constraints are imposed. Under such per-antenna power budget, the relay processing design is studied recently for single source-destination pair [5] and for multicasting scenario [13], both with a single multi-antenna relay.

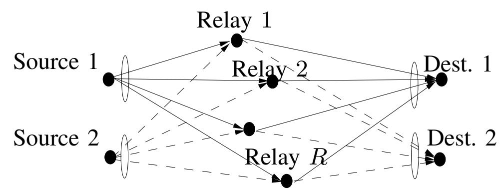

Fig. 1: The multiuser peer-to-peer relay network.

In this work, we consider a MUP2P AF relay network with multiple relays assisting user pairs' transmissions. A general setting of multi-antenna relays is considered, which was not studied in the existing works. Furthermore, we focus on per-antenna relay power, and aim at the relay processing design to minimize the per-antenna relay power under each user pair's SNR target requirement. The problem is inherently non-convex and is in general NP-hard. We develop an approximate solution using the Lagrange dual approach. Through a sequence of transformations, we obtain the solution of each relay processing matrix, which has a semi-closed form structure. To determine the solution, a set of dual variables are computed by solving a semi-definite programming (SDP) problem. We also consider the traditional semi-definite relaxation (SDR) approach to solve this problem, which is used in [10] for total power minimization. We analyze the two approaches in terms of algorithm complexity and performance. Our analysis and simulations show that the semi-closed form solution under the dual approach has significantly lower computational complexity, as compared with the SDR approach. In terms of performance, in some cases, the solution is optimal, and both approaches can obtain it at the same time. although the SDR approach is not always straightforward in extracting the solution, besides the much higher computational complexity. However, the SDR approach tends to provide better performance when the solution is approximate. We further propose a combined method of the two approaches to trade-off performance and computational complexity. Simulation results show the effectiveness of such combined method.

Notations: Kronecker product is denoted as $\otimes$. Hermitian and transpose are denoted as $(\cdot)^H$ and $(\cdot)^T$, respectively. $(\cdot)^\dagger$ is the pseudo-inverse of a matrix. The semi-definite matrix $\mathbf{A}$ is denoted as $\mathbf{A} \succeq 0$. The vectorization vec($\mathbf{A}$) vectorize matrix $\mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_N]$ to a vector$[\mathbf{a}_1^T, \cdots, \mathbf{a}_N^T]^T$

2. System Model

We consider a system with K source-destination pairs communicating through R AF relays. All sources and destinations are equipped with a single antenna, as shown in Fig. 1. Each relay m is equipped with N antennas, and processes the incoming signals using the $N \times N$ processing matrix $\mathbf{W}_m$ before forwarding to the

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destinations. The $N \times 1$ channel vectors between the kth source and mth relay and between mth relay and kth destination are denoted as $\mathbf{h}_{1,km}$ and $\mathbf{h}_{2,mk}$, respectively. The received signal at destination k is given by

$$y_{d,k} = \sum_{m=1}^{R} \mathbf{h}_{2,mk}^T \mathbf{W}_m \left(\sum_{l=1}^{K} \mathbf{h}_{1,lm} \sqrt{P_0} s_l \right) + \sum_{m=1}^{R} \mathbf{h}_{2,mk}^T \mathbf{W}_m \mathbf{n}_{r,m} + n_{d,k},$$

where s_l is the signal sent from source k with $\mathbb{E}|s_k|^2 = 1$, P_0 is the transmit power, and $\mathbf{n}_{r,m}$ and $n_{d,k}$ are the complex AWGN at relay m with covariance $\sigma_{r,m}^2 \mathbf{I}$ and at destination k with variance $\sigma_{d,k}^2$, respectively. The received signal $y_{d,k}$ consists of desired signal s_k, interference from other sources s_l, $l \neq k$, and the noises from the relays and at the receiver. Define the vectorized relay processing matrices as $\mathbf{w} = [\mathbf{w}_1^H, \cdots, \mathbf{w}_R^H]^H$, where $\mathbf{w}_m = \text{vec}(\mathbf{W}_m^H)$. Define the channel vector of the kth source-destination pair through relay m as $\mathbf{h}_{k(m)} = \mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,km}$, and through all relays as $\mathbf{h}_k = [\mathbf{h}_{k(1)}^H \cdots \mathbf{h}_{k(R)}^H]^H$. Define the interfering channel vector of source l to destination k through relay m as $\mathbf{h}_{k,l(m)} = \mathbf{h}_{2,mk} \otimes \mathbf{h}_{1,lm}$, and through all relays as $\mathbf{h}_{k,l} = [\mathbf{h}_{k,l(1)}^H \cdots \mathbf{h}_{k,l(R)}^H]^H$. Define the interfering channel matrix for the kth pair as $\mathbf{G}_{k^-} = [\mathbf{h}_{k,1} \cdots \mathbf{h}_{k,k-1}, \mathbf{h}_{k,k+1} \cdots \ \mathbf{h}_{k,K}]$. Finally, for the kth pair, define amplified noise covariance matrix from relay m as $\mathbf{F}_{k(m)} = \mathbf{h}_{2,mk} \mathbf{h}_{2,mk}^H \otimes \mathbf{I} \sigma_{r,m}^2$, and from all relays as $\mathbf{F}_k = \text{diag}(\mathbf{F}_{k(1)} \cdots \mathbf{F}_{k(R)})$. Using these quantities, the received SNR for the kth pair can be written as

$$\text{SNR}_k = \frac{\left|\mathbf{h}_k^H \mathbf{w}\right|^2 P_0}{\left\|\mathbf{G}_{k^-}^H \mathbf{w}\right\|^2 P_0 + \left\|\mathbf{F}_k^{\frac{1}{2}} \mathbf{w}\right\|^2 + \sigma_{d,k}^2}. \quad (1)$$

The power usage at antenna i of relay m, denoted as $P_{m,i}$, is given by

$$P_{m,i} = \left[\sigma_{r,m}^2 \mathbf{W}_m \mathbf{W}_m^H + P_0 \mathbf{W}_m \left(\sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H \right) \mathbf{W}_m^H \right]_{ii} \quad (2)$$

Let γ_k be the SNR target for the destination k, our goal is to design $\{\mathbf{W}_m\}$ to minimize the antenna power consumption at each relay, subject to receiver SNR target constraints. This problem can be expressed as

$$\min_{\{\mathbf{W}_m\}} \max_{m,i} \ P_{m,i} \quad (3)$$
$$\text{subject to} \ \ \text{SNR}_k \geq \gamma_k, \ \forall k. \quad (4)$$

The above min-max power problem is equivalent to the following problem

$$\min_{\{\mathbf{W}_m\}} P_r \ \text{ subject to } \ (4) \text{ and } P_{m,i} \leq P_r, \forall i, m \quad (5)$$

where P_r is the per-antenna power constrait.

3. Proposed Solution for Power Minimization

Denote $\mathbf{W}_m^H = [\mathbf{w}_{m,1}, \cdots \mathbf{w}_{m,N}]$. Then, $P_{m,i}$ in (2) can be rewritten as

$$P_{m,i} = \mathbf{w}_{m,i}^H \left(\sigma_{r,m}^2 \mathbf{I} + P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H \right) \mathbf{w}_{m,i}. \quad (6)$$

Define $\mathbf{D}_m \triangleq \sigma_{r,m}^2 \mathbf{I} + P_0 \sum_{k=1}^{K} \mathbf{h}_{1,km} \sum_{k=1}^{K} \mathbf{h}_{1,km}^H$. Using the vectorized $\mathbf{w}$, the Lagrangian of the optimization problem (5) is given by

$$L(\mathbf{w}, \{\mathbf{\Lambda}_m\}, \boldsymbol{\nu}) = P_r - P_r \text{tr}(\mathbf{\Lambda}) + \sum_{m=1}^{R} \mathbf{w}_m^H [\mathbf{\Lambda}_m \otimes \mathbf{D}_m] \mathbf{w}_m - \sum_{k=1}^{K} \nu_k \left[\left|\mathbf{h}_k^H \mathbf{w}\right|^2 \frac{P_0}{\gamma_k} - (\left\|\mathbf{G}_{k^-}^H \mathbf{w}\right\|^2 P_0 + \left\|\mathbf{F}_k^{\frac{1}{2}} \mathbf{w}\right\|^2 + \sigma_{d,k}^2) \right]$$

where $\mathbf{\Lambda}_m \triangleq \text{diag}(\lambda_{m1} \cdots \lambda_{mN})$, for $m = 1, \cdots, R$, with $\lambda_{m,i}$ being the Lagrange multiplier related to the ith antenna power constraint of relay m, and $\boldsymbol{\nu} = [\nu_1, \cdots, \nu_K]^T$ with ν_k being Lagrange multipliers associated with the SNR constraint of destination k, for $k = 1, \cdots, K$. Define $\mathbf{R}_m \triangleq \mathbf{\Lambda}_m \otimes \mathbf{D}_m$, $\mathbf{R} \triangleq \text{diag}(\mathbf{R}_1, \cdots, \mathbf{R}_R)$, and $\mathbf{\Lambda} \triangleq \text{diag}(\mathbf{\Lambda}_1 \cdots \mathbf{\Lambda}_R)$. The Lagrangian above can be rewritten as

$$L(\mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) = P_r(1 - tr(\mathbf{\Lambda})) + \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 + \mathbf{w}^H (\mathbf{R} + \sum_{k=1}^{K} \nu_k \left[(P_0 \mathbf{G}_{k^-} \mathbf{G}_{k^-}^H + \mathbf{F}_k) - \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \right]) \mathbf{w}.$$

Then, the dual problem of the optimization problem (5) is given by

$$\max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \min_{\mathbf{w}} \ L(\mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) \quad (7)$$
$$\text{subject to} \ \ \mathbf{\Lambda} \succeq 0, \boldsymbol{\nu} \succeq 0. \quad (8)$$

Examining the expression of $L(\mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu})$, we can show that the above dual problem is equivalent to the following problem with two new added constraints

$$\max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \min_{\mathbf{w}} \ L(\mathbf{w}, \mathbf{\Lambda}, \boldsymbol{\nu}) \quad (9)$$
$$\text{subject to} \ \ (8), \text{ and}$$
$$\text{tr}(\mathbf{\Lambda}) \leq 1 \quad (10)$$
$$\mathbf{\Sigma} \succeq \sum_{k=1}^{K} \nu_k \frac{P_0}{\gamma_k} \mathbf{h}_k \mathbf{h}_k^H \quad (11)$$

where $\mathbf{\Sigma} \triangleq \mathbf{R} + \sum_{k=1}^{K} \nu_k (P_0 \mathbf{G}_{k^-} \mathbf{G}_{k^-}^H + \mathbf{F}_k)$. Solving the inner minimization over $\mathbf{w}$, we have

$$\max_{\mathbf{\Lambda}, \boldsymbol{\nu}} \ \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 \ \text{ subject to } \ (8), (10), \text{ and } (11). \quad (12)$$

To solve the above optimization problem, we first examine the constraint in (11). We will need the following lemma [13].

Lemma 1 ([13]): Let $\mathbf{A}$ be an $n \times n$ positive definite matrix and $\mathbf{B}$ be an $n \times n$ positive semi-definite matrix. Then,

$$\mathbf{A} \succcurlyeq \mathbf{B} \Leftrightarrow 1 - \sigma_{\max} \left(\mathbf{A}^{\frac{\dagger}{2}} \mathbf{B} \mathbf{A}^{\frac{\dagger}{2}} \right) \geq 0 \quad (13)$$

In addition, we have the following lemma.

Lemma 2: At optimality of the optimization problem (9), $\mathbf{\Sigma}$ is positive definite.

Proof: Omitted due to page limitation.

Using Lemmas 1 and 2, we can replace the constraint in (11) by (15), and the optimization problem (12) is equivalent to the following

$$\max_{\mathbf{\Lambda},\boldsymbol{\nu}} \ \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 \tag{14}$$

subject to (8), (10), and

$$\sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K}\nu_k \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right) \leq 1. \tag{15}$$

The above optimization problem is further equivalent to the following problem

$$\max_{\mathbf{\Lambda}} \min_{\boldsymbol{\nu}} \ \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 \tag{16}$$

subject to (8), (10), and

$$\sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K}\nu_k \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right) \geq 1 \tag{17}$$

where we change the maximization to minimization over $\boldsymbol{\nu}$, and flip the inequality in (15) to (17). To see the equivalence, we note that, for any given $\mathbf{\Lambda}$, both optimization problems (14) and (16) are equivalent. This is because to reach the optimality, they both require the constraints (15) and (17) to be met with equality, *i.e.,*

$$\sigma_{\max}\left(\mathbf{\Sigma}^{\frac{\dagger}{2}}(\sum_{k=1}^{K}\nu_k \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H)\mathbf{\Sigma}^{\frac{\dagger}{2}}\right) = 1 \tag{18}$$

with the optimal $\boldsymbol{\nu}^o$ being the root of the above equation.

We now show that the optimization problem (16) is equivalent to the following problem

$$\max_{\mathbf{\Lambda}} \min_{\boldsymbol{\nu},\mathbf{w}} \ \sum_{k=1}^{K} \nu_k \sigma_{d,k}^2 \tag{19}$$

subject to (8), (10), and

$$\frac{P_0 \sum_{k=1}^{K} \frac{\nu_k}{\gamma_k}\left|\mathbf{h}_k^H\mathbf{w}\right|^2}{\mathbf{w}^H\mathbf{\Sigma}\mathbf{w}} \geq 1. \tag{20}$$

For a given $\mathbf{\Lambda}$, we look at the inner minimization of (19). At the optimality, the constraint in (20) is attained with equality as follow

$$\min_{\mathbf{w}} \frac{P_0 \sum_{k=1}^{K} \frac{\nu_k}{\gamma_k}\left|\mathbf{h}_k^H\mathbf{w}\right|^2}{\mathbf{w}^H\mathbf{\Sigma}\mathbf{w}} = 1. \tag{21}$$

which is a generalized eigenvalue problem. Therefore, the optimal $\tilde{\mathbf{w}}$ to (19) has the following structure

$$\tilde{\mathbf{w}} = \Sigma^{o\frac{\dagger}{2}}\mathcal{P}\left(\mathbf{\Sigma}^{o\frac{\dagger}{2}}\left[\sum_{k=1}^{K}\frac{\nu_k^o}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H\right]\mathbf{\Sigma}^{o\frac{\dagger}{2}}\right) \tag{22}$$

where $\boldsymbol{\nu}^o$ is the optimal value for $\boldsymbol{\nu}$, and $\mathbf{\Sigma}^o$ is $\mathbf{\Sigma}$ under the optimal $\mathbf{\Lambda}^o$ and $\boldsymbol{\nu}^o$. The final solution $\mathbf{w}^o$ will have the form

$$\mathbf{w}^o = \beta\tilde{\mathbf{w}} \tag{23}$$

where β is a scaling factor to ensure that the SNR constraint (4) is met, for all k. This means

$$\frac{\beta^2\left|\mathbf{h}_k^H\tilde{\mathbf{w}}\right|^2 P_0}{\beta^2 P_0\left\|\mathbf{G}_{k-}^H\tilde{\mathbf{w}}\right\|^2 + \beta^2\left\|\mathbf{F}_k^{\frac{1}{2}}\tilde{\mathbf{w}}\right\|^2 + \sigma_{d,k}^2} \geq \gamma_k, \forall k. \tag{24}$$

Thus, the optimal β is obtained as

$$\beta = \sqrt{\sigma_{d,k}^2 / \min_k \{g_k(\tilde{\mathbf{w}})\}} \tag{25}$$

where

$$g_k(\tilde{\mathbf{w}}) = \frac{P_0}{\gamma_k}\left|\mathbf{h}_k^H\tilde{\mathbf{w}}\right|^2 - P_0\left\|\mathbf{G}_{k-}^H\tilde{\mathbf{w}}\right\|^2 - \left\|\mathbf{F}_k^{\frac{1}{2}}\tilde{\mathbf{w}}\right\|^2.$$

To determine $\mathbf{w}^o$ in (23), we need to obtain the optimal $\mathbf{\Lambda}^o$ and $\boldsymbol{\nu}^o$, which can be obtained from the optimization problem (12). The dual problem (12) can be transformed into an SDP as

$$\min_{\mathbf{x}} \ \boldsymbol{\sigma}^T\mathbf{x} \tag{26}$$

subject to $\mathbf{b}^T\mathbf{x} \preceq 1, \ \mathbf{x} \succeq 0,$

$$\sum_{m=1}^{R}\sum_{i=1}^{N} x_{(m-1)N+i}\mathbf{D}_{m,i} + \sum_{k=1}^{K} x_{RN+k}\mathbf{T}_k \preceq 0$$

where $\boldsymbol{\sigma} \triangleq [\mathbf{0}_{RN\times1}^T, -\sigma_{dk}^2\mathbf{1}_{K\times1}^T]^T$, $\mathbf{b} \triangleq [\mathbf{1}_{RN\times1}^T, \mathbf{0}_{K\times1}^T]^T$, and $\mathbf{x} = [x_1, \cdots, x_{RN+K}]^T \triangleq [\lambda_1, \cdots \lambda_{RN}, \nu_1, \cdots \nu_K]^T$. The last constraint above corresponds to the constraint in (11), where $\mathbf{D}_{m,i}$ is a block diagonal matrix with RN diagonal blocks of size $N\times N$, with the $(m-1)N+i$ diagonal block being $\mathbf{D}_m$ (defined below (6)) and the rest being zero. Finally, $\mathbf{T}_k$ is an $RN^2 \times RN^2$ matrix, defined as $\mathbf{T}_k \triangleq \frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H - (P_0\mathbf{G}_{k-}\mathbf{G}_{k-}^H + \mathbf{F}_k)$, for $k = 1, \cdots, K$.

The above SDP problem can be efficiently solved using standard SDP software, such as SeDuMi [14].

4. Comparison with the SDR Approach

The power minimization problem in (5) can be solved using the SDR approach. To see this, we write all the constraints in trace form. The per-antenna power $P_{m,i}$ in (6) is rewritten as $P_{m,i} = \mathrm{tr}(\mathbf{w}_{m,i}^H\mathbf{D}_m\mathbf{w}_{m,i}) = \mathrm{tr}(\mathbf{D}_{m,i}\mathbf{X})$, where $\mathbf{X} \triangleq \mathbf{w}\mathbf{w}^H$. The SNR constraint in (4) can be rewritten as

$$\mathbf{w}^H\left(\frac{P_0}{\gamma_k}\mathbf{h}_k\mathbf{h}_k^H - [\mathbf{G}_{k-}\mathbf{G}_{k-}^H P_0 + \mathbf{F}_k]\right)\mathbf{w} = \mathrm{tr}(\mathbf{T}_k\mathbf{X}) \geq \sigma_{d,k}^2$$

The optimization problem (5) can be reformulated as the following

$$\min_{\mathbf{X}} \ P_r \tag{27}$$

$$\text{subject to} \quad \mathrm{tr}(\mathbf{T}_k\mathbf{X}) \geq \sigma_{d,k}^2, \forall k, \quad \mathrm{tr}(\mathbf{D}_{m,i}\mathbf{X}) \leq P_r, \forall m, i \tag{28}$$

$$\mathrm{rank}(\mathbf{X}) = 1, \ \mathbf{X} \succcurlyeq 0.$$

By removing the rank constraint, the above non-convex problem is relaxed to the following SDP problem

$$\min_{\mathbf{X}} \ P_r \ \text{ subject to (28) and } \mathbf{X} \succcurlyeq 0. \tag{29}$$

For a fixed P_r, the above problem is an SDP feasibility problem. Thus, we can solve (29) using a bi-section search as an outer loop over an SDP feasibility problem w.r.t. P_r. After obtaining the optimal $\mathbf{X}^o$ from (29), depending on rank($\mathbf{X}^o$), we can extract the solution $\mathbf{w}$ from $\mathbf{X}^o$ either directly (rank($\mathbf{X}^o$) = 1), or through some randomization methods (rank($\mathbf{X}^o$) > 1) [15].

The computational complexity of the proposed solution in Section 3 is much lower than that of the SDR approach. To see this, $\mathbf{w}$ is directly obtained from the semi-closed form solution (23) which requires solving the SDP problem (29). For the SDR approach, obtaining $\mathbf{w}$ requires to iteratively solve the SDP feasibility problem (29) multiple times. The complexity in solving each SDP problem can also be obtained by examining the size of each SDP [16]. The complexity per iteration for the SDP problem (26) is $\mathcal{O}((RN+K)^2(RN^2)^2)$. For the SDP feasibility problem (29) with a given P_r, the complexity per iteration is $\mathcal{O}((RN^2)^4(RN^2)^2)$. Thus, the overall complexity of the proposed solution is significantly lower than the SDR approach. We will also show the actual processing time in Section 5 to demonstrate this.

Regarding the performance, it is known that the dual problem (7) and the relaxed SDP problem (29) provide the the same lower bound to the original primary problem (5) [15]. This means that, in some cases, if the optimal solution can be obtained by one approach, it can be obtained by both approaches at the same time. The proposed approach however has significantly lower computational complexity. When the solution is non-optimal, simulations demonstrate that the SDR approach has a better performance Therefore, in practice, we suggest to combine the two approaches to trade-off performance and complexity. Specifically, we use a threshold to decide when to use dual or SDR approach:

1) Compute $\mathbf{w}^o$ using (23).
2) Let $P_r(\mathbf{w}^o)$ be the per-antenna power in (5) under $\mathbf{w}^o$, and $P_r^{lw}(\mathbf{w}^o)$ be the optimal value of (7). Compute dual gap $G^{\text{d}} \triangleq P_r(\mathbf{w}^o)/P_r^{lw}(\mathbf{w}^o)$ and compare it with the threshold, if G^{d} is less than the threshold, then $\mathbf{w}^o$ is the final solution. Otherwise, go to Step 3.
3) Use the SDR approach to produce a solution $\mathbf{w}$.

Note that the approximate performance bound for the SDR approach has been analyzed in the literature [15]. We can use the bound to set the threshold to determine when to use the SDR approach.

5. Numerical Results

We study the performance of the described approaches through simulations. We assume the channel vectors $h_{1,km}$ and $h_{2,mk}$ are i.i.d. Gaussian with unit variance. We set noise power at relays and destinations to be equal $\sigma_{r,m}^2$=$\sigma_{d,k}^2, \forall m,k$. The source transmission power over noise power is set to be $P_0/\sigma_{r,m}^2 = 0$dB.

We compare the performance of the dual approach, the SDR approach, and combined method. Similar to the gap G^{d} defined for the proposed dual solution, we define $G^{\text{SDR}} \triangleq P_r(\mathbf{w}^{\text{SDR}})/P_r^{lw}(\mathbf{X}^o)$, where $P_r^{lw}(\mathbf{X}^o)$ is the result from (29), and $\mathbf{w}^{\text{SDR}}$ is the solution extracted from $\mathbf{X}^o$. The gap G^{com} under the combined method can be similarly obtained. As mentioned earlier, the dual approach and SDR approach attain the same lower bound. In Fig. 2, we plot the CDF of $G^{\text{d}}, G^{\text{SDR}}$ and G^{com} under the three methods, respectively. We set $R = 2$, $N = 6$, $K = 2, 4, 8$, and $\gamma_k = 4$dB, $\forall k$. The same set of 2000 channel realizations are used for each method. The gap being 0dB indicates the optimal solution is obtained. We can see the percentages of 0dB gap in three approaches are identical, verifying that the optimal solutions are obtained by these approaches at the same time. When the solution is suboptimal, we observe that the tail distribution of G^{SDR} is tighter than that of G^{d}. Therefore, the SDR approach produces a tighter approximate solution than the dual approach in this case. The corresponding average processing time of each method is shown in Fig. 3. We see that the dual approach uses significantly shorter time than the SDR approach to compute the solution. From Figs. 2 and 3, we see that the combined method can effectively trade-off the performance and complexity, with the performance between the two approaches and the processing time slightly worse than that of the dual approach. The threshold for the combined method can be adjusted to trade-off the performance and the complexity. Using the combined method, we also study the resulting total relay power obtained vs. γ_k under various K in Fig. 4. We set $R = 2$, $N = 4$, and γ_k's are equal for all k. As K increases, the interference from other sources at the relays increases. We see the effect of such interference on the total relay power consumption, especially at the high SNR target.

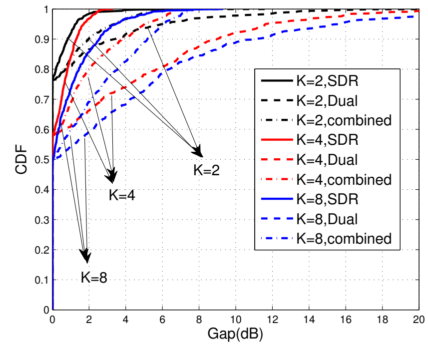

Fig. 2: Gap CDF ($R = 2, N = 6, \gamma_k = 4$dB)

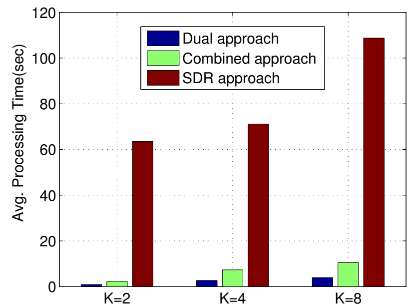

Fig. 3: Average processing time ($R = 2, N = 6, \gamma_k = 4$dB)

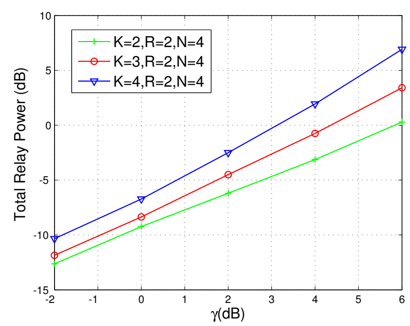

Fig. 4: Total relay power vs γ_k for $K = 2, 3, 4$.

6. Conclusion

In this paper, we considered the design of distributed multi-antenna multi-relay beamforming in a MUPSP AF relay network to minimize the per-antenna relay power usage. We developed an approximate solution through the Lagrange dual domain, and obtained a semi-closed form solution for each relay processing matrix. We also considered the SDR approach. Compared with the traditional SDR approach, the proposed solution has significantly low computational complexity. The advantage of such solution is apparent when the optimal solution can be obtained by both approaches. Since the SDR approach has better performance when the solution is suboptimal, we proposed a combined method to trade-off performance and complexity. Simulations showed the effectiveness of the combined method.

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