# WIDE-BAND MOVING SOURCE PASSIVE LOCALIZATION IN HIGHLY CORRUPTIVE ENVIRONMENTS

Antonio Napolitano

Department of Engineering, University of Napoli "Parthenope", Italy and Consorzio Nazionale Interuniversitario per le Telecomunicazioni (CNIT), Italy

### ABSTRACT

A passive localization algorithm is proposed for moving sources emitting man-made signals. The method models the received signals on two sensors as (singularly) almostcyclostationary. It is not based on the usual so called narrowband assumption that limits bandwidth, data-record length, and relative radial speed between source and sensors. Thus, unlike previous techniques, it statistically characterizes the signals on the two sensors as jointly spectrally correlated rather than as jointly almost cyclostationary. The algorithm estimates time-scale ratio, frequency-difference-of-arrival, and time-delay-of-arrival of the source signal impinging on the two sensors. It is highly tolerant to noise and interference and outperforms classical cyclostationarity-based techniques for large data-record lengths.

*Index Terms*— Cyclostationarity; Spectrally correlated signals; Doppler effect.

## 1. INTRODUCTION

The problem of passively locating a moving source emitting a man-made signal has a variety of applications. They include navigation, tracking and monitoring of moving objects for surveillance, and locating hostile jamming emitters. The estimation of the time-delay-of-arrival (TDOA) of the wavefronts impinging on two sensors allows one to estimate the direction of the source. Doppler measurements allow to estimate the source radial speed with respect to the sensors.

If the relative radial speed of the source with respect to a sensor can be assumed constant within the observation interval, then the complex envelope of the received signal is an amplitude-scaled, time-scaled, time-delayed, and frequencyshifted version of the complex-envelope signal emitted by the source [17, Sec. 7.3], [26, pp. 239-242]. The time-scale factor in the argument of the complex envelope can be assumed unity (and hence neglected) under the so called "narrow-band condition", that is, if the product of signal bandwidth and data-record length is much smaller than the ratio of medium propagation speed and relative radial speed between transmitter and sensor [17, Sec. 7.5], [26, pp. 239-242]. In such a case, the Doppler effect is just a frequency shift of the carrier.

If the source signal is almost-cyclostationary (ACS) [9], under the narrow-band condition, cyclostationarity-based algorithms for estimating TDOA [3], [8], [14], and TDOA and frequency-difference-of arrival (FDOA) [11], [12] of the wavefronts impinging on two sensors have been successfully exploited to provide accurate estimates in severe noise and interference environments, when the disturbance signals completely overlap in time and frequency domains the signals-of-interest (SOIs) on both sensors. These techniques have satisfactory performance at very low signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR), provided that a cycle frequency of the SOI exists which is not shared with the disturbance signals and a sufficiently large observation interval is available.

The narrow-band condition puts a limit on the maximum relative radial speed between transmitter and receiver and/or the signal bandwidth and/or the maximum data-record length that can be adopted for cyclic statistic estimates. The limit on the data-record length in turn puts a limit on the minimum SNR and SIR for which satisfactory performance can be achieved, specially for very fast moving sources like satellites, airplanes, helicopters, and missiles.

In this paper, the case of a rapidly moving source emitting a wide-band ACS signal is considered. Unlike all previously proposed cyclostationarity-based techniques, it is not assumed here that the narrow-band condition is satisfied. A technique proposed in the radar context in [19] is adapted to estimate time-scale ratio (TSR) and FDOA of the signals collected on two sensors. Then, exploiting a novel class of stochastic processes, the spectrally correlated (SC) processes [16], [17, Chap. 4], [18], the signals received on the two sensors are jointly statistically characterized as jointly SC. Thus, a new interference tolerant algorithm for estimating TDOA and complex-gain ratio (CGR) is proposed. The algorithm exploits the signal-selectivity properties typical of cyclostationarity-based techniques considered in a more general sense for the (jointly) SC signals. The proposed technique is named wide-band spectral coherence alignment (WB-SPECCOA). It generalizes to the wide-band case the SPECCOA method with compensated frequency shift [11]

This work is partially supported by CNIT (AIBER project).

derived under the narrow-band (NB) condition and from now referred to as NB-SPECCOA, which in turn generalizes the SPECCOA method [8] derived for fixed sources.

The signal parameter estimation techniques in [6], [19], and [20] are derived when the narrow-band condition is not fulfilled. In [6] and [19], unlike in the WB-SPECCOA method proposed here, only auto cyclic statistics are used without exploiting the beneficial effects of cross-correlating signals. In addition, in [6] and [19], constraints on the parameters to be estimated must be satisfied to avoid ambiguous estimates. The technique in [20] exploits a known reference signal.

Since the constraint imposed by the narrow band condition does not need to be satisfied, the proposed WB-SPECCOA technique provides satisfactory performance in scenarios characterized by higher mobility, larger bandwidths, and larger data-record lengths with respect to those allowed for classical cyclostationarity-based algorithms [3], [8], [11], [12], [14]. Consequently, satisfactory performance can be achieved at lower values of SNR and SIR. Simulation results show the effectiveness of the proposed WB-SPECCOA method and its better performance with respect to competitive methods when the narrow-band condition is not satisfied.

The paper is organized as follows. In Section 2, the source location problem is described. In Section 3, the transmitted and received signals are statistically characterized. TSR, FDOA, TDOA, and CGR estimation procedures are outlined in Section 4. Numerical results are reported in Section 5 and conclusions are drawn in Section 6.

### 2. SOURCE LOCATION PROBLEM

Let x(t) be the complex envelope of the signal transmitted by a source in relative motion with respect to two sensors and let  $r_1(t)$  and  $r_2(t)$  denote the received complex-envelope signals.

If the relative radial speeds of the source with respect to the sensors can be assumed constant within the observation interval, then it results  $r_i(t) = b_i x(s_i(t-\tau_i))e^{j2\pi\nu_i t} + n_i(t)$ , i = 1, 2, where  $b_i$  are complex gains,  $s_i$  time-scale factors,  $\tau_i$ time delays, and  $\nu_i = (s_i - 1)f_c$  frequency shifts, with  $f_c$  the carrier frequency. Signals  $n_i(t)$  are disturbance. Assuming the useful signal on the first sensor as the reference signal, that is,  $x_1(t) \triangleq b_1 x(s_1(t-\tau_1))e^{j2\pi\nu_1 t}$ , we have

$$r_1(t) = x_1(t) + n_1(t)$$
(1a)

$$r_2(t) = y_1(t) + n_2(t) = b x_1(s(t - \tau_0)) e^{j2\pi\nu t} + n_2(t)$$
 (1b)

In (1b), the time-scale ratio (TSR) *s*, frequency-difference of arrival (FDOA)  $\nu$ , time-delay-of-arrival (TDOA)  $\tau_0$ , and complex-gain ratio (CGR) *b* are defined as  $s \triangleq s_2/s_1$ ,  $\nu \triangleq \nu_2 - s\nu_1$ ,  $\tau_0 \triangleq \tau_2 - \tau_1/s$ ,  $b \triangleq (b_2/b_1) e^{j2\pi\nu_1 s\tau_0}$ .

Using estimates of these quantities and geometry, the direction of arrival of the wavefronts impinging on the two sensors and the source speed can be estimated [21], [23]. Finally, note that the "narrow-band condition" that allows to consider  $s_i \simeq 1$  in the argument of the complex envelope received signals is [17, Sec. 7.5.1]

$$BT \ll 1/|1 - s_i| \simeq c/|\mathbf{v}_i| \tag{2}$$

where B is the signal bandwidth, T the data-record length, c the medium propagation speed, and  $v_i$  is the relative radial speed between the source and the *i*th sensor.

# 3. STATISTICAL CHARACTERIZATION OF TRANSMITTED AND RECEIVED SIGNALS

Let the source signal x(t) be ACS. That is, its autocorrelation and conjugate autocorrelation functions are almost-periodic functions of time [9]. The coefficients and frequencies of the (generalized) Fourier series expansion of the (conjugate) autocorrelation function are referred to as (conjugate) cyclic autocorrelation functions and (conjugate) cycle frequencies. The almost-periodicity in the time domain reflects, in the frequency domain, into correlation between spectral components that are separated by quantities equal to the (conjugate) cycle frequencies. Thus, the Loève bifrequency spectrum [15] of x(t) is

$$E\left\{X(f_1) X^{(*)}(f_2)\right\} = \sum_{\alpha \in \mathcal{A}} S^{\alpha}_{xx^{(*)}}(f_1) \,\delta(f_2 - (-)(\alpha - f_1))$$
(3)

where X(f) is the Fourier transform of x(t) defined in a distributional sense [10, Chap. 3], [17, Secs. 1.1.2, 4.2.1]. In (3),  $\delta(\cdot)$  denotes Dirac delta, (\*) is an optional complex conjugation, (-) is an optional minus sign linked to (\*),  $\mathcal{A}$  is the countable set of the (conjugate) cycle frequencies, and the (conjugate) cyclic spectra  $S^{\alpha}_{xx^{(*)}}(f)$  are the Fourier transforms of the (conjugate) cyclic autocorrelation functions. Both functions with optional conjugation (\*) present and absent are necessary for a complete second-order characterization of the complex-valued signal x(t) [1], [22]. From (3) it follows that for ACS signals the support of the Loève bifrequency spectrum is contained in lines with slopes  $\pm 1$ .

Let  $\alpha_0$  and  $\beta_0$  be a cycle frequency and a conjugate cycle frequency, respectively, of x(t). If x(t) and  $n_i(t)$  are zero-mean and do not exhibit joint cyclostationarity, it can be shown that the (conjugate) cyclic spectra of  $r_1(t)$  and  $r_2(t)$ can be expressed as

$$S_{r_1r_1^*}^{\alpha_1}(f) = S_{x_1x_1^*}^{\alpha_1}(f) + S_{n_1n_1^*}^{\alpha_1}(f)$$
(4a)

$$S_{r_2r_2^*}^{\alpha_2}(f) = \frac{|b|^2}{|s|} e^{-j2\pi\alpha_1s\tau_0} S_{x_1x_1^*}^{\alpha_1}\left(\frac{f-\nu}{s}\right) + S_{n_2n_2^*}^{\alpha_2}(f)$$
(4b)

$$S_{r_1r_1}^{\beta_1}(f) = S_{x_1x_1}^{\beta_1}(f) + S_{n_1n_1}^{\beta_1}(f)$$
(4c)

$$S_{r_2r_2}^{\beta_2}(f) = \frac{b^2}{|s|} e^{-j2\pi\beta_1 s\tau_0} S_{x_1x_1}^{\beta_1}\left(\frac{f-\nu}{s}\right) + S_{n_2n_2}^{\beta_2}(f) \quad (4d)$$

$$\alpha_i = s_i \alpha_0 \qquad \beta_i = s_i \beta_0 + 2\nu_i \qquad i = 1, 2 \tag{4e}$$

where  $S_{x_1x_1^*}^{\alpha}(f)$  and  $S_{x_1x_1}^{\beta}(f)$  are the cyclic spectrum and the conjugate cyclic spectrum, respectively, of  $x_1(t)$ , and  $S_{n_in_i^*}^{\alpha}(f)$  and  $S_{n_in_i}^{\beta}(f)$  those of  $n_i(t)$ , i = 1, 2.

A novel cross-statistical characterization of  $r_1(t)$  and  $r_2(t)$  is presented here by resorting to the new class of SC processes [16], [17, Chap. 4]. By denoting with upper-case letter the Fourier transform defined in a distributional sense of the corresponding lower-case letter denoting a signal, and using (1a) and (1b), the Loève bifrequency cross-spectrum [15] of  $r_1(t)$  and  $r_2(t)$  can be expressed as

where

$$\mathbb{E}\left\{Y_{1}(f_{1}) X_{1}^{(*)}(f_{2})\right\} = \frac{b}{|s|} e^{-j2\pi(f_{1}-\nu)\tau_{0}} \\
 \sum_{\alpha_{1}\in\mathcal{A}_{1}} S_{x_{1}x_{1}^{(*)}}^{\alpha_{1}}\left(\frac{f_{1}-\nu}{s}\right) \delta\left(f_{2}-(-)\left(\alpha_{1}-\frac{f_{1}-\nu}{s}\right)\right) \quad (6)$$

with  $A_1$  set of (conjugate) cycle frequencies of  $x_1(t)$ .

From (6), it follows that when  $s \neq 1$ , the Loève bifrequency cross-spectrum of  $x_1(t)$  and  $y_1(t)$  has support contained in lines with slopes different from  $\pm 1$ . That is, even if both signals  $x_1(t)$  and  $y_1(t)$  are (singularly) ACS, they are not jointly ACS but, rather, jointly SC. Jointly SC signals have Loève bifrequency cross-spectrum with spectral masses concentrated on a countable set of curves in the bifrequency plane [17, Chap. 4]. Signals  $y_1(t)$  and  $x_1(t)$  are jointly SC with support lines with (non unit) slopes  $\pm 1/s$ . The density of spectral cross-correlation along the support line  $f_2 =$  $(-)(\alpha_1 - (f_1 - \nu)/s)$  is

$$S_{y_1 x_1^{(*)}}(f_1) \triangleq \frac{b}{|s|} e^{-j2\pi(f_1 - \nu)\tau_0} S_{x_1 x_1^{(*)}}^{\alpha_1} \left(\frac{f_1 - \nu}{s}\right) .$$
(7)

### 4. PROPOSED METHOD

In this section, the proposed algorithm is presented to estimate TSR, FDOA, TDOA and CGR. Note that, for the localization purpose, the CGR is not of interest. However, its estimate is obtained as by-product of the TDOA estimation. Only the phase of the CGR will be considered since of its interest in synchronization.

The parameters  $s_i$  and  $\nu_i$  can be estimated starting from the technique proposed in [19]. Let us define

$$\lambda_{r_i r_i^{(*)}}(\alpha) \triangleq \int_{\mathbb{R}} \left| \widehat{S}_{r_i r_i^{(*)}}^{\alpha}(f) \right|^2 \mathrm{d}f \tag{8}$$

where  $\widehat{S}_{r_i r_i^{(*)}}^{\alpha}(f)$  denotes the (conjugate) frequency-smoothed cyclic periodogram obtained observing signals in [0, T] [5], [7], [13]. Let us assume that the values of  $s_i$  and  $\nu_i$  are such that for some cycle frequency interval  $J_{\alpha_0}$  around  $\alpha_0$  and conjugate cycle frequency interval  $J_{\beta_0}$  around  $\beta_0$  the signals  $x_1(t)$  and  $y_1(t)$  have only one cycle frequency in  $J_{\alpha_0}$  and only one conjugate cycle frequency in  $J_{\beta_0}$ . Moreover, let us assume that  $S_{n_i n_i^*}^{\alpha}(f) = 0$  for  $\alpha \in J_{\alpha_0}$  and  $S_{n_i n_i}^{\beta}(f) = 0$  for  $\beta \in J_{\beta_0}$ . The following

$$\widehat{\alpha}_{i} = \arg \max_{\alpha \in J_{\alpha_{0}}} \lambda_{r_{i}r_{i}^{*}}(\alpha) \quad \widehat{\beta}_{i} = \arg \max_{\beta \in J_{\beta_{0}}} \lambda_{r_{i}r_{i}}(\beta) \quad (9)$$

provide consistent estimates for  $\alpha_i$  and  $\beta_i$  [4]. Then, accounting for (4e), estimates of  $s_i$  and  $\nu_i$  can be obtained by  $\hat{s}_i = \hat{\alpha}_i / \alpha_0$  and  $\hat{\nu}_i = (\hat{\beta}_i - \hat{s}_i \beta_0)/2$ , and estimates of the TSR and FDOA are given by

$$\widehat{s} \triangleq \widehat{s}_2/\widehat{s}_1 = \widehat{\alpha}_2/\widehat{\alpha}_1 \quad \widehat{\nu} \triangleq \widehat{\nu}_2 - \widehat{s}\widehat{\nu}_1 = (\widehat{\beta}_2 - \widehat{s}\widehat{\beta}_1)/2.$$
 (10)

Let us now make the mild assumption that the densities of spectral correlation of  $E\{N_2(f_1) X_1^{(*)}(f_2)\}$ ,  $E\{X_1((f_1 - \nu)/s) N_1^{(*)}(f_2)\}$  and  $E\{N_2(f_1) N_1^{(*)}(f_2)\}$  are zero along the support line  $f_2 = (-) (\alpha_1 - (f_1 - \nu)/s)$ . Thus, the spectral (cross-)correlation densities in (7) can be replaced by those of the noisy signals (see (1a) and (1b)) and we have

$$S_{r_2 r_1^{(*)}}(f_1) = \frac{b}{|s|} e^{-j2\pi(f_1 - \nu)\tau_0} S_{r_1 r_1^{(*)}}^{\alpha_1} \left(\frac{f_1 - \nu}{s}\right) \quad (11)$$

The parameters  $\tau_0$  and b can be estimated by minimizing the square error (for functions of  $f_1 \in L^2(\mathbb{R})$ ) between estimates (denoted by an hat) of the left- and right-hand sides of (11). That is,

$$(\hat{b}, \hat{\tau}_0) = \arg\min_{(\zeta, \tau)} \left\| \widehat{S}_{r_2 r_1^{(*)}}(f_1) - \frac{\zeta}{|\widehat{s}|} e^{-j2\pi (f_1 - \widehat{\nu})\tau} \widehat{S}_{r_1 r_1^{(*)}}^{\widehat{\alpha}_1} \left( \frac{f_1 - \widehat{\nu}}{\widehat{s}} \right) \right\|_2^2.$$
 (12)

By equating to zero the gradient of the norm in (12) which is function of the complex variable  $\zeta$  and of the real variable  $\tau$ , and considering  $\zeta$  and  $\zeta^*$  as independent variables [2], leads one to consider the function

$$F(\tau) \triangleq \frac{e^{-j2\pi\widehat{\nu}\tau}}{|\widehat{s}|} \int_{\mathbb{R}} \widehat{S}_{r_2 r_1^{(*)}}(f) \, \widehat{S}_{r_1 r_1^{(*)}}^{\widehat{\alpha}_1} \left(\frac{f-\widehat{\nu}}{\widehat{s}}\right)^* e^{j2\pi f\tau} \mathrm{d}f$$
(13)

and take as estimates of TDOA and angle of the CGR

$$\widehat{\tau}_0 = \arg \max_{\tau} |F(\tau)| \qquad \widehat{\phi} = \angle F(\widehat{\tau}_0) .$$
(14)

The estimate  $\widehat{S}_{r_2r_1^{(*)}}(f)$  is made by the cross-periodogram frequency smoothed along the estimated support line  $f_2 = (-)(\widehat{\alpha}_1 - (f_1 - \widehat{\nu})/\widehat{s})$  [17, Secs. 4.6, 4.7].

The estimation procedure of (14) is named WB-SPECCOA. It reduces to the NB-SPECCOA technique [11] in the narrowband case, that is, if  $s_i = 1$  can be assumed in the argument of the complex envelopes.

### 5. NUMERICAL RESULTS

The moving source is a low earth orbit (LEO) satellite at altitude h = 200 km and with orbital speed  $v_o =$ 28061.5 km h<sup>-1</sup>. The satellite transmits a binary directsequence spread-spectrum (DSSS) signal with number of chip per bit  $N_c = 64$ , chip period  $T_c = 0.06 \,\mu s$ , bit period  $T_p = N_c T_c$ , and carrier frequency  $f_c = 2$  GHz. Assuming an approximate bandwidth  $B \simeq 1/T_c = 16.5$  MHz, the complex-envelope received signals are uniformly sampled with sampling frequency  $f_s = 4B = 66$  MHz, which is the minimum required sampling frequency to avoid aliasing in second-order cyclic statistics of the sampled signal [9], [17, Sec. 1.3.9]. The distance between sensors is 100 km. The values of TSR, FDOA, and TDOA are  $s = 1 + 10^{-5}$ ,  $\nu = 21 \text{ kHz} = 0.00029/T_s$ , and  $\tau_0 = -0.8 \text{ ms}$  which corresponds to  $\tau_0 \mod T_p = -15.5T_s$ , where  $T_s = 1/f_s$  and  $a \mod b$  is the modulo operation with values in (-b/2, b/2]. These are typical values when the observation angles from sensors and from satellite are not too close to 0,  $\pi/2$ , or  $\pi$ .

Each disturbance term  $n_1(t)$  and  $n_2(t)$  contains circular additive white Gaussian noise (AWGN) with SNR = 0 dB in the band  $(-f_s/2, f_s/2)$ . In addition, the same jamming binary phase-shift keying (BPSK) signal with carrier frequency  $f_c$  and symbol period  $T_{pi} = 5T_s$  impinges on each sensor with different time-scale, frequency shift, and delay. On each sensor SIR = -3 dB and the power spectrum of the interferer BPSK signal completely overlaps that of the useful DSSS signal. In addition, the interfering terms on the two sensors are correlated. In the experiments,  $\alpha_0 = \beta_0 = 1/T_p$ .

The performance of the proposed WB-SPECCOA method, in terms of normalized sample root mean-squared error (rmse) of estimated parameters is compared with that of the NB-SPECCOA method. This latter method is chosen for comparison since, among the cyclostationarity-based techniques, SPECCOA and NB-SPECCOA are the most robust against noise and interference [3], [11], and since WB-SPECCOA reduces to NB-SPECCOA under the narrow-band condition. Moreover, the classical estimation methods based on the narrow-band cross-ambiguity function (NB-CAF) [26, Chap. 10] and the wide-band cross-ambiguity function (WB-CAF) [24], [25], [27] are also considered since in AWGN and for high SNR the former is optimal in the narrow-band case and the latter is suboptimal close to optimal in the wide-band case. In Figure 1, the sample rmse of (a)  $\hat{s}/|s|$ , (b)  $\hat{\nu}T_p$ , (c)  $\hat{\tau}_0/T_p$ , and (d)  $\hat{\phi}/2\pi$ , with  $\hat{\phi}$  phase of  $\hat{b}$ , are reported as function of the number  $N_b$  of processed bits. One hundred Monte Carlo trials are carried out to evaluate sample statistics.

It results  $T = N_b N_c T_c \simeq N_b N_c / B$ , and the narrow-band condition (2) reduces to  $N_b N_c \ll 1/|1-s|$ , that is,  $N_b \ll 10^5/64 \simeq 2^{10.6}$ .

From Figs. 1 (a)–(d) it follows that the proposed WB-SPECCOA method outperforms the competitors when the narrow-band condition is not satisfied ( $N_b \ge 2^{10}$ ). In con-

trast, when this condition is satisfied, NB-SPECCOA exhibits better performance. The bad performance of the WB-CAF method is a consequence of bias due to the presence of the strong correlated interferers on the two sensors.



**Fig. 1.** Normalized sample rmse of estimated parameters as function of the number  $N_b$  of processed bits. ( $\Box$ ) WB-SPECCOA; (\*) NB-SPECCOA; (o) WB-CAF; ( $\nabla$ ) NB-CAF. (a) rmse of  $\hat{s}/|s|$ ; (b) rmse of  $\hat{\nu}T_p$ ; (c) rmse of  $\hat{\tau}_0/T_p$ ; (d) rmse of  $\hat{\phi}/2\pi$ .

#### 6. CONCLUSION

The WB-SPECCOA method is proposed for localizing a moving source emitting an almost-cyclostationary signal. It is based on noisy measurements of the source signal impinging on two sensors. The method does not assume the narrowband condition be satisfied and models the signals on the two sensors as jointly spectrally correlated. It provides estimates of the TSR, FDOA, TDOA and CGR of the signals collected on the two sensors. It has good performance with data-record lengths that can be significantly larger than those that can be adopted by the classical cyclostationarity-based methods that assume the narrow-band condition is satisfied. Thus it can be suitably exploited at values of SNR and SIR lower than those for which classical methods exhibit good performance. Simulation experiments have shown the effectiveness of the proposed method to provide reliable estimates in the presence of severe noise and interference environments outperforming the most robust classical cyclostationarity-based method for FDOA and TDOA estimation.

### 7. REFERENCES

•

- T. Adali, P. J. Schreier, and L. L. Scharf. Complex-valued signal processing: The proper way to deal with impropriety. *IEEE Transactions on Signal Processing*, 59(11):5101–5125, November 2011.
- [2] D. H. Brandwood. A complex gradient operator and its application in adaptive array theory. *IEE Proceedings, Parts F and H*, 130(1):11–16, February 1983.
- [3] C.-K. Chen and W. A. Gardner. Signal-selective timedifference-of-arrival estimation for passive location of manmade signal sources in highly corruptive environments. Part II: Algorithms and performance. *IEEE Transactions on Signal Processing*, 40:1185–1197, 1992.
- [4] P. Ciblat, P. Loubaton, E. Serpedin, and G. B. Giannakis. Asymptotic analysis of blind cyclic correlation-based symbolrate estimators. *IEEE Transactions on Information Theory*, 48(7):1922–1934, July 2002.
- [5] A. V. Dandawaté and G. B. Giannakis. Nonparametric polyspectral estimators for kth-order (almost) cyclostationary processes. *IEEE Transactions on Information Theory*, 40(1):67–84, January 1994.
- [6] V. De Angelis, L. Izzo, A. Napolitano, and M. Tanda. Cyclostationarity-based parameter estimation of wide-band signals in mobile communications. In *IEEE Statistical Signal Processing Workshop (SSP'05)*, Bordeaux, France, July 17-20 2005.
- [7] D. Dehay and H. L. Hurd. Representation and estimation for peridically and almost periodically correlated random processes. In W. A. Gardner, editor, *Cyclostationarity in Communications and Signal Processing*, chapter 6, pages 295–328. IEEE Press, 1994.
- [8] W. A. Gardner and C.-K. Chen. Signal-selective timedifference-of-arrival estimation for passive location of manmade signal sources in highly corruptive environments. Part I: Theory and method. *IEEE Transactions on Signal Processing*, 40:1168–1184, 1992.
- [9] W. A. Gardner, A. Napolitano, and L. Paura. Cyclostationary: Half a century of research. *Signal Processing*, 86:639–697, 2006.
- [10] I. M. Gel'fand and N. Ya. Vilenkin. Generalized Functions, Applications of Harmonic Analysis, volume 4. Academic Press, New York, NJ, 1964.
- [11] Z. T Huang, Y. Y. Zhou, W. L. Jiang, and Q. Z. Lu. Joint estimation of Doppler and time-difference-of-arrival exploiting cyclostationary property. *IEE Proceedings - Radar, Sonar and Navigation*, 149(4):161–165, August 2002.
- [12] Zhitao Huang, Yiyu Zhou, and Wenli Jiang. TDOA and Doppler estimation for cyclostationary sgnals based on multicycle frequencies. *IEEE Transactions on Aerospace and Electronic Systems*, 44(4):1251–1264, October 2008.

- [13] H. L. Hurd and J. Leśkow. Strongly consistent and asymptotically normal estimation of the covariance for almost periodically correlated processes. *Statist. & Decision*, 10:201–225, 1992.
- [14] Yang Liu, Tianshuang Qiu, and Hu Sheng. Time-differenceof-arrival estimation algorithms for cyclostationary signals in impulsive noise. *Signal Processing*, 92(9):2238 – 2247, 2012.
- [15] M. Loève. *Probability Theory*. Van Nostrand, Princeton, NJ, 1963.
- [16] A. Napolitano. Uncertainty in measurements on spectrally correlated stochastic processes. *IEEE Transactions on Information Theory*, 49(9):2172–2191, September 2003.
- [17] A. Napolitano. Generalizations of Cyclostationary Signal Processing – Spectral Analysis and Applications. John Wiley & Sons Ltd – IEEE Press, 2012.
- [18] A. Napolitano. Generalizations of cyclostationarity: A new paradigm for signal processing for mobile communications, radar, and sonar. *IEEE Signal Processing Magazine*, 30(6):53– 63, 2013.
- [19] A. Napolitano and K. Doğançay. On waveform design in interference-tolearnt range-Doppler estimation for wideband multistatic radars. In XIX European Signal Processing Conference (EUSIPCO 2011), Barcelona, Spain, August 29– September 2 2011.
- [20] A. Napolitano and I. Perna. On synchronization of Dopplerstretched GPS signals. In XXI European Signal Processing Conference (EUSIPCO 2013), Marrakech, Morocco, September 9–13 2013.
- [21] A.H. Sayed, A. Tarighat, and N. Khajehnouri. Network-based wireless location: Challenges faced in developing techniques for accurate wireless location information. *IEEE Signal Processing Magazine*, 22(4):24–40, 2005.
- [22] P. J. Schreier and L. L. Scharf. Stochastic time-frequency analysis using the analytic signal: Why the complementary distribution matters. *IEEE Transactions on Signal Processing*, 51(12):3071–3079, December 2003.
- [23] Petre Stoica and Jian Li. Lecture notes source localization from range-difference measurements. *IEEE Signal Processing Magazine*, 23(6):63–66, 2006.
- [24] D. A. Swick. An ambiguity function independent of assumption about bandwidth and carrier frequency. Technical Report 6471, Naval Research Laboratory, Washington D.C., USA, December 1966.
- [25] D. A. Swick. A review of wideband ambiguity functions. Technical Report 6994, Naval Research Laboratory, Washington D.C., USA, December 1969.
- [26] H. L. Van Trees. Detection, Estimation, and Modulation Theory. Part III. John Wiley & Sons, Inc., New York, 1971.
- [27] L. G. Weiss. Wavelets and wideband correlation processing. *IEEE Signal processing Magazine*, 11(1):13–22, January 1994.