

# A NONLINEAR SECOND-ORDER DIGITAL OSCILLATOR FOR VIRTUAL ACOUSTIC FEEDBACK

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## ABSTRACT

The guitar feedback effect, or howling, is well known to the general public and identified with many rock music genres and it is the only case of acoustic feedback employed for musical purposes. Virtual Acoustic Feedback (VAF), is regarded as the extension of this phenomenon to any instrument or sound source by means of virtual acoustics and is meant to enrich the sound palette of a musician. The study of the acoustic feedback as a musical tool and computational techniques for its emulation have been scarcely addressed in literature. In this paper a nonlinear feedback oscillator is proposed and its properties derived. The oscillator does not necessarily need to be connected to a virtual instrument, thus enables to process any kind of pitched real-time input.

**Index Terms**— acoustic feedback, digital oscillator, non-linear filtering, digital audio effect

## 1. INTRODUCTION

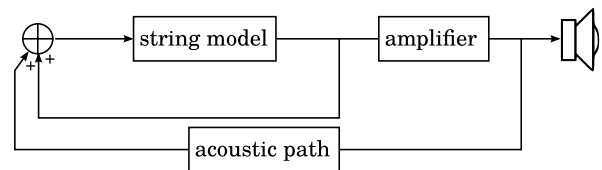
In the past the music industry has proposed commercial products inspired by or emulating the guitar howling effect. Products inspired by this effect generally employ electromagnetic stimuli to strings, in order to sustain or affect their sound. The howling effect, in deed, is based on the stimulation of a vibrating string by pressure waves, generated by the personal amplification system, that sum in phase to it.

Triggering howling needs high sound pressure levels and experience. For this reason several hardware devices have been developed to obtain easily controllable feedback electromagnetic feedback (such as the vibesware GR<sup>1</sup>) or comparable timbre quality (such as the e-bow<sup>2</sup>). On the digital side there are also a few virtual feedback simulators but their architecture, is generally not publicly disclosed.

The first algorithm for virtual feedback that can be found in the academic literature is the one from Sullivan [1], recently reimplemented by Smith [2]. Related literature includes devices for instrument augmentation and feedback

control, e.g. for metal strings by means of electromagnetic devices [3, 4]. Recently an acoustic control system to actuate feedback between a guitar and amplifier has been proposed [5].

Sullivan's method to emulate howling consists of a tunable delay line fed back with positive sign to a Karplus-Strong [6] string model as shown in Figure 1. The wave propagation delay line resonates at a frequency  $f_o = F_s/N$  (where  $F_s$  is the sample rate and  $N$  is the delay line length), and is closely related to well-known Digital Waveguide (DWG) oscillators [7, 8, 9]. This structure has desirable properties, such as easy stability control, but has no control over the harmonic content. Furthermore, it has no straightforward extension to the case of external input sources. One software product based on Sullivan's work, documented in patent [10], requires the implementation of a virtual string model, that needs to be tuned after analysis on the incoming signal. One downside of Sullivan's algorithm is, thus, the need for a virtual string model adapting to the guitar strings used as acoustic source, for the algorithm to work properly, posing problems in its real-time parametrization.



**Fig. 1.** Overview of the circuit used by Sullivan[1].

In this paper the authors propose a nonlinear digital oscillator able to simulate acoustic feedback. The howling harmonic and other operating parameters are user-tunable. The oscillator topology is derived from that of analog positive feedback oscillators, with careful selection of zeros and poles to ensure computability in the discrete-time domain and avoid frequency deviation. The oscillator can be employed either with virtual instruments or with external input sources.

<sup>1</sup><http://www.vibesware.com/>

<sup>2</sup><https://ebow.com/>

## 2. ACOUSTIC FEEDBACK

Being the most well known musical use of acoustic feedback, the guitar howling will be taken from now on as a reference and inspiration for the design of a novel VAF digital effect. The typical setup, involves an electric guitar as sound source and a nonlinear amplifier with loudspeaker. A constructive feedback is generated when the sound pressure waves couple with one of the guitar strings. For the coupling to be remarkable the travelling waves must be in phase with the string and their amplitude high enough to overcome the string mechanical resistance and sensibly excite the string (either fretted or open). The coupled string acts as a resonator. As a corollary, it must be noted that howling can rise only at a multiples of the fundamental  $F_0$  of the string.

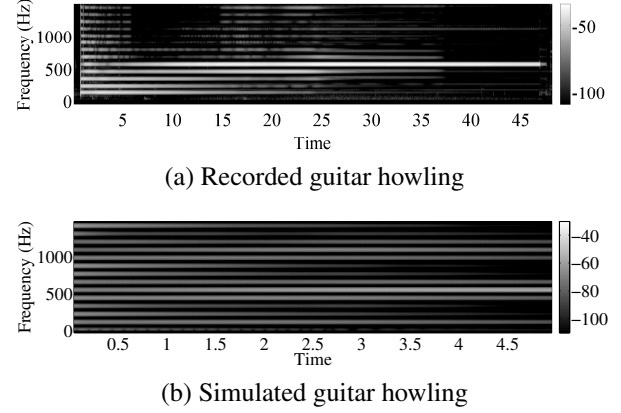
The system response from the guitar to the loudspeaker can be considered slowly time-varying (unless time-varying effects are employed). The only portion of the system which can vary remarkably is the path between the musician and the loudspeaker. In a closed or even semi-anechoic environment there will be several paths able to sum in phase with a specific string. The howling will take place if at least one of the paths will violate the Barkhausen stability criterion [11], i.e. for a certain frequency  $\omega_0$  :

$$A(j\omega_0)G(j\omega_0) = 1 \Rightarrow \begin{cases} |A(j\omega_0)G(j\omega_0)| = 1 \\ \angle A(j\omega_0)G(j\omega_0) = 0 \end{cases} \quad (1)$$

where  $A(j\omega)$  is the direct path transfer function (instrument to loudspeaker) and  $G(j\omega)$  the feedback path transfer function (loudspeaker to instrument).

If several paths are unstable, most likely one feedback path (i.e. one howling frequency) will prevail among the others, since, in unstable conditions, small deviations in the loop gain or phase delay will result in substantial amplitude difference between the paths after a sufficiently large amount of time. This has been experienced by computer simulations based on Sullivan's method. Tests have been conducted using Sullivan's method to gather insight on the guitar feedback phenomenon. Figure 2 compares a recorded A2 tone with howling rising at the 5th partial, and a simulated counterpart of similar characteristics employing a DWG guitar model as the sound source.

To complete our overview on the howling effect, a large database of guitar tones and howling was recorded in an anechoic chamber, with an electric guitar and overdriven transistor amplifier, in order to analyze which settings influence the onset of the feedback and which partial is more likely to be fed back. The tones recorded spanned from E2 to E5. Some notes required very high pressure levels to trigger howling, thus, protective headphones were required. Depending on the note, howling was triggered for partials from  $F_0$  up to the sixth. Overall, the lowest howling frequency obtained was at 440 Hz, the highest at 2637 Hz. Distance and position had an



**Fig. 2.** Comparison of the spectrograms of a recorded guitar howling (a), and a simulation using Sullivan's method (b). Both tones are A2 (110Hz) with howling occurring at the 5th partial.

effect on the triggered partial and required volume. Finally, an interesting finding is that octave-spaced notes trigger the same howling frequency, if all the other conditions do not change.

### 2.1. Desired Features

A VAF digital effect must have:

- stable oscillation with peak amplitude limiting,
- precisely tunable frequency of oscillation  $f_o$  at a multiple of the incoming fundamental frequency  $F_0$ ,
- emulation of nonlinearities (e.g. distortion, strings nonlinearities, etc.),
- selectable rise time.

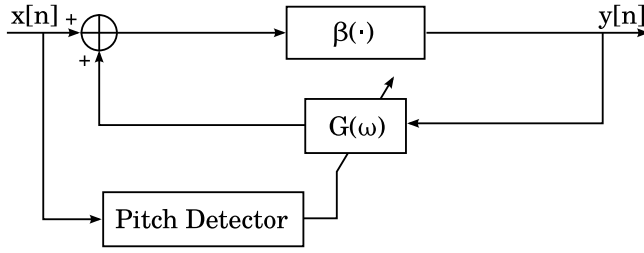
## 3. PROPOSED OSCILLATOR

The problem at hand can be posed in terms of oscillators. In literature there are examples of digital oscillators mainly devoted to virtual analog modelling [12] or physical modelling of passive structures [13, 14]. While the former are generally complex and computationally expensive, the latter do not apply to the current case, which is not passive and may exhibit growth as well as decay. The following section, thus, proposes an oscillator model capable of emulating acoustic feedback taking inspiration from the howling scenario described in Section 2.

### 3.1. Second-Order Digital Nonlinear Oscillator

In order to fulfil Section 2.1 requirements, a positive feedback oscillator, depicted in Figure 3, can be employed. This oscillator consists of a selective bandpass filter in positive feedback  $G(\omega)$ , to select the desired harmonic and a memoryless

nonlinear component  $\beta(\cdot)$  to engage oscillation and prevent instabilities. The latter also adds distortion to the incoming signal as in real guitar amplifiers.



**Fig. 3.** General scheme of a digital oscillator with nonlinearity  $\beta(\cdot)$  and bandpass transfer function  $G(z)$ .

### 3.2. Linear Filter Design

The nonlinear component of the oscillator will be now neglected (i.e.  $\beta = 1$ ) in order to design the coefficients of the linear part and obtain the overall transfer function  $H(z)$  of the oscillator.

The bandpass filter must be selective enough to reject all harmonics beside the desired one. Second-order designs are sufficient for this task. However, when placed in a feedback loop, problems of computability may arise. Let  $G(z) = B(z)/A(z)$  be the bandpass transfer function and  $H(z) = N(z)/D(z)$  be the entire oscillator transfer function. If the bandpass direct path filter coefficient  $b_0$  is not null, the output  $y[n]$ , passing through the bandpass direct path, is fed back without any delay, making the system output impossible to compute in a discrete time environment. By observing that the oscillator always has a direct signal path to the output,  $n_0 = 1$ , the original solution proposed here is to design the bandpass filter to have  $B(z)$  with first coefficient  $b_0 = 0$ . This ensures computability and does not affect the oscillation frequency of the oscillator. The frequency response of the oscillator in Figure 3 is that of a peaking filter.

In general the oscillator poles will differ from the bandpass poles, hence the filter center frequency  $f_c$  and the oscillator frequency  $f_o$  will differ. The oscillator transfer function, is:

$$H(z) = \frac{A(z)}{A(z) - B(z)}. \quad (2)$$

The higher the bandpass bandwidth, the higher the  $B(z)$  coefficients will be, thus the difference between  $f_o$  and  $f_c$  will increase. To prevent this drift, the  $H(z)$  can be designed first, and then the  $G(z)$  evaluated accordingly. The proposed method consists in the design of a peaking Butterworth design centered at the desired  $f_o$  and with desired  $BW$ , yielding numerator and denominator  $N(z)$ ,  $D(z)$ . The  $G(z)$  will be then evaluated as:

$$A(z) = N(z); \quad (3)$$

$$B(z) = N(z) - D(z). \quad (4)$$

The Butterworth peaking design ensures the first coefficient of the numerator  $n_0 = 1$ , thus, from Eqns.(2,3) the first coefficient of  $B(z)$  will be always  $b_0 = 0$ , ensuring computability of the oscillator.

The oscillator must be causal and stable. Stability is always guaranteed, provided the initial filter design is stable. Proof of this can be gathered if considering the poles of  $H(z)$ , given by  $D(z) = A(z) - B(z)$ . The bandpass poles are always inside the unit circle by design, and only approach unity with  $BW \rightarrow 0$ , while its zeros can always be designed to be nonnegative. The roots of  $D(z)$  are, thus, always inside the unit circle.

### 3.3. Nonlinear Oscillator Properties

In electronic oscillator design practice [15], a positive feedback oscillator makes use of a frequency independent amplifier, with nonlinear function  $\beta(\cdot)$ , typically a saturating nonlinearity, such as  $\tanh$ . The amplifier behaviour at small signals can be considered linear, i.e. a constant gain  $A_s$ , which saturates, i.e. reduces up to zero for increasingly large signals. It is frequent to study such a circuit as a quasi-linear system, i.e. study how the linear system transfer function is affected by the nonlinearity [16].

The nonlinearity, in fact, has an impact on the oscillator poles position in the complex plane. As a corollary, the more the signal amplitude increases, the more the oscillator behaviour deviates from the linear one. Specifically the  $f_o$  may shift from the desired value depending on the signal amplitude, i.e. the loop gain.

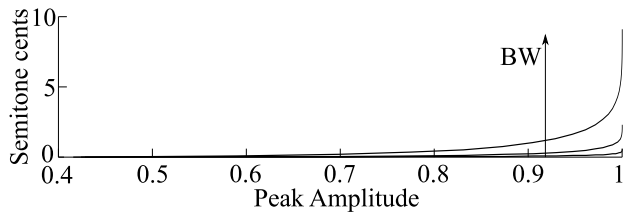
Let the variable gain imposed by  $\beta(x)$  be considered constant for an approximately linear region, and  $\beta$  be that constant gain, the oscillator transfer function is

$$H_{nl}(z) = \frac{\beta A(z)}{A(z) - \beta B(z)}, \quad (5)$$

i.e. the poles frequency deviates from the linear case as a function of the gain  $\beta$ , which in turn depends on the input signal amplitude.

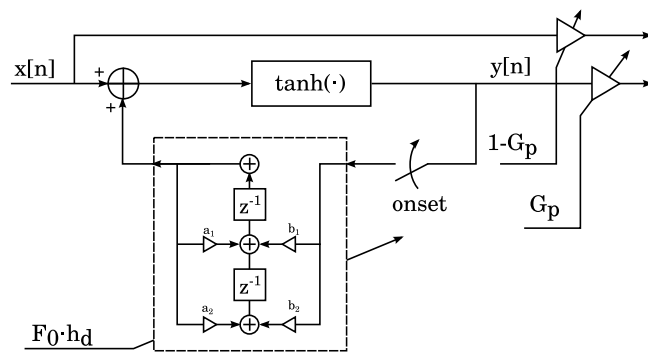
The roots of the denominator can be evaluated for increasing values of  $\beta$ . Figure 4 shows the skew in cents of semitone at increasing values of signal peak amplitude and increasing bandwidth. This frequency skew is not desirable, but for any input peak amplitude and BWs of up to 1/5 the frequency skew is lower or comparable to the *just-noticeable difference* [17], i.e. the minimum pitch interval that can be discriminated by the human ear. If the frequency estimation method is sufficiently reliable the bandwidth can be generally chosen large enough to compensate for the estimate tolerance.

Finally, the amplitude, rise time and slope can be set by the musician with the use of a gain  $G_p$  cascaded to the oscillator and its complementary  $1 - G_p$  cascaded to the dry signal path. The oscillator loop is closed only when an onset is triggered and a pitch detected, as the positive feedback oscillator



**Fig. 4.** Nonlinear oscillator  $f_o$  skew in cents of semitone, with respect to a desired  $f_o$  of 500 Hz, function of the bandwidth of the  $G(\omega)$  and the input signal peak amplitude. The three curves correspond to a bandwidth of 25 Hz, 50 Hz and 100 Hz.

can start oscillation with small input signals. The proposed oscillator is depicted in Figure 5.



**Fig. 5.** The proposed digital second-order oscillator.  $h_d$  is the desired partial for howling onset.

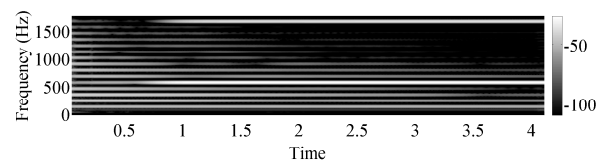
Since the  $\tanh(\cdot)$  function is odd, only odd harmonics will take place in the distorted signal. In musical applications, a distortion function should also generate even harmonics. In principle additional distortion components can be applied to the output signal  $y[n]$ . However, if the howling harmonic distortion is required to keep low, any memoryless waveshaping function can be used, provided it is bounded. Performed experiments show that an asymmetrical bounded function such as that from Doidic et al. ([18], Eq. (3)) poses no problems to stability and sustained oscillation. When distortion modelling employing components with memory, is desired to be placed inside the loop further stability and frequency skew studies are required. One rather simple method for this is that of the describing function [19].

#### 4. SIMULATIONS AND REAL-TIME CONSIDERATIONS

In a real scenario the oscillator needs note information to tune on the desired partial, thus, when the sound source is external a frequency estimator algorithm is needed to extract pitch information. Real-time onset detection is needed as well[20].

Given the low computational cost and latency of the oscillator, the choice of the pitch estimation algorithm is critical, given the real-time constraints on latency and computational cost. A number of good algorithms are available in literature. Two parametric pitch detection algorithms, developed by Christensen et al. [21], have been compared to a simpler autocorrelation-based one, SNAC (Specially Normalized Autocorrelation)[22]. The first two are based on subspace orthogonality (see [21] Ch.4.7) and shift invariance (see [21] Ch.4.9). From a first evaluation SNAC proved to be much more efficient, sufficiently accurate in tracking pitch, including the subtle changes due to vibratos and bending. A trade-off between lower pitch bound and latency must be found. The other two methods, were computationally heavy and provided scarce accuracy with small frame sizes (those required by real-time processing). On the other side they provided no false detections with the addition of a MAP-criterion based decision algorithm (see [21], Ch. 2.6). Real-time capable algorithms with good accuracy need to be carefully evaluated.

A spectrogram obtained from simulations is shown in Figure 6. Virtual acoustic feedback can be triggered on a wide class of pitched instruments, including human voice. More research material and audio examples are available at the paper companion page<sup>3</sup>.



**Fig. 6.** Spectrogram obtained from a simulated howling. A2 tone (110Hz) with howling at 550Hz (5th partial).

#### 5. CONCLUSIONS

This paper describes a digital oscillator able to simulate virtual acoustic feedback. The effect can be employed with any pitched sound source and does not need source modelling. The oscillator has a very low computational cost, almost negligible when a frequency estimator algorithm is added to extract pitch information for the oscillator tuning. The oscillator has good stability properties, and enables frequency, amplitude and rise time control. Further extension of the study may be done to provide modelling of distortion or other nonlinearities (such as those related to the physics of the involved instrument) without compromising stability and tunability, and avoiding possible aliasing. Subjective assessment of the sound quality is still required[23]. An implementation on an embedded target is ongoing [24].

<sup>3</sup>a3lab.dii.univpm.it/projects/vaf

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