PHASE CONSTRAINED COMPLEX NMF: SEPARATING OVERLAPPING PARTIALS IN MIXTURES OF HARMONIC MUSICAL SOURCES

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ABSTRACT

This paper examines complex non-negative matrix factorization (CMF) as a tool for separating overlapping partials in mixtures of harmonic musical sources. Unlike non-negative matrix factorization (NMF), CMF allows for the development of source separation procedures founded on a mixture model rooted in the complex-spectrum domain (in which the superposition of overlapping sources is preserved). This paper introduces a physically motivated phase constraint based on the assumption that the source's pitch is sufficient in specifying the phase evolution of the harmonics over time, uniting sinusoidal modelling of acoustic sources with the CMF analysis of their spectral representations. The CMF-based separation procedure, armed with this novel phase constraint, is demonstrated to offer a superior performance to NMF when employed as a tool for separating overlapping partials in the acoustic test cases considered.

Index Terms— non-negative matrix factorization, complex non-negative matrix factorization, phase constraints, source separation, harmonic resolution

1. INTRODUCTION

Non-negative matrix factorization (NMF) [1] has been demonstrated to be an effective tool for performing single-channel musical source separation (SC-MSS) when applied to a time-frequency matrix representation (e.g., a magnitude/power spectrogram) of a musical mixture [2]. This approach, however, incorrectly models the spectrogram, X, of the mixed sources as the sum of the spectrograms, S_p , of the unmixed sources, disregarding any phase discrepancies between the overlapping spectra. This inequality can be expressed as follows:

$$X = |\mathbb{X}| = |\sum_{p=1}^{P} \mathbb{S}_{p}| \neq \sum_{p=1}^{P} |\mathbb{S}_{p}| = \sum_{p=1}^{P} S_{p}$$
(1)

Here, X and S_p represent the complex valued Short-Time Fourier Transform (STFT) of the mixture and p^{th} source (of a *P* source mixture), respectively. It is recognized in the literature, notably [3], that this mixture model assumption limits the use of NMF as a tool for resolving the amplitudes of the unmixed sources in regions of high-energy spectral overlap, for which consideration of the phase difference between overlapping sources is essential.

Complex Matrix Factorization (CMF) [4] was proposed in 2009 to address this shortcoming of NMF by incorporating the previously excluded phase information directly into the factorization framework. To date, nearly all research concerning CMF-based singlechannel source separation deals exclusively with acoustic mixtures of speech samples [4], [5], [6], [7]. There exists a substantial gap in the literature investigating the use of CMF as a tool for SC-MSS (note that [8], [9], and [10], do examine the use of CMF applied to musical mixtures but in the context of multi-channel source separation). In this paper, we specifically consider the task of separating overlapping partials in single-channel mixtures of harmonic musical sources using CMF, a problem which could not be properly addressed using the previously established NMF-based source separation procedures. Moreover, recent literature [6] suggests that one of the largest areas of improvement for the CMF algorithm is in the estimation of the phase parameter. The work presented here attempts to improve upon the phase estimation of the overlapping partials in mixtures of harmonic musical sources through the development of a physically motivated phase evolution constraint.

The remainder of the paper is organized as follows: Section 2 reviews the relevant background literature concerning NMF and CMF. Section 3 outlines the newly developed phase evolution constraint. Section 4 presents the results of two experiments conducted as a means of comparing NMF to CMF under the newly proposed phase constraint. Finally, Section 5 concludes and discusses future directions.

2. BACKGROUND

2.1. Non-negative Matrix Factorization

NMF consists of approximately factorizing a given $N \times M$ nonnegative data matrix, X, into an $N \times K$ non-negative basis matrix, W, and $K \times M$ non-negative activation matrix, H. NMF is typically expressed as follows:

Given :
$$X \in \mathbb{R}_{\geq 0}^{N \times M}$$
 and $K \in \mathbb{N}_{>0}$
Factorize : $X \approx \widehat{X} = WH$ (2)
Subject to : $W \in \mathbb{R}_{\geq 0}^{N \times K}$ and $H \in \mathbb{R}_{\geq 0}^{K \times M}$

As was first examined in [11], when the data matrix, X, corresponds to a non-negative spectrogram of a musical passage, the *ideal* non-negative factorization results in matrix factor, W, whose column vectors correspond to spectral templates of the generative musical structures (e.g., sustained tones, transients, and noise). These spectral profiles combine in a purely additive fashion, according to the non-negative temporal activations, specified by the rows of H, to form an approximation of the original time-frequency representation of the musical passage.

For the purposes of this paper we will specifically focus on sparse NMF, as specified by the following optimization problem:

Given :
$$X \in \mathbb{R}_{>0}^{N \times M}$$
 and $K \in \mathbb{N}_{>0}$

$$\begin{aligned} \text{Minimize} &: \frac{1}{2} \sum_{n,m} |[X]_{n,m} - [\widehat{X}]_{n,m}|^2 + \lambda \sum_{k,m} |[H]_{k,m}|^g \\ \text{Subject to} &: \sum_{n} [W]_{n,k}^2 = 1 \; (\forall k = 1, ..., K), W \in \mathbb{R}_{\geq 0}^{N \times K} \\ H \in \mathbb{R}_{\geq 0}^{K \times M} \end{aligned}$$
(3)

The first term in the minimization of Equation 3 corresponds to the distance induced by the square of the Frobenius norm, a quantifiable measure for the quality of approximation of $\hat{X} = WH$ to X, whereas the second term corresponds to the sparsity factor, penalizing larger values of the elements of H. Here, λ is a weighting parameter corresponding to the relative importance of the sparsity factor on the overall optimization and g is a parameter influencing the shape of the sparsity distribution. A solution to the optimization problem of Equation 3 can be found in [12].

2.2. Complex Non-negative Matrix Factorization

CMF can be defined by the following optimization problem:

Given :
$$\mathbb{X} \in \mathbb{C}^{N \times M}$$
 and $K \in \mathbb{N}_{>0}$
Minimize : $\frac{1}{2} \sum_{n,m} |[\mathbb{X}]_{n,m} - [\widehat{\mathbb{X}}]_{n,m}|^2 + \lambda \sum_{k,m} |[H]_{k,m}|^g$
Subject to : $\sum_n [W]_{n,k} = 1 \; (\forall k = 1, ..., K), W \in \mathbb{R}_{\geq 0}^{N \times K}$
 $H \in \mathbb{R}_{\geq 0}^{K \times M}$ and $\Phi \in \mathbb{R}^{N \times K \times M}$

$$(4)$$

where $[\widehat{X}]_{n,m} = \sum_{k=1}^{K} [W]_{n,k} [H]_{k,m} \exp(i[\Phi]_{n,k,m})$. Note that the inclusion of the phase term, $\exp(i[\Phi]_{n,k,m})$ prevents CMF from being expressed as a true factorization. However, it is referred to as such due to the similarities in structure shared with NMF. A solution to the optimization problem of Equation 4 can be found in [4], and

3. PHASE EVOLUTION CONSTRAINT

an implementation of the solution can be found online at [13].

3.1. Phase Evolution Assumptions

The following model/assumptions guided the development of the phase evolution constraint. Note that similar assumptions are also adopted in the work of [3].

A:1 Assume that the fundamental frequency of each source is known.

A:2 Assume that each source is well modelled as a sum of sinusoids:
$$\tilde{s}_p(t) = \sum_{r=1}^{R_p} A_{p,r} \exp(i(2\pi f_{0_p} rt + \phi_{0_{p,r}})))$$
, where f_{0_p}

corresponds to the fundamental frequency (in Hz) of the p^{th} source, r corresponds to the harmonic number starting from

r = 1 (the fundamental) to R_p (the total number of harmonics considered for source p), where $f_{0_p}R_p < \frac{Fs}{2}$, $A_{p,r}$ and $\phi_{0_{p,r}}$ corresponds to the amplitude and initial phase, respectively, of the r^{th} harmonic of the p^{th} source, and t corresponds to the continuous time variable.

- A:3 Assume that the energy of a given harmonic does not extend beyond the frequency bins which fall under the main lobe of the Fourier transform of the analysis window centered about the frequency of that harmonic.
- A:4 Assume that the number of sources is known and that only one component is set to be extracted per source (i.e., K = P).

3.2. Phase Evolution Cost Function

Consider the following form of the STFT:

$$[\mathbb{X}]_{n,m} = \sum_{l=0}^{\tilde{N}-1} x(l+mL)w(l) \exp(\frac{-i2\pi ln}{\tilde{N}})$$
(5)

where w is a suitably chosen window function of length \tilde{N} , where $\tilde{N} = 2(N-1)$ and L is the frame shift in samples. Applying this form of the STFT to the mixture of sources modelled according to A:2, while assuming A:1 - A:4, we arrive at the proposed phase evolution cost function:

$$C(\Phi) = \sum_{n,p,r,m} \mathbb{1}_{\mathcal{N}_{p,r}} |\exp(i[\Phi]_{n,p,m}) - \exp(i[\Phi]_{n,p,m-1}) \exp(i2\pi f_{0_p} r L T)|^2$$
(6)

where T represents the sampling period, $\mathcal{N}_{p,r}$ is the set of all frequency bins, n, which fall under the main lobe of the Fourier transform of the analysis window centered about the frequency $f_{0_p}r$ and $\mathbb{I}_{\mathcal{N}_{p,r}}$ is an indicator function specifying the membership of the n^{th} frequency bin to the set $\mathcal{N}_{p,r}$. Intuitively, minimizing the cost function presented in Equation 6 is equivalent to specifying that the phase of the harmonics of the sources evolve according to the sinusoidal model assumed in A:2.

3.3. CMF under Phase Evolution Constraints

The new CMF model under phase evolution constraints can now be stated as follows:

Given :
$$\mathbbm{X} \in \mathbbm{C}^{N \times M}$$
 , $P \in \mathbbm{N}_{>0}$ and $f_{0_p}(p=1...P)$

$$\begin{aligned} \text{Minimize} &: \frac{1}{2} \sum_{n,m} |[\mathbb{X}]_{n,m} - [\widehat{\mathbb{X}}]_{n,m}|^2 + \lambda \sum_{p,m} |[H]_{p,m}|^g \\ &+ \sigma \sum_{n,p,r,m} \mathbb{1}_{\mathcal{N}_{p,r}} |\exp(i[\Phi]_{n,p,m}) \\ &- \exp(i[\Phi]_{n,p,m-1}) \exp(i2\pi f_{0_p} r L T)|^2 \end{aligned}$$

$$\begin{aligned} \text{Subject to} &: \sum_{n} [W]_{n,p} = 1 \; (\forall p = 1, ..., P), W \in \mathbb{R}_{\geq 0}^{N \times P} \\ &H \in \mathbb{R}_{\geq 0}^{P \times M} \text{ and } \Phi \in \mathbb{R}^{N \times P \times M} \end{aligned}$$

$$\end{aligned}$$

where σ is a weighting parameter corresponding to the relative importance of the phase evolution cost function on the overall optimization. The form of the phase evolution cost function is such that the auxiliary function optimization techniques, as employed in [4], [6] and [7], are not required. However due to lack of continuity, convergence of the optimization algorithm is not guaranteed. In practice, however, convergence of the optimization algorithm was observed for the experiments described in Section 4. The CMF algorithm, with the inclusion of the phase evolution constraint, is described in Algorithm: 1.

4. EXPERIMENTATION AND RESULTS

Two experiments were conducted to investigate the behaviour of CMF with the newly proposed phase evolution constraint as a tool for harmonic resolution of overlapping musical sources. Two acoustic piano note samples (D4, B4) and one classical guitar note sample (C4) were considered. The three recordings were taken from [14], down sampled to a rate of Fs = 11025 Hz and truncated in time so that each sample was exactly one second long. Both mixtures were three seconds long and consisted of a piano note played in isolation (either D4 or B4) followed by the (C4) guitar note played in isolation followed by both the guitar note and the chosen piano note played simultaneously (100% overlap). Magnitude spectrograms were created for each source using a 46 ms long modified Hann window, as defined in [15], and an 11 ms long frame shift. Estimates for the fundamental frequencies of the C4 guitar source and the D4/B4 piano sources were taken to be C4: 261.63 Hz, D4: 293.66Hz, and B4: 493.88Hz. As such, given the size and shape of the analysis window used, strong spectral overlap exists between the first harmonic of the C4 guitar source and the first harmonic of the D4 piano source. Similarly, strong spectral overlap exists between the second harmonic of the C4 guitar source and the first harmonic of the B4 piano source. By design, both mixtures provided good test cases in which a strong violation of the NMF mixture model assumption exists. Source separation was performed using either NMF or CMF to decompose the spectrograms of each mixture. The number of extracted components was set to K = P = 2, one for each source, and the sparsity parameters were set to $\lambda = 0.01$ and g = 1. The source to distortion ratio (SDR), source to interference ratio (SIR), and source to artifact ratio (SAR) performance measures [16] were used as a means of quantifying the overall separation performance.

4.1. Experiment 1

The first experiment examined the performance of the CMFbased source separation procedure for varying degrees of the phase evolution weight, σ . The value of the phase evolution weight was set to be a function of $[W]_{n,p}$ and $[H]_{p,m}$ as follows: $\sigma(n,p,m)=\hat{\sigma}[W]_{n,p}[H]_{p,m}$ where $\hat{\sigma}$ varied within the range of: $\hat{\sigma} \in \{0, 0.001, 0.01, 0.1, 1\}$. Although defining σ in this way is not theoretically justified as it introduces a dependence on $[W]_{n,p}$, and $[H]_{p,m}$, which was not accounted for during the optimization of the phase evolution cost function, this definition of σ allowed for a normalization of the terms found within the phase parameter update (providing a more intuitive weighting of the phase evolution constraint's contribution to the phase update), and was observed to yield superior results in practice. For each level of the modified phase evolution weight, 40 source separation tasks were performed (2 note pairings \times 20 random initializations). The factorizations were set to terminate after 100 iterations of the algorithm. The results of the first experiment are represented in the box-plots of Figure 1. Each box plot is made up of a central line indicating the median of the data, upper and lower box edges indicating the 25^{th} and 75^{th} percentiles, vertical whiskers extending to the minimum and maximum extrema, and red stars indicating the outliers. A data point is considered to be an outlier if it lies beyond 1.5 times the interquartile range, either below or above the first and third quartiles, respectively. Each box-plot summarizes the pooled performance measures obtained from each source, for both mixtures over all 20 trials. The results indicate an increase in median SDR, SIR, and SAR performance measures and

Algorithm 1 CMF with Phase Evolution Constraint Input: $\mathbb{X} \in \mathbb{C}^{N \times M}$ and $P \in \mathbb{N}_{>0}$ and $f_{0_p}(p = 1...P)$ **Output:** W, H, Φ s.t
$$\begin{split} & [\mathbb{X}]_{n,m} \approx \sum_{p=1}^{P} [W]_{n,p} [H]_{p,m} \exp(i[\Phi]_{n,p,m}) \\ & W \in \mathbb{R}_{\geq 0}^{N \times P}, H \in \mathbb{R}_{\geq 0}^{P \times M} \text{ and } \Phi \in \mathbb{R}^{N \times P \times M} \end{split}$$
Initialize W, H, Φ , such that $W \in \mathbb{R}_{>0}^{N \times P}$, $H \in \mathbb{R}_{>0}^{P \times M}$ and $\Phi = \frac{X}{|X|}$ while stopping criteria not met do Compute B
$$\begin{split} [B]_{n,p,m} &= \frac{[W]_{n,p}[H]_{p,m}}{\sum\limits_{p} [W]_{n,p}[H]_{p,m}} \\ \textbf{Compute } \bar{\mathbb{X}} \end{split}$$
 $[\bar{\mathbf{X}}]_{n,n,m} = [W]_{n,p}[H]_{p,m} \exp(i[\Phi]_{n,p,m})$ $+[B]_{n,p,m}([X]_{n,m}-[\widehat{X}]_{n,m})$ Compute \bar{H} $\left[\bar{H}\right]_{n,m} = H_{p,m}$ Compute Φ $[\Phi]_{n,p,m} = Arg \left(\frac{[\bar{\mathbf{X}}]_{n,p,m}}{[B]_{n,p,m}} [W]_{n,p} [H]_{p,m} \right)$ $+\sigma \sum_{r} \mathbb{1}_{\mathcal{N}_{p,r}} \left(\exp(i[\Phi]_{n,p,m-1}) \exp(i2\pi f_{0_p} r LT) \right)$ $+\exp(i[\Phi]_{n,p,m+1})\exp(-i2\pi f_{0_p}rLT))$ Compute W $[W]_{n,p} = \frac{\sum\limits_m [H]_{p,m} \Re[(\frac{\widehat{\mathbb{X}}_{n,p,m}}{[E]_{n,p,m}}) \exp(-i[\Phi]_{n,p,m})]}{(\sum\limits_m [H]_{p,m}^2)(\frac{1}{[E]_{n,p,m}})}$ Project W onto non-negative orthant Normalize $[W]_{n,p} = \frac{[W]_{n,p}}{\sum [W]_{n,p}}$ Compute *H*
$$\begin{split} [H]_{p,m} &= \frac{\sum\limits_{n}^{[W]_{n,p}} \Re[(\frac{\bar{\mathbf{x}}_{n,p,m}}{|\overline{B}|_{n,p,m}}) \exp(-i[\Phi]_{n,p,m})]}{(\sum\limits_{n}^{[W]_{n,p}^2})(\frac{1}{|\overline{B}|_{n,p,m}}) + \lambda g(|\overline{H}]_{p,m})^{g-2}} \end{split}$$
Project H onto non-negative orthant iter = iter + 1end while

a decrease in spread of the results for higher values of $\hat{\sigma}$, suggesting a positive trend in separation performance given a stronger presence of the phase evolution constraint. Specifically, an increase in median SDR, SIR, and SAR of 3.7dB, 9.2dB, and 2.6dB, respectively, was observed when using a phase evolution weight of $\hat{\sigma} = 0.1$ compared to having the phase evolution weight set to $\hat{\sigma} = 0$. Based on these results, the value of $\hat{\sigma}$ was set as 0.1 for the second experiment.

4.2. Experiment 2

A second experiment was conducted to compare the NMF-based and the CMF-based separation procedures. Three variants of CMF were considered: 1) CMF(EP) (without the phase evolution constraint), where EP stands for "estimated phase"; 2) CMF(MP) (with the phase evolution constraint), where MP stands for "modelled phase"; and 3) CMF(OP) (in which the phase is updated according to the true phase of the unmixed sources), where OP stands for "oracle phase".



Fig. 1: SDR(left), SIR(center), SAR(right) measures obtained for the CMF-based separation procedures as a function of $\hat{\sigma}$.

The second experiment was similar in nature to the first experiment, however, instead of varying the phase evolution constraint weight, the matrix factorization techniques varied. The results of the second experiment are summarized in Figure 2. The box-plots indicate that the CMF(MP)-based separation procedure produced an increase in median SDR, SIR, and SAR of 2.8dB, 10.5dB, and 0.63dB, respectively, over the NMF-based separation procedure. The CMF(MP)-based separation procedure in median SDR, SIR, and SAR of 2.8dB, 10.5dB, and 0.63dB, respectively, over the NMF-based separation procedure. The CMF(MP)-based separation procedure in accordance with the results obtained in the first experiment. The spread of the results for the CMF(MP)-based separation procedures was also indicative of a better overall separation performance compared to the NMF-based and the CMF(EP)-based separation procedures. The CMF(MP)-based separation procedures.

Further analysis was conducted to investigate how each factorization method was able to resolve the spectral magnitudes of the source estimates from the mixtures. Magnitude profiles were created by multiplying the spectral templates, which resulted in the highest SDR performance measure for each matrix factorization-based separation procedures, by their corresponding activation weights. Figure 3, compares the spectral magnitude of the NMF-based, CMF(EP)based, CMF(OP)-based, and CMF(MP)-based source estimates for the (B4) piano source across the 24th frequency bin (corresponding to the overlapping first harmonic of the (B4) piano source and second harmonic of the (C4) guitar source) against the true magnitude spectrum of the unmixed piano source, and the mixture of the sources. Also included is the magnitude spectrum of the true unmixed guitar source and the mixture that would occur if the first harmonic of both sources were in phase (for which the NMF mixture model assumption would hold true). For clarity, only the magnitude profiles corresponding to the last second of the mixture are depicted in Figure 3. A noticeable improvement is observed in the spectral magnitude of the (B4) piano source estimate using the CMF(MP)-based separation procedures over the NMF-based and the CMF(EP)-based separation procedures both in terms of proximity to the spectral magnitude of the true piano source and in terms of a reduction of the oscillating amplitude effect due to phase discrepancies present over time.

5. CONCLUDING REMARKS

In this paper, we examined CMF as a tool for separating overlapping partials in mixtures of harmonic musical sources. A novel phase evolution constraint was developed and demonstrated to of-



Fig. 2: SDR(left), SIR(center), SAR(right) measures obtained for the separation procedures as a function of the factorization method employed.



Fig. 3: Magnitude profile analysis of the (B4) piano source estimate across the 24^{th} frequency bin (corresponding to the overlapping first harmonic of the (B4) piano source and second harmonic of the (C4) guitar source).

fer an improvement in separation performance when incorporated into the CMF framework. Two experiments were conducted, investigating the behaviour of a CMF-based separation procedure, both with and without the proposed phase evolution constraint. It was observed that by applying CMF(MP) to mixtures containing regions of strong harmonic overlap between sources, the resolved magnitude of the extracted source more closely matched the true magnitude of the unmixed source, outperforming both the NMF-based and CMF(EP)based separation procedures.

Future research will focus on the improvement of the proposed phase constraint to include a more theoretically justified amplitudebased weighting, and a convergence guaranteed update algorithm. Additionally, a relaxation of the assumptions upon which the phase evolution constraint was developed will also be subject to further investigation.

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