CHANNEL-AWARE DISTRIBUTED DYNAMIC SPECTRUM ACCESS VIA LEARNING-BASED HETEROGENEOUS MULTI-CHANNEL AUCTION

Marjan Zandi, Min Dong, and Ali Grami

Department of Electrical, Computer and Software Engineering University of Ontario Institute of Technology, Ontario, Canada Email: {marjan.zandi, min.dong, ali.grami}@uoit.ca

ABSTRACT

We consider the design of a distributed online learning and access mechanism for dynamic spectrum access, where channel availability statistics are unknown to each secondary user (SU). Unlike existing distributed access policies, we explore the instantaneous channel gain of SUs' channels for multi-user multi-channel diversity gain. We consider an auction-based approach. For the primary channels with heterogeneous statistics, we apply the unit demand auction [1] to determine each SU's selection of a primary channel based on its instantaneous rate over each channel. We further propose a learning based unit demand (LBUD) auction, where each SU only bids for the *M*-best channels estimated by itself through distributed learning. The new mechanism not only reduces communication overhead, but also improves the throughput performance when the primary channels have dissimilar availability statistics. In addition, we show that the LBUD auction preserves the strong property of unit demand auction, *i.e.* it is dominant strategy incentive compatible. To improve the convergence speed of the iterative procedure of channel allocation in the auction, we also propose an adaptive price increment algorithm. Simulations show the effectiveness of our proposed auction mechanism in throughput gain by exploring instantaneous channel fade.

1. INTRODUCTION

One of the main challenges in cognitive radio networks is to design dynamic spectrum access mehanisms for efficient utilization of the spectrum. A hierarchical cognitive radio network consists of primary users who are licensed to use the spectrum and the SUs who opportunistically use the idle channels that are not occupied by the primary users. The channel availability statistics of the primary network are typically unknown to the SUs. Through limited spectrum sensing, the SUs search for idle channels and make decisions based on their observation histories for channel access. Thus, the challenges in designing a distributed policy for spectrum access among SUs involve not only online learning of the primary channel statistics using local sensing observations, but also the distributed mechanism to resolve collisions among SUs.

Consider a cognitive radio network with N independent channels and M SUs, where $N \ge M$. Several decentralized learning and access policies have been recently developed for distributed dynamic spectrum access [2]–[7] by formulating the problem as decentralized multi-arm bandit (MAB) problem [8]. These policies use different mechanisms to resolve collision among SUs for their access to the M most available primary channels. In addition, these existing access policies only rely on the estimated mean channel availabilities through the learning process to determine access division without exploring the instantaneous channel fade conditions.

Auction design has been recently considered as a potential spectrum access mechanism in cognitive radio networks. Different from the MAB formulation, an auction-based access approach was proposed [9] where a repeated auction, based on the second-price auction [10], is considered by exploring instantaneous channel fading gain. A repeated auction game is considered as a game with incomplete information. There are several studies that have been conducted in area of the repeated auction game where auction mechanism is used as a tool for

assigning the objects among multiple bidders [11] and [12]. The authors of [13] further considered the case where heterogeneous items are auctioned to multiple bidders with limited budgets. For spectrum access without the presence of primary users, [14] proposed a distributed auction method among users for channel access. It is a modification of the Bertsekas auction algorithm [15] for distributed channel assignment.

In this paper, we consider an auction-based approach and explore the instantaneous channel fade for multi-user multi-channel diversity gain. For the heterogeneous primary channels, we apply a unit demand auction (also known as Demange-Gale-Sotomayor (DGS) auction) to determine each SU selection of a primary channel based on SU's instantaneous rate over each channel. We further propose a learning based unit demand (LBUD) auction, where each SU only bids for the M-best channels estimated by each SU through distributed learning. The new mechanism not only reduces communication overhead, but also improves the throughput performance when the primary channels have dissimilar availability statistics. In addition, we show that the LBUD auction preserves the strong property of the DGS auction, *i.e.*, it is dominant strategy incentive compatible (DSIC) (its definition is given in Section 4.2). To improve the convergence speed of the iterative procedure for channel allocation in the auction, we also propose an adaptive algorithm to adjust the price increment in each iteration. Simulations show the effectiveness of our proposed auction mechanism in throughput gain by exploring instantaneous channel fade.

Note that the problem considered in [9] is different from ours in the sense that each SU is allowed to win multiple channels depending on the auction outcome, instead of each SU selecting one channel to access. In addition, the second-price auction is a suboptimal auction for bidding heterogenous multiple channels. Our proposed auction mechanism is also different from Bertsekas auction [15] which does not consider bidders' incentives and thus is not DSIC. In addition, it cannot handle the scenario with the presence of primary users.

To the best of our knowledge, we are first to apply unit demand auction designed for bidding heterogeneous objects to the distributed dynamic access problem. In addition, for our proposed LBUD auction, we combine distributed learning outcome into the auction mechanism to explore both channel availability statistics and instantaneous fading gain among SUs, which is not considered in either existing decentralized MAB policies or auction-based access mechanisms.

2. NETWORK MODEL

We consider a spectrum consisting of N orthogonal channels that are licensed to a slotted primary network. A secondary network with M ($M \leq N$) users independently search for the instantaneous idle channels among these N channels. Denote $X_i(n)$ as the availability state of the *i*th primary channel at slot n, with $X_i(n) = 1$ when channel i is available, and 0 otherwise. We assume the channel availability $X_i(n)$ evolves as an i.i.d. Bernoulli random process over time, with $X_i(n) \sim \text{Bernoulli}(\theta_i)$, where $\theta_i = E[X_i(n)], \forall n$, for $i = 1, \dots, N$. We assume θ_i 's are all distinct and are unknown to the SUs. Let $\boldsymbol{\theta} \triangleq [\theta_1, \dots, \theta_N]^T$.

At the beginning of each slot n, each SU selects a channel to sense, and accesses the channel if it is available. Perfect channel sensing is assumed. Each SU can use the sensing outcome and history to learn the availability statistics over time. Let $T_i^j(n)$ denote the number of times that the SU *j* senses channel *i* up to time slot *n*. For SU *j* selecting channel *i* to sense, it records the value of $X_i(n)$ as $X_i^j(T_i^j(n))$. The sensing observation history of channel *i* up to time slot *n* for SU *j* is denoted by $\mathbf{X}_i^j(n) \stackrel{\Delta}{=} [X_i^j(1), \cdots, X_i^j(T_i^j(n))]^T$. SU *j* estimates θ_i of channel *i* at time *n* using the sample mean of the observations from $\mathbf{X}_i^j(n)$ as

$$\hat{\theta}_{i}^{j}(T_{i}^{j}(n)) = \frac{1}{T_{i}^{j}(n)} \sum_{k=1}^{T_{i}^{j}(n)} X_{i}^{j}(k).$$
(1)

Let $h_i^j(n)$ be the channel gain for SU *j* communicating to its (secondary) destination using channel *i* at time slot *n*. The corresponding instantaneous achievable rate of SU *j* is denoted as $R_i^j(n)$, where $R_i^j(n) = \log\left(1 + P_j |h_i^j(n)|^2 / \sigma^2\right)$ with P_j and σ^2 are the transmit power at SU *j* and receiver noise variance, respectively. We assume perfect knowledge of $h_i^j(n)$ at each SU¹, and $R_i^j(n)$ is known at each SU *j*. The expected throughput obtained under a given access mechanism is obtained as

$$\frac{1}{n}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k\in\mathcal{I}_{i}^{j}(n)}\theta_{i}E\left[R_{i}^{j}(k)\right]$$
(2)

where $\mathcal{I}_i^j(n)$ denotes the set of time slots up to the current slot *n* that SU *j* has been the sole user of the channel *i*. Our problem is to design a distributed online learning and access mechanism among SUs, utilizing the instantaneous secondary link gain, to maximizes SUs' throughput, with minimum exchange of information among SUs.

3. DYNAMIC ACCESS VIA MULTI-CHANNEL AUCTION

One major challenge faced in distributed access among SUs to the primary network is the possibility of collisions among SUs and how to resolve them when they happen. We consider an auction-based access mechanism, in which each SU performs online learning of the primary channels distributively while selection of an access channel is handled by a coordinator. Such an auction mechanism will result in a collision free access.

Let S denote the set of SUs and C the set of the primary channels. Consider SUs as the bidders and the primary channels as the objects of the auction. We first consider SUs bid for all the channels in C. In Section 4, we modify our auction mechanism to consider bidding of a subset of primary channels. At the beginning of time slot n, SU j sends a confidential bidding vector of all the channels to the coordinator, denoted as $\mathbf{m}^{j}(n) \stackrel{\Delta}{=} [m_{1}^{j}(n), \cdots, m_{N}^{j}(n)]^{T}$. If SU j decides not to participate in bidding of channel *i*, then $m_i^j(n) = 0$. We also define $\mathbf{m}^{-j}(n)$ as the bidding vectors of SU j's opponents, *i.e.*, $\mathbf{m}^{-j}(n) =$ $\{\mathbf{m}^k(n)|k \in S \setminus j\}$. Note that since each channel *i* has distinct mean availability statistic θ_i , they are considered as heterogeneous types that SUs bid for. Based on SUs' bids, the coordinator (or auctioneer as the trusted third party) will assign a channel to each SU. We consider a unit demand auction (also called DGS auction [1]), which is first proposed in economics for bidding heterogenous objects. It is a generalization of the second-price auction [16] (bidding for one object) to handle bidding of multiple heterogenous objects with unit demand. The DGS auction preserves some nice properties of the second-price auction. It is a weakly dominant strategy² which leads to a dominant strategy equilibrium in which the payoff of each bidder is maximized. Note that the dominant strategy equilibrium is a Nash equilibrium, but not vice versa. In addition, the DGS auction reaches the minimum price equilibrium [1].

Under the DGS allocation strategy, based on the bids from SUs, the coordinator allocates the channels to SUs. Denote $A_i(n)$ the allocation

¹Note that channel *i* indicates the frequency channel SU *j* occupies, while $h_i^j(n)$ is the channel gain over the link between the secondary transceiver, which can be measured by SU *j*.

²In game theory, a player's strategy is called a weakly dominant strategy if it is at least as good as any other strategy for that player irrespective of what other players' strategies are.

of channel i among SUs at time slot n and $\mathbf{A}^{j}(n)$ the allocation of channels for SU j at time slot n, respectively. They are given by $\mathbf{A}_i(n) \stackrel{\Delta}{=} \{A_i^1(n), \cdots, A_i^M(n) | A_i^j(n) \in \{0,1\} \land \sum_{j=1}^M A_i^j(n) \le 1\},\$ and $\mathbf{A}^{j}(n) \stackrel{\Delta}{=} \{A_{1}^{j}(n), \cdots, A_{N}^{j}(n) | A_{i}^{j}(n) \in \{0,1\} \land \sum_{i=1}^{N} A_{i}^{j}(n) \leq \{0,1\} \land \sum_{i$ 1}, where $A_i^j(n)$ is the indicator of channel *i* allocation to SU *j* at time slot n. At most one channel will be assigned to each SU, and at most one SU can be assigned to each channel. In addition, if an SU does not bid for a channel, it will not be assigned to that channel, *i.e.*, $A_i^j(n)|(m_i^j(n)=0)=0$. A reservation price $P_{\min,i}$ is given to each channel *i*, indicating the minimum price the coordinator accepts for a specific channel. Let $\mathbf{P}_{\min} \stackrel{\Delta}{=} [P_{\min,1}, \cdots, P_{\min,N}]^T$. The auction mechanism determines the channel assignment for each user through an iterative procedure described below: Let $P_i^l(n)$ denote the price for channel i at time slot n at iteration l. Let $\mathbf{P}^{l}(n) \stackrel{\Delta}{=}$ $[P_1^l(n), \cdots, P_N^l(n)]^T$. We summarize the unit demand (UD) auction mechanism for access decision as follows:

- The coordinator initializes the price vector P⁰(n) to the reservation price vector P_{min};
- SU j ∈ S observes its current valuation of each channel, *i.e.*, R^j_i(n), ∀i. Since bidding truthfully is a weakly dominant strategy in the UD auction, we set m^j_i(n) = R^j_i(n); SU j sends m^j(n) to the coordinator;
- The coordinator obtains the *demand set* for each SU j, denoted as D^j(P^l(n)). It is defined as the set of channels that maximizes SU j's current payoff

$$\mathcal{D}^{j}(\mathbf{P}^{l}(n)) = \arg\max_{i}(R_{i}^{j}(n) - P_{i}^{l}(n)).$$
(3)

- 4) Let D(P^l(n)) ≜ [D¹(P^l(n)), ..., D^M(P^l(n))]^T. The coordinator follows an iterative procedure to check whether there is any *overdemanded sets* (defined below) among the demand sets:
 i) If there is no overdemanded set of channels: Set A^j_i(n) = 1, where i ∈ D^j(P^l(n)) is picked randomly if |D^j(P^l(n))| > 1,³ and A^j_{i-}(n) = 0, for i⁻ ∈ C \{i}. The allocation process is completed.
 - ii) If there are overdemanded sets of channels:
 - a) The coordinator collects all the overdemanded sets into a set \mathcal{O} . Let \mathcal{S}^o be the set of SUs whose $\mathcal{D}^j(\mathbf{P}^l(n))$ is an overdemanded set, then

$$\mathcal{O} = \{\mathcal{D}^{j}(\mathbf{P}^{l}(n)) : j \in \mathcal{S}^{o}\}.$$
(4)

b) The coordinator finds the minimal overdemanded set $\mathcal{D}^{\min}(\mathbf{P}^l(n))$ (defined below) from \mathcal{O} . For channel $i \in \mathcal{D}^{\min}(\mathbf{P}^l(n))$, update the price vector $P_i^l(n)$: $P_i^{l+1}(n) = P_i^l(n) + \Delta P_{n,l}$, where $\Delta P_{n,l}$ is the price increment at iteration l and time slot n. Return to Step 3.

The overdemanded set and minimal overdemanded set are defined as follows [1]:

Overdemanded set: Define the demanders of $\mathcal{D}^{j}(\mathbf{P}^{l}(n))$ as $\mathcal{B}(\mathcal{D}^{j}(\mathbf{P}^{l}(n))) = \{k : \mathcal{D}^{j}(\mathbf{P}^{l}(n)) \cap \mathcal{D}^{k}(\mathbf{P}^{l}(n)) \neq \emptyset, \forall k \in \mathcal{S}\}$. Define the exclusive demanders of $\mathcal{D}^{j}(\mathbf{P}^{l}(n))$ as

$$\mathcal{B}^{E}(\mathcal{D}^{j}(\mathbf{P}^{l}(n))) = \{k : \mathcal{D}^{k}(\mathbf{P}^{l}(n)) \subseteq \mathcal{D}^{j}(\mathbf{P}^{l}(n)), \forall k \in \mathcal{S}\}$$
(5)

Then, $\mathcal{D}^j(\mathbf{P}^l(n))$ is overdemanded (or weakly overdemanded) at price $\mathbf{P}^l(n)$ if

$$\mathcal{D}^{j}(\mathbf{P}^{l}(n)) \subset \mathcal{C} \text{ and } |\mathcal{B}^{E}(\mathcal{D}^{j}(\mathbf{P}^{l}(n))| > (\text{or } \geq)|\mathcal{D}^{j}(\mathbf{P}^{l}(n))|.$$
 (6)

Minimal Overdemanded set: For each set \mathcal{D} in \mathcal{O} , for any strict subset of \mathcal{D} , *i.e.*, $\mathcal{D}' \subset \mathcal{D}$, \mathcal{D}' is not an overdemanded set, then \mathcal{D} is a minimal overdemanded set.

It is shown that the above auction mechanism leads to a minimal price equilibrium [1]. That is, let $\mathbf{P}^*(n)$ be the price obtained at the end of the auction, and $\mathbf{q}(n)$ be any other competitive price vector at

 $^{|| \}cdot |$ denotes the cardinality of a set.

time slot n. Then, $\mathbf{P}^*(n)$ would satisfy $\mathbf{P}^*(n) \leq \mathbf{q}(n)$. In addition, since the auction is shown to be weakly dominant strategy, each SU obtains its maximal payoff regardless of other bidders' strategies. In the above auction mechanism, each SU uses its instantaneous rate over a channel to bid, thus the channel assignment from the auction depends on the instantaneous channel condition for each SU over the primary channels. Thus, the channel assignment is opportunistic, and the obtained throughput at the secondary network gains from such opportunistic allocation, in addition to be collision free.

4. MULTI-CHANNEL AUCTION VIA DISTRIBUTED LEARNING

In the UD auction described above, each SU bids for all N channels. There are two drawbacks for this approach: First, there are a total MN bids submitted to the coordinator which incurs large overhead. Second, the payoff used in determining the channel allocation only reflects each SU's instantaneous rate over the channels, but does not take into account the different mean channel availability among these heterogeneous channels. Thus, it could result in more throughput loss. To overcome these drawbacks, we propose an adaptive auction mechanism, named learning-based unit demand (LBUD) auction. In this auction, SU i will adaptively choose the best M channels to bid.

To determine the best M channels, recall that each SU learns the unknown mean availability θ_i of channel *i* from its sensing history based on (1). The online learning algorithm UCB1 [17] is a samplemean based index policy for a single user learning and access of Nchannels, which is shown to be order-optimal in terms of the learning efficiency. Existing decentralized policies [2]-[4] extend the UCB1 algorithm to the decentralized scenario for distributed learning at each SU. In the UCB1 algorithm, each SU j ranks channel i based on an index, defined as

$$I_i^j(n) \stackrel{\Delta}{=} \hat{\theta}_i^j(T_i^j(n)) + \sqrt{2\log n/T_i^j(n)}.$$
(7)

Each SU j computes its index vector $\mathbf{I}^{j}(n) \stackrel{\Delta}{=} [I_{1}^{j}(n), \cdots, I_{N}^{j}(n)]^{T}$ based on its own sensing observation history.

We first define the *M*-best channels as those channels whose θ_i 's are among the M highest ones. We also define the estimated M-best channels as those channels whose indexes in (7) are among the top Mranked. For SU j, let $C_M^j(n)$ denote the set of indexes of the estimated M-best channels for SU j at time slot n

$$\mathcal{C}_{M}^{j}(n) = \left\{ i : I_{i}^{j}(n) \in \left\{ I_{(1)}^{j}(n), \cdots, I_{(M)}^{j}(n) \right\} \right\}$$
(8)

where $\{I_{(i)}^{j}(\cdot)\}$ is the ordered statistics of $\{I_{i}^{j}(\cdot)\}$ with $I_{(1)}^{j}(\cdot) >$ $\cdots > I_{(N)}^{j}(\cdot)$. At each time slot n, SU j updates its estimated *M*-bast channel set $C_M^j(n)$, and form the bidding vector for those channels $\mathbf{m}_M^j(n) = [m_{k_1}^j(n), \cdots, m_{k_M}^j(n)]^T$, where $k_i \in C_M^j(n)$. The coordinator performs an auction-based allocation using these bidding vectors from SUs. The demand set $\mathcal{D}_{M}^{j}(\mathbf{P}^{l}(n))$ for SU j in this case is given by

$$\mathcal{D}^{j}_{M}(\mathbf{P}^{l}(n)) = \underset{i \in \mathcal{C}^{j}_{M}(n)}{\arg\max(m^{j}_{i}(n) - P^{l}_{i}(n))},$$
(9)

which is used in the iterative procedure of the LBUD auction to determine channel allocation to M SUs. The LBUD auction mechanism is described in Algorithm 1. In Section 4.2, we show that using the true valuation, *i.e.*, $m_i^j(n) = R_i^j(n)$, will lead to the maximum payoffs among SUs. Therefore, in the following, we assume $m_i^j(n) = R_i^j(n)$.

Note that, in the LBUD auction, each SU only submits M bids along with the channel index set C_M^j . The total overhead is M^2 as compared with MN in the UD auction in Section 3. For $N \gg M$, the reduction in communication overhead is significant. Performancewise, each SU only bids among its estimated M-best channels, and at the same time, the channel allocation is based on instantaneous fading gain. Thus, the auction mechanism not only ensures good selection of channels in mean sense, but also can enjoy the gain from opportunistic channel selection. Note that existing distributed learning and access

Algorithm 1 : Learning-Based Unit Demand (LBUD) Auction

- 1) Init: Set the price vector $\mathbf{P}^0(n) \leftarrow \mathbf{P}^{\min}$
- 2) SU j updates $\hat{\theta}_i^j(n)$ using (1), $\forall i$, and obtain $\mathcal{C}_M^j(n)$ using (8).
- 3) SU j observes its current valuation (rate) $R_i^j(n)$, $\forall i$ and sends a confidential bidding vector $\mathbf{m}_{M}^{j}(n)$.
- 4) The coordinator obtains the demand set $\mathcal{D}_{M}^{j}(\mathbf{P}^{l}(n))$ for SU j using (9)
- 5) The coordinator checks whether there is any overdemanded sets among the demand sets $\{\mathcal{D}_{M}^{j}(\mathbf{P}^{l}(n))\}$
 - a) The coordinator obtains the exclusive $\mathcal{B}^{E}(\mathcal{D}^{j}_{M}(\mathbf{P}(n)))$ of $\mathcal{D}^{j}_{M}(\mathbf{P}(n))$ as in (5). demanders
 - b) The coordinator checks whether $\mathcal{D}_{M}^{j}(\mathbf{P}^{l}(n)), \forall j$, is a overdemanded set or not as in (6) If there is no overdemanded set, for each j, pick channel irandomly from $\mathcal{D}^{j}(\mathbf{P}(n))$ as the winning channel for j. The allocation process is completed. If there is overdemanded sets,
 - The coordinator forms Set \mathcal{O} as in (4).
 - It finds the minimal overdemanded set $\mathcal{D}_{M}^{\min}(\mathbf{P}^{l}(n))$ from \mathcal{O} , and updates the price vector $\mathbf{P}^{l}(n)$, for $i \in$ $\mathcal{D}_{M}^{\min}(\mathbf{P}^{l}(n))$, as $P_{i}^{l+1}(n) = P_{i}^{l}(n) + \Delta P_{n,l}$. • Set iteration $l \leftarrow l+1$;return to Step 4.

policies [2]-[4] only ensure good channel selection in the mean sense without considering instantaneous channel gain of SUs' communication links. For the LBUD auction, the winning channel will be selected only among those highly available channels. When the mean primary channel availability values in θ are relatively spread, the LBUD auction outperforms the UD auction in Section 3. This is because bidding only among the M-best channels prevents SUs to access the channels which are less likely to be available even though the SU's instantaneous rates over these channels are high. However, when values in θ are similar, the UD auction may outperform the LBUD auction, due to the "multichannel diversity" gain from the opportunistic selection of M among N channels. To cover a broad range of the distribution of primary channel statistics, in practice, we should consider both two mechanism to adapt to different types of mean channel availability distribution.

4.1. Adaptive Price Increment $\Delta P_{n,l}$

The iterative procedure in both UD and LBUD auctions requires the price update with the price increment $\Delta P_{n,l}$. The setting of $\Delta P_{n,l}$ is important as it directly affects the convergence behavior of the auction. Note that, the DGS auction [1] is originally proposed for integer valuations and prices. Due to this, the price increment in the iteration procedure is fixed to $\Delta P_{n,l} = 1$, and the convergence is shown with this unit increment. In our case, the valuations (instantaneous rates) are real numbers. To accommodate this, we can set $\Delta P_{n,l}$ to be the minimum difference of instantaneous rates of all SUs' channels, i.e.,

$$\Delta P_{n,l} = \min_{j \in \mathcal{S}} \min_{i \neq m} \left| R_i^j(n) - R_m^j(n) \right|.$$
(10)

Note that, in this case, $\Delta P_{n,l}$ is fixed for each auction procedure, but varies from slot to slot.

Using the price increment suggested above may lead to slow convergence. To improve the convergence speed, we propose an algorithm which sets $\Delta P_{n,l}$ adaptively in each iteration. Specifically, at each iteration *l*:

- 1) For SU $j \in S$: Let $\rho_i^j(n,l) = R_i^j(n) P_i^l(n)$ be the payoff of SU j for channel i at iteration l. Find the two channels i and i' with two highest (distinct) payoffs, and compute their payoff difference $\omega^j(n,l) = |\rho_i^j(n,l) - \rho_{i'}^j(n,l)|.$
- Set the price increment $\Delta P_{n,l}$ to be the minimum $\omega^j(n,l)$ among 2) all the SUs:

$$\Delta P_{n,l} = \min_{j \in \mathcal{S}} \{ \omega^j(n,l) \}.$$
(11)

It can be seen that, the price increment is adaptively set based on the current payoffs among the SUs. Simulations show that the proposed adaptive algorithm for price increment substantially improves the convergency speed of the DGS and the LBUD algorithm.

4.2. Property of the LBUD Auction

An auction mechanism is said to be *incentive compatible* (IC) if all bidders will receive the maximum payoffs when their bids reflect their valuation truthfully. Furthermore, a strategy is called DSIC, if each bidder achieves its maximum payoff by bidding truthfully irrespective of the other bidders bid truthfully or not. Bidding truthfully simplifies decision making of the SUs in an auction. It is shown that bidding truthfully is a dominant strategy in the DGS auction [1], therefore the UD auction described in Section 3 is a DSIC mechanism. Here, we show that the proposed LBUD auction is also DSIC.

Proposition 1: The proposed LBUD auction is DSIC.

5. SIMULATION RESULTS

We first show the improvement of convergence speed using the adaptive price increment algorithm for $\Delta P_{n,l}$ proposed in Section 4.1. We set $\boldsymbol{\theta} = [0.3, 0.34, 0.5, 0.6, 0.67, 0.91, 0.2, 0.8, 0.7]$, M = 4 and N = 9. In Fig. 1, using the LBUD auction, we plot the CDFs of the number of iterations under the baseline increment method in (10) and under our proposed price increment algorithm. As we see, adaptively updating $\Delta P_{n,l}$ improves significantly the convergence speed as compared with the baseline increment method.

Next, we compare the performance under the UD auction and LBUD auction. We assume SUs' channel gain $h_i^j(n)$'s are zero-mean Gaussian with variance σ_h^2 and i.i.d. across channels, among SUs and over time slot *n*. We set the average received SNR $P_j E |h_i^j(n)|^2 / \sigma^2 = 8$ dB, $\forall j$.

We first show the benefit of exploiting the instantaneous channel gains of SUs in the channel auction. Fig. 2 shows the average throughput per SU under the proposed LBUD auction for N = 15 and M = 2, 4, 6, 8. We set the mean channel availabilities among channels to be similar, *i.e.*, $\theta = [0.71, 0.715, 0.72, 0.725, 0.73, 0.735, 0.74, 0.745, 0.75, 0.755, 0.76, 0.765, 0.77, 0.78, 0.79]. We see that the average throughput increases as <math>M$ increases. This is because each SU will be assigned one of M channels, and with more SUs, the channel selection can take advantage from the instantaneous channel gain, and thus increases the multi-user diversity gain.

In Fig. 3, we compare the average throughput under the UD auction and the LBUD auction, for different mean channel availability $\boldsymbol{\theta}$ as follows; Case 1: $\boldsymbol{\theta} = [0.3, 0.34, 0.5, 0.6, 0.67, 0.91, 0.2, 0.8, 0.7]$, Case 2: $\boldsymbol{\theta} = [0.1, 0.2, \cdots, 0.9]$, and Case 3: $\boldsymbol{\theta} = [0.71, 0.72, \cdots, 0.79]$. We set N = 9 and M = 4. As we see, there is a trade-off between the gain of selecting channels among higher mean availabilities and the gain of multi-channel diversity, for different $\boldsymbol{\theta}$ distributions. When $\boldsymbol{\theta}$ is relatively spread, the gain of selecting channels with higher mean channel availability outweighs the loss of multi-channel diversity due to only bidding among *M*-best channels, and the LBUD auction outperforms the UD auction. When values in $\boldsymbol{\theta}$ are similar, the gain of choosing among *M* channels with highest mean availabilities diminishes, and the UD auction outperforms the LBUD auction due to the gain of multi-channel diversity.

Finally, we compare the performance of the UD and LBUD auction mechanisms with existing distributed access policies, the ρ^{RAND} [3] and the DLF [4] policies. Case 1 is used for θ distribution. The latter two policies are distributed with no central coordinator, thus there are collisions but with less overhead. In addition, the channel selections of these two policies only rely on mean channel statistics but do not utilize the instantaneous channel gain of SUs. Fig. 4 compares the average throughput among these schemes. As it can be seen, both two auction mechanisms substantially outperform the ρ^{RAND} and the DLF policies.

6. CONCLUSION

In this work, we investigated the design of distributed online learning and access mechanism for dynamic spectrum access. We considered an auction-based approach to avoid collision among SUs, and explored the



Fig. 1. CDF of iteration (M = 4, N = 9, SNR = 8dB)



Fig. 2. Average Throughput vs. time slot (N = 15, SNR = 8dB).



Fig. 3. Average Throughput vs. time slot (M = 4, N = 9, SNR = 8dB)



Fig. 4. Average Throughput vs. time slot (M = 4, N = 9, SNR = 8dB)

instantaneous channel fade of SUs for multi-user multi-channel diversity gain. For the heterogeneous primary channels, we applied the UD auction to determine each SU's channel access selection. Such auction has been shown to be a weakly dominant strategy that maximizes each SU's payoff. We further proposed the LBUD auction, where each SU distributively learns the primary channel mean availability, and only bids for its estimated *M*-best channels. The LBUD auction not only reduces communication overhead of required bidding data over the UD auction, but also incorporates both the mean channel availability statistics of the primary channels and instantaneous channel fade of SUs to improve the throughput performance. Simulations show the effectiveness of our proposed auction mechanisms.

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