OPPORTUNISTIC USER SCHEDULING IN MIMO COGNITIVE RADIO NETWORKS

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ABSTRACT

This paper studies multiuser diversity of uplink MIMO cognitive radio network and proposes a two-stage opportunistic user scheduling scheme. In the first stage, a cognitive beamforming design is proposed to ensure the interference caused by secondary signals is canceled or minimized on the spatial dimensions occupied by primary MIMO system. Then, some secondary users that cause minimal interference leakage at primary system are pre-selected as candidate users. In the second stage, some candidate users that produce maximum sum secondary rate are further selected for uplink scheduling. The proposed scheme enables the secondary link to take advantage of multiuser diversity while ensuring that the interference on primary link is within a certain threshold. Analytical results show that the sum rate of secondary uplink scales as $N_s \log\log K$ for K secondary users and N_s antennas on secondary receiver for very large K.

***Index Terms*—** Opportunistic user scheduling, cognitive radio network, multiuser diversity

1. INTRODUCTION

Cognitive radio (CR) is a promising solution for efficient utilization of radio resources. The main idea of cognitive radio is to allow a class of radio devices, called secondary users (SUs), to opportunistically access certain portions of spectrum, called white spaces, that are not occupied by licensed users [1]. The white space can be associated with a specific frequency carrier, time slot, or spatial direction [2]. In applications where the primary link occupies entire frequency band and is intensively active, an efficient way to improve the spectrum efficiency is to explore the white spaces in spatial domain. Moreover, the performance of the network in such cases can be significantly boosted if a large number of SUs are in presence. Some SUs can be opportunistically selected to obtain the multi-user diversity gain [3]. In general, the selection of SUs is under one of the following two principles: 1. the SUs who generate minimum interference on primary receiver (PR) are selected [4]; 2. the SUs who achieve the highest throughput for the secondary network are selected, while keeping the interference on PR under certain constraints [5–8].

In [4], an opportunistic spatial orthogonalization scheme was proposed for the CR network where there exists multiple point-to-point secondary links. One or several secondary links whose interference channels are most orthogonal to the primary link's channel are selected. In [5], the secondary network is a SIMO multiple access channel (MAC). Combining user selection with power allocation, an optimization problem is formulated to maximize the sum rate of secondary network. In [6], at first each ST adjusts its power so that the interference generated on PR is within a certain interference temperature constraint. Then, the secondary link with the highest signal-to-interference-plus-noise-ratio (SINR) is selected. [6] showed that with sufficiently large power on each ST, multiuser diversity gain can be obtained. [7] investigated the multiuser diversity gain of the cognitive radio network where the secondary networks are multiple access channel, broadcast channel and parallel channels, respectively. In [8], a two-stage user scheduling scheme was proposed for a downlink secondary network. At first, SUs whose channels are nearly orthogonal to the primary link channel are selected so as to minimize the interference on PR. Then, some SUs whose channels are nearly orthogonal to each other are scheduled. In [9], both primary link and secondary link are point-to-point MIMO channels, and the signals of secondary link are opportunistically aligned on the unused spatial dimensions of primary link. This work is extended to multiple secondary links in [10]. In [11], a two-cell uplink network was studied, where an opportunistic interference alignment (OIA) scheme was proposed to achieve optimal degrees of freedom. [12] introduced the OIA scheme for multi-cell uplink networks where each user is equipped with single antenna. The OIA was later extended to multiple-antenna case in [13]. Further, the multiuser diversity gain of OIA schemes in multi-cell uplink network was analyzed in [14] and [15] in terms of degrees of freedom and sum rate, respectively.

In this paper, we investigate the CR network in which the secondary link is a multi-user uplink MIMO network. Each SU has independent message intended to the secondary receiver (SR). The primary system isa point-to-point MIMO, whose capacity is maximized by implementing a water-filling power allocation scheme. As some spatial directions (SDs) are left unused by primary users, the signals from SUs could be aligned on the unused SDs as much as possible. A two-stage opportunistic user scheduling scheme is proposed, which enables the secondary link to take advantage of multiuser diversity while ensuring that the interference of primary link is under a certain threshold. Specifically, we first select some SUs that cause the interference at primary receiver (PR) less than a predetermined threshold. These pre-selected SUs are referred to as candidate users. In the second stage, N_s users are selected from the candidate users according to semi-orthogonal user selection (SUS) algorithm [16, 17] to transmit signals to SR, where N_s denotes the number of antennas on SR. It is shown that with large K, (the total number of SUs) the sum rate of secondary link scales as $N_s \log\log K$, i.e., the multiuser diversity gain can be obtained. Meanwhile, the interference caused on the primary link is constrained below a preset threshold.

The rest of the paper is organized as follows. In Section 2, system model is introduced. In Section 3, a two-stage opportunistic user scheduling scheme is proposed and the capacity scaling law is derived. Simulation results are presented in Section 4. Section 5

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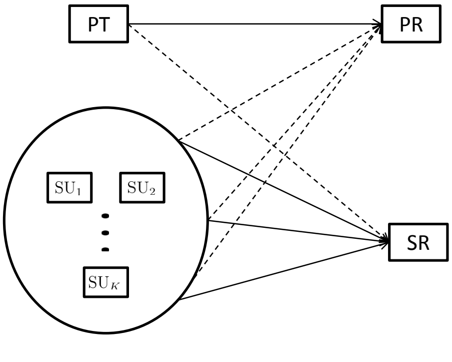

Fig. 1. The cognitive radio network where the primary link is a point-to-point MIMO channel and the secondary link is a multi-user MIMO uplink with K secondary users.

concludes the paper.

2. SYSTEM MODEL

The system model is illustrated in Fig. 1, where the primary link is a point-to-point MIMO channel and the secondary link is a multi-user uplink. In the primary link, the primary transmitter (PT) and PR are equipped with M_p and N_p antennas, respectively. Without secondary link, the received signals on PR can be expressed as

$$\mathbf{y}_p \quad = \quad \mathbf{D}_p\mathbf{H}_{pp}\mathbf{V}_p\mathbf{P}^{\frac{1}{2}}\mathbf{m}_p + \mathbf{D}_p\mathbf{z}_p \tag{1}$$

where $\mathbf{H}_{pp} \in \mathbb{C}^{N_p\times M_p}$ denotes the channel between PT and PR, and $\mathbf{V}_p \in \mathbb{C}^{M_p\times M_p}$ and $\mathbf{D}_p \in \mathbb{C}^{N_p\times N_p}$ are the precoding matrix on PT and post-processing matrix on PR, respectively. $\mathbf{m}_p \in \mathbb{C}^{M_p\times 1}$ denotes the original message vector sent from PT, in which the first s elements are the symbols to be transmitted and the last $M_p - s$ elements are zeros. The value of s equals the number of non-zero elements in the main diagonal of $\mathbf{P}$, where $\mathbf{P} = \mathrm{diag}\begin{pmatrix} p_1 & \cdots & p_{M_p}\end{pmatrix}$ denotes the power allocation matrix on PT and $\mathbf{z}_p$ denotes the noise with variance $\sigma_p^2\mathbf{I}_{N_p}$.

Let $\mathbf{H}_{pp} = \mathbf{U}_{pp}\mathbf{\Lambda}_{pp}\mathbf{V}_{pp}^H$ be a singular value decomposition (SVD) of $\mathbf{H}_{pp}$, where $\mathbf{U}_{pp} \in \mathbb{C}^{N_p\times N_p}$ and $\mathbf{V}_{pp} \in \mathbb{C}^{M_p\times M_p}$ are two unitary matrices, and $\mathbf{\Lambda}_{pp} \in \mathbb{C}^{N_p\times M_p}$ with main diagonal $\begin{pmatrix} \lambda_{p,1} & \cdots & \lambda_{p,\min\{N_p,M_p\}}\end{pmatrix}$ and zeros on its off-diagonal. Using water-filling power allocation, the primary link maximizes capacity by choosing $\mathbf{V}_p = \mathbf{V}_{pp}$ and $\mathbf{D}_p = \mathbf{U}_{pp}^H$, $\mathbf{P} = \mathrm{diag}\begin{pmatrix} p_1 & \cdots & p_{M_p}\end{pmatrix}$ with

$$p_k = (a - \frac{\sigma_p^2}{\lambda_{p,\,k}^2})^+, \;\; k = 1, 2, \cdots M_p \tag{2}$$

where $(A)^+$ denotes $\max\{A,\ 0\}$, and the constant a (water-level) is set to satisfy the power constraint. If $p_k > 0$, it means the kth sub-channel is used by the primary link (known as used SDs). Accordingly, (1) can be written as

$$\mathbf{y}_p = \mathbf{\Lambda}_{pp}\mathbf{P}^{\frac{1}{2}}\mathbf{m}_p + \mathbf{D}_p\mathbf{z}_p \tag{3}$$

where the equivalent channel $\mathbf{\Lambda}_{pp}\mathbf{P}^{\frac{1}{2}} \in \mathbb{C}^{N_p\times M_p}$ is a diagonal matrix whose main diagonal contains s nonzero entries and $n = N_p - s$ zero entries. The value of s is equal to the number of SDs that are used by the primary link and n is the number of unused SDs.

The secondary link is a multi-user uplink with K SUs and one SR, where the SR is equipped with N_s antennas and each SU is equipped with M_s antennas ($M_s < N_p$). In secondary link, the SUs are opportunistically selected to transmit signals by circumventing the SDs being used by the primary link. Let $\mathbf{H}_{pi} \in \mathbb{C}^{N_p\times M_s}$ and $\mathbf{H}_{si} \in \mathbb{C}^{N_s\times M_s}$ denote the channel between SU i and PR and the channel between SU i and SR, respectively. We assume that PT and PR only know $\mathbf{H}_{pp}$, and SU i only knows $\mathbf{H}_{pi}$ and $\mathbf{H}_{si}$. Single data stream is assumed for each secondary user.

3. OPPORTUNISTIC USER SCHEDULING SCHEME

In this section, we propose a two-stage secondary user scheduling scheme. In the first stage, the SUs who may cause interference on PR less than certain threshold are selected, which are referred to as the candidate users. In the second stage, N_s candidate users are further selected according to a SUS-based algorithm to transmit signals.

3.1. Stage 1: Selection of candidate users

We first introduce the design of transmitted signals on each SU. Then, the criteria of candidate users is given.

The received signal on PR can be expressed as

$$\mathbf{y}_p = \mathbf{\Lambda}_{pp}\mathbf{P}^{\frac{1}{2}}\mathbf{m}_p + \underbrace{\mathbf{D}_p\sum_{i=1}^{l}\mathbf{H}_{pi}\mathbf{v}_i\mathbf{P}_i^{\frac{1}{2}}m_i}_{SU\mathrm{interference}} + \mathbf{D}_p\mathbf{z}_p \tag{4}$$

where $\mathbf{v}_i$ and m_i denote the precoding vector and original message of SU i, respectively. $\mathbf{P}_i$ denotes the transmitting power on SU i. Note that the first s rows of $\mathbf{D}_p$ correspond to the sub-channels that are used by the primary link, which are not supposed to be interfered by the signals of SUs. Hence, the power of interference that is caused by SU i on the used sub-channels can be expressed as

$$|\mathbf{I}_i|^2 = |\mathbf{D}_p(s)\mathbf{H}_{pi}\mathbf{v}_i|^2 \cdot \mathbf{P}_i$$

where $\mathbf{D}_p(s) \in \mathbb{C}^{s\times N_p}$ denotes the first s rows of $\mathbf{D}_p$, $|\mathbf{A}|$ denotes the L_2-norm of vector $\mathbf{A}$.

Let $\mathbf{G}_i \in \mathbb{C}^{s\times M_s} = \mathbf{D}_p(s)\mathbf{H}_{pi}$. If $s < M_s$, $\mathbf{G}_i$ is a 'fat' matrix, which means $\mathbf{v}_i$ can be set as the null space of $\mathbf{G}_i$ to ensure that $\mathbf{G}_i\mathbf{v}_i = 0$, i.e., the interference on the "in use" sub-channels are zero. If $s \geq M_s$, $\mathbf{G}_i$ becomes a square or 'tall' matrix, which makes $\mathbf{G}_i\mathbf{v}_i = 0$ impossible almost for sure. In that case, $\mathbf{v}_i$ needs to be designed such that $|\mathbf{G}_i\mathbf{v}_i|^2$ is minimized. Let $\mathbf{G}_i = \mathbf{\Omega}_i\mathbf{\Sigma}_i\mathbf{\Phi}_i^H$ denote the SVD of $\mathbf{G}_i$, where $\mathbf{\Omega}_i \in \mathbb{C}^{s\times s}$ and $\mathbf{\Phi}_i \in \mathbb{C}^{M_s\times M_s}$ consist of orthonormal columns, and $\mathbf{\Sigma}_i \in \mathbb{C}^{s\times M_s}$ has main diagonal $(\lambda_{i,1}, \cdots, \lambda_{i,M_s},\ 0, \cdots, 0)$ and zeros on off-diagonal. In addition, $\lambda_{i,1} \geq \lambda_{i,2} \geq \cdots \geq \lambda_{i,\min\{s,M_s\}}$. Then, $\mathbf{v}_i$ should be set as the last column of $\mathbf{\Phi}_i$, such that

$$|\mathbf{I}_i|^2 = |\mathbf{G}_i\mathbf{v}_i|^2\mathbf{P}_i = \lambda_{i,M_s}^2|\mathbf{v}_i|^2\mathbf{P}_i = \lambda_{i,M_s}^2\mathbf{P}_i \tag{5}$$

$\lambda_{i,M_s}^2\mathbf{P}_i$ would be the smallest interference power that SU i can have, which is referred to as the "interference leakage" of SU i.

As we can see, in the case of $s \geq M_s$, interference on PR is inevitable with the presence of secondary link. To protect the performance of primary link while improving the spectrum efficiency of the network, we let the total interference power on PR be lower than a predetermined threshold, γ_{th},

$$\sum_{i=1}^{l} |\mathbf{I}_i|^2 = \sum_{i=1}^{l} \lambda_{i,M_s}^2 \mathbf{P}_i \leq \gamma_{th} \tag{6}$$

(6) can be guaranteed by setting

$$\lambda_{i,M_s}^2 \leq \frac{\gamma_{th}}{\mathbf{P}_i N_s} \triangleq \gamma'_{th} \tag{7}$$

Hence, SU i would be deemed as a candidate user if (7) can be satisfied.

3.2. Stage 2: Selection of active users

Assume N_c candidate users are selected in stage 1. Then, further N_s of them are selected to transmit signals to SR according to a selection algorithm which will be explained later. The received signals on SR can be expressed as

$$\begin{aligned}\mathbf{y}_r &= \underbrace{\begin{bmatrix} \mathbf{H}_{s1}\mathbf{v}_1 & \cdots & \mathbf{H}_{sl}\mathbf{v}_l \end{bmatrix}}_{\mathcal{H}} \mathbf{P}_i^{\frac{1}{2}} \mathbf{m}_s + \mathbf{I}_p + \mathbf{z}_s \\ &= \begin{bmatrix} \mathfrak{h}_1 & \cdots & \mathfrak{h}_l \end{bmatrix} \mathbf{P}_i^{\frac{1}{2}} \mathbf{m}_s + \mathbf{I}_p + \mathbf{z}_s \end{aligned} \tag{8}$$

where $\mathbf{I}_p$ denotes the interference term from primary link, $\mathbf{z}$ denotes the noise on SR. Since $\mathbf{v}_i$ has already been designed in the first stage, $\mathfrak{h}_i \in \mathbb{C}^{N_s \times 1} = \mathbf{H}_{si}\mathbf{v}_i, (i = 1, 2, \cdots l)$ can be seen as the equivalent channel between SU i and SR.

Next, we explain how to select these l candidate users. As $K >> N_s$, for large K it has $N_c > N_s$ almost surely. Then, N_s candidate users are selected finally, based on a SUS-based algorithm, which is shown in **Algorithm 1**.

Algorithm 1 Semi-orthogonal user selection

Step 1. Initialization, $S = \varnothing, i = 1$.
The candidate SUs are denoted by
$Q_1 = \{k \mid \lambda_{k,M_s}^2 \leq \gamma'_{th}, k = 1, \cdots, N_c\}$,
Step 2. Selecting the first user
1) The first selected SU is determined by
$S(1) = \arg\max_{k \in Q_1} |\mathfrak{h}_k|^2$, where $\mathfrak{h}_k \triangleq \mathbf{H}_{sk}\mathbf{v}_k, \mathfrak{h}_{(1)} = \mathfrak{h}_{S(1)}$
2) Define $\mathbf{g}_{(1)} = \mathfrak{h}_{(1)}$
Step 3. Semiorthogonal user selection.
While $i < N_s,\ i = i + 1$
a) $Q_i = \{\forall k \in Q_{i-1},\ k \neq S(i-1) \mid \frac{\mathbf{g}_{(i-1)}^H \mathfrak{h}_k}{|\mathfrak{h}_k||\mathbf{g}_{(i-1)}|} < \alpha\}$
If $|Q_i| \leq N_s - i + 1$, break While, end
b) each user $k \in Q_i$ calculates $\mathbf{g}_k$, which is the component of $\mathfrak{h}_k$ that orthogonal to span$\{\mathbf{g}_{(1)}, \cdots, \mathbf{g}_{(i-1)}\}$, i.e.,
$\mathbf{g}_{(k)} = \mathfrak{h}_k - \sum_{j=1}^{i-1} \frac{\mathbf{g}_{(j)}^H \mathfrak{h}_k}{|\mathbf{g}_{(j)}|^2} \mathbf{g}_{(j)}$.
c) $S(i) = \arg\max_{k \in Q_i} |\mathbf{g}_{(i)}|^2, \quad \mathfrak{h}_{(i)} = \mathfrak{h}_{S(i)}, \quad \mathbf{g}_{(i)} = \mathbf{g}_{S(i)}$.

The algorithm can be explained as follows. In **step 2**, the user who has the largest equivalent channel gain $|\mathfrak{h}_i|^2$ is selected and the first basis vector $\mathbf{g}_{(1)}$ is set accordingly. In **step 3**, we apply the SUS [16] to select other candidate users. Since **step 3** is a loop, we first explain **step 3**-b). In b), we project equivalent user channels in Q_i to the orthogonal complement of span$\{\mathbf{g}_{(1)}, \cdots, \mathbf{g}_{(i-1)}\}$. Next in c), the user with the largest projected norm is selected as $S(i)$, which determines $\mathfrak{h}_{(i)}$ and $\mathbf{g}_{(i)}$. Now, we explain **step 3**-a). This step guarantees that the equivalent channels in Q_i are already semi-orthogonal to $\mathbf{g}_{(1)}, \cdots, \mathbf{g}_{(i-1)}$, which implies that $\mathfrak{h}_i \approx \mathbf{g}_i$ (α is a small positive constant). This algorithm enables secondary link to take advantages of multiuser diversity and leads to the establishment of **Theorem 1**, which is described as follows,

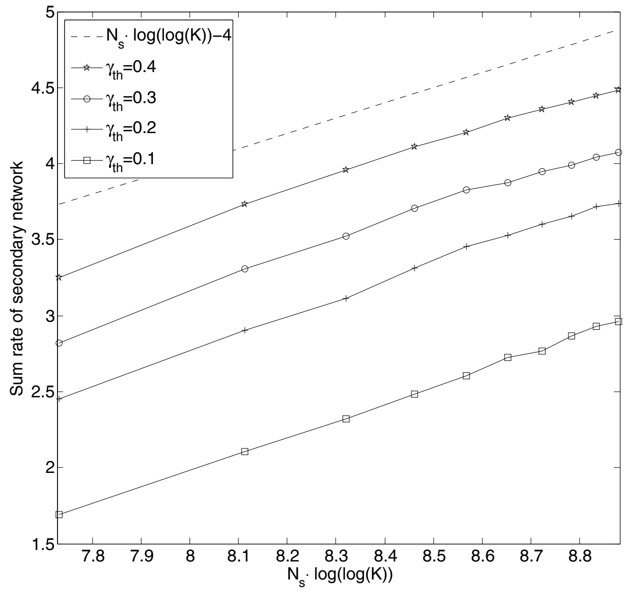

Fig. 2. The sum rate of secondary link as a function of $N_s \log(\log K)$, with different value of γ_{th}. $N_p = N_s = M_p = 4$, $s = M_s = 3$, $\alpha = 0.3$.

Theorem 1 *In the cognitive radio network where the primary link is a point-to-point MIMO channel and the secondary link is a multi-user uplink with K SUs, each SU is equipped with M_s antennas and PT, PR, and SR are equipped with M_p, N_p and N_s antennas, respectively. Based on the proposed two-stage user selection scheme, the sum rate of secondary link scales as $N_s \log\log K$ for K users with $K \to \infty$.*

The proof is given in Appendix.

4. SIMULATION RESULTS

Fig. 2 shows the sum rate of secondary link as a function of $N_s \log(\log K)$ with different thresholds. It is assumed that PR, PT and SR are all equipped with 4 antennas, and each SU is equipped with 3 antennas. The number of dimensions on PR that are occupied by primary link is equal to 3, i.e., $s = M_s = 3$. In addition, we assume each SU has the same transmitting power, i.e., $\mathbf{P}_i = \mathbf{P}$. Hence, the threshold of each SU, γ'_{th}, can be determined according to (7).

First of all, it is shown that higher threshold leads to higher average sum rate of secondary link. This is because a higher threshold results in more candidate users selected from **step 3** of **Algorithm 1**. As a result, the rate is boosted with multiuser diversity gain. Next, for the fixed threshold, we can see that with large K, the slope of each simulation curve almost matches the slope of $N_s \log(\log K)$, which confirms the results in **Theorem 1**.

5. CONCLUSION

In this paper, a two-stage opportunistic user scheduling scheme has been proposed for a cognitive radio network where the secondary link is a multi-user MIMO uplink network. In the first stage, the secondary users that cause interference leakage at the primary user less than a predetermined threshold are selected as candidate users. Then, N_s users should be selected among all the candidate users with an aim of achieving the highest rate. Analytical and simulation results both show that the sum rate of secondary link scales as $N_s \log(\log K)$ with sufficiently large K SUs.

6. APPENDIX

The received signals on SR is given as (8), which are post-processed by a zero-forcing filter $\mathbf{U} \in \mathbb{C}^{l\times N_s} = \begin{bmatrix} \mathbf{u}_1^T & \cdots & \mathbf{u}_l^T \end{bmatrix}^T$, i.e., $\mathbf{U} = (\mathcal{H}^H\mathcal{H})^{-1}\mathcal{H}^H$.

Note that each row of $\mathbf{U}$ satisfies the zero-forcing condition $\mathbf{u}_i\mathfrak{h}_j = 0$ for $i \neq j$, where $\mathfrak{h}_j = \mathbf{H}_{sj}\mathbf{v}_j$. Hence, the instantaneous rate of the ith signal stream of secondary link can be expressed as

$$\begin{aligned} R_i &= \log(1+\frac{\mathbf{P}_i}{|\mathbf{u}_i\mathbf{z}_s|^2+|\mathbf{u}_i\mathbf{I}_p|^2}) \approx \log(\frac{\mathbf{P}_i}{|\mathbf{u}_i\mathbf{z}_s|^2+|\mathbf{u}_i\mathbf{I}_p|^2}) \\ &\overset{a}{\geq} \log(\frac{\mathbf{P}_i}{|\mathbf{u}_i|^2(1+|\mathbf{I}_p|^2)}) = \log(\frac{\mathbf{P}_i\frac{1}{|\mathbf{u}_i|^2}}{1+|\mathbf{I}_p|^2}) \end{aligned} \tag{9}$$

where $\overset{a}{\geq}$ is due to Cauchy-Schwarz inequality and the assumption that $\mathbf{z}_s$ has unit variance.

Accordingly, the lower bound of the average rate of the ith signal stream can be expressed as

$$\mathrm{E}\{R_i\} \geq \log(\mathbf{P}_i) + \mathrm{E}\{\log(\frac{1}{|\mathbf{u}_i|^2})\} - \mathrm{E}\{\log(1+|\mathbf{I}_p|^2)\} \tag{10}$$

Since $\mathbf{I}_p$ is from primary link which is not affected by the selection of SUs, $\mathrm{E}\{\log(1+|\mathbf{I}_p|^2)\}$ can be seen as a constant. Hence, (10) can be written as $\mathrm{E}\{R_i\} \geq \mathrm{E}\{\log(\frac{1}{|\mathbf{u}_i|^2})\} + C$, where $C = \log(\mathbf{P}_i) - \mathrm{E}\{\log(1+|\mathbf{I}_p|^2)\}$. Further, we can get

$$\frac{1}{|\mathbf{u}_i|^2} = \frac{1}{[(\mathcal{H}^H\mathcal{H})^{-1}]_{i,i}} \tag{11}$$

where $[\mathbf{B}]_{i,j}$ denotes the element in the ith row and jth column of matrix $\mathbf{B}$.

Denote $h_{ij} = \frac{\mathbf{g}_{(j)}^H\mathfrak{h}_{(i)}}{|\mathbf{g}_{(j)}|}$ and $e_{ij} = \frac{h_{ij}}{|\mathbf{g}_{(i)}|}$. Then, we have $\mathfrak{h}_{(i)} = \mathbf{g}_{(i)} + \sum_{j=1}^{i-1}\mathbf{g}_{(j)}e_{ij}$. Hence, $\mathcal{H}$ can be written as

$$\begin{aligned}\mathcal{H} &= \begin{bmatrix} \frac{\mathbf{g}_{(1)}}{|\mathbf{g}_{(1)}|} & \cdots & \frac{\mathbf{g}_{(N_s)}}{|\mathbf{g}_{(N_s)}|} \end{bmatrix} \begin{bmatrix} 1 & e_{21} & e_{31} & \cdots & e_{N_s1} \\ 0 & 1 & e_{32} & \cdots & e_{N_s2} \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & 1 & e_{N_s,N_s-1} \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \\ &\quad \begin{bmatrix} |\mathbf{g}_{(1)}| & & \\ & \ddots & \\ & & |\mathbf{g}_{(N_s)}| \end{bmatrix} \\ &\triangleq \mathbf{W}\cdot\mathbf{E}\cdot\mathbf{R}\end{aligned}$$

Since $\mathcal{H} \in \mathbb{C}^{N_s\times N_s}$ is invertible almost for sure, we have $\mathbf{U} = \mathcal{H}^{-1} = \mathbf{R}^{-1}\mathbf{E}^{-1}\mathbf{W}^H$. According to (11), we can get

$$\frac{1}{|\mathbf{u}_i|^2} = \frac{1}{[(\mathbf{R}^H\mathbf{E}^H\mathbf{W}^H\mathbf{W}\mathbf{E}\mathbf{R})^{-1}]_{i,i}} = \frac{|\mathbf{g}_{(i)}|^2}{[(\mathbf{E}^H\mathbf{E})^{-1}]_{i,i}} \tag{12}$$

Note that $\mathbf{E}$ is an upper triangular matrix, and its inverse is also an upper triangular matrix with ones in the diagonal. Accordingly,

$$\begin{aligned} [(\mathbf{E}^H\mathbf{E})^{-1}]_{i,i} &= [\mathbf{E}^{-1}\cdot(\mathbf{E}^{-1})^H]_{i,i} = \sum_{k=1}^{N_s}|[\mathbf{E}^{-1}]_{ik}|^2 \\ &= |[\mathbf{E}^{-1}]_{ii}|^2 + \sum_{k=1,k\neq i}^{N_s}|[\mathbf{E}^{-1}]_{ik}|^2 \\ &= 1 + \sum_{k=1,k\neq i}^{N_s}|[\mathbf{E}^{-1}]_{ik}|^2 \end{aligned} \tag{13}$$

Since $[(\mathbf{E}^*\mathbf{E})^{-1}]_{i,i} \geq 1$, we have

$$\frac{1}{|\mathbf{u}_i|^2} < |\mathbf{g}_{(i)}|^2 \tag{14}$$

Next, we investigate the lower bound of $\frac{1}{|\mathbf{u}_i|^2}$, i.e., the upper bound of $\sum_{k=1,k\neq i}^{N_s}|[\mathbf{E}^{-1}]_{ik}|^2$ in (13). Note that $\mathbf{E}$ can be written as $\mathbf{I}+\mathbf{F}$, where

$$\mathbf{F} = \begin{bmatrix} 0 & e_{21} & e_{31} & \cdots & e_{N_s1} \\ 0 & 0 & e_{32} & \cdots & e_{N_s2} \\ \vdots & \cdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & e_{N_s,N_s-1} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{15}$$

Since $\mathbf{F}^{N_s} = \mathbf{0}$, we can get $\mathbf{E}^{-1} = \sum_{n=0}^{N_s-1}(-\mathbf{F})^n$ [16], which leads to

$$|[\mathbf{E}^{-1}]_{ik}|^2 = |[\sum_{n=0}^{N_s-1}(-\mathbf{F})^n]_{ik}|^2 \leq \sum_{n=0}^{N_s-1}|[(\mathbf{F})^n]_{ik}|^2 \tag{16}$$

Then, note that $[(\mathbf{F})]_{ik} = |e_{ki}|^2 < 1$ and $|[\mathbf{E}^{-1}]_{ik}|^2 \leq \sum_{n=0}^{N_s-1}|[(\mathbf{F})^n]_{ik}|^2 \leq N_s$ for small value of α. According to (13) we have $[(\mathbf{E}^H\mathbf{E})^{-1}]_{i,i} \leq 1 + (N_s-1)\cdot N_s$, which means

$$\frac{1}{|\mathbf{u}_i|^2} \geq \frac{|\mathbf{g}_{(i)}|^2}{1+(N_s-1)\cdot N_s} \tag{17}$$

From (14) and (17) we can see that $\frac{1}{|\mathbf{u}_i|^2}$ scales as $|\mathbf{g}_{(i)}|^2$, which means the lower bound of the average rate of each signal stream, $\mathrm{E}\{\log\frac{1}{|\mathbf{u}_i|^2}\} + C$, can be approximated as $\mathrm{E}\{\log|\mathbf{g}_{(i)}|^2\} + C'$, where C' is a constant. According to the algorithm, $\mathbf{g}_{(i)}$ is chosen as the one with the maximum $|\mathbf{g}_k|^2$ in Q_i. Since the variables in Q_i are i.i.d, it was proved in [16, 18–20] that $\mathrm{E}\{|\mathbf{g}_{(i)}|^2\} \approx \log|Q_i|$. Further, it has $\log|Q_i| \approx \log(\beta_i K) = \log K + c_0$ for large K, where $c_0 = \log\beta_i$ is a constant related to γ_{th} and $0 < \beta_i \leq 1$. (Smaller γ_{th} leads to smaller β_i). This implies that $\mathrm{E}\{\log\frac{1}{|\mathbf{u}_i|^2}\}$ scales as $\log\log K$ for sufficiently large K and small γ_{th}. Since there are total N_s signal streams for SR, the sum rate of secondary link scales as $N_s\log\log K$ for large K.

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