COGNITIVE TRANSMIT BEAMFORMING FROM BINARY LINK QUALITY FEEDBACK FOR POINT TO POINT MISO CHANNELS

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ABSTRACT

Transmit beamforming is an effective way to enhance transmission range and quality of service while limiting interference to other co-channel systems, thus facilitating easier coexistence. Transmit beamforming requires channel state information to be acquired at the receiver and fed back to the transmitter. This in turn requires a relatively complex receiver, agreement on a training protocol, and a cold-start training period during which no payload is sent to the receiver. This article explores how the transmitter can learn to beamform on-the-fly from very low-rate channel quality indicator bits fed back from the receiver, while transmitting payload at the same time. The setup is tuned to low-latency scenarios where the receiver has limited capabilities, and is paired up opportunistically with the transmitter. Leveraging the Analytic Center Cutting Plane Method (ACCPM), an online channel correlation matrix learning method is developed, based on one-bit Signal to Noise Ratio (SNR) feedback from the receiver. The method is shown to asymptotically achieve the maximum possible SNR at the receiver (attained with perfect knowledge of the correlation matrix), *starting from no channel state information*. A Maximum Likelihood (ML) formulation is also developed for the case when there are feedback errors, and conditions for its asymptotic convergence are derived.

Keywords: Transmit beamforming, spatial channel correlation, online learning, cutting plane method, maximum likelihood.

1. INTRODUCTION

Transmit beamforming is a communication technique using multiple transmit antennas to steer radiated power towards directions that provide good quality of service (QoS) to a desired receiver [1]. The resulting link gain can be used to boost the reach or the information rate, while limiting interference to nearby co-channel systems - thereby facilitating coexistence, which is crucial for frequency reuse and dynamic spectrum access applications. The price paid is the need for channel estimation at the receiver, and channel state feedback to the transmitter.

In order to mitigate the signaling overhead, the channel correlation matrix is what is usually estimated at the receiver (instead of the instantaneous channel vectors) and fed back to the transmitter. This enables long-term average Signal to Noise Ratio (SNR) optimization, but still requires a receiver capable of performing relatively sophisticated computations. Another issue is that a cold-start (or periodic re-training) implies a black-out period, during which no payload is sent to the receiver, which is a concern for low-latency applications such as voice and streaming. Finally, the traditional approach requires prior agreement on a training protocol, which may not be possible with legacy systems or when the transmitter and the receiver are paired up opportunistically.

In the special case of Multiple-Input Single-Output (MISO) systems (multiple antennas at the transmitter and a single antenna at the receiver), the average received SNR can be maximized by aligning the transmit beamforming vector with the direction of the principal eigenvector of the channel correlation matrix [2]. Existing approaches for designing long-term transmit beamforming vectors assume knowledge of the channel correlation matrix (or its principal eigenvector) at the transmitter, obtained through receiver-side estimation and feedback to the transmitter. Taking advantage of the linear relationship between the received data correlation matrix and the channel correlation matrix, a linear least-squares channel correlation matrix estimator for Multiple-Input Multiple-Output (MIMO) systems was proposed in [3], assuming training symbols are available. After estimating the channel correlation matrix, the receiver may compute its principal eigenvector using, e.g., the power method, and feed back a quantized version of the principal eigenvector instead of the channel correlation matrix. A codebook design criterion for directly quantizing transmit beamforming vectors using Grassmanian line packing has been proposed by Love *et al.* [4], where bounds were derived on the codebook size needed for a given capacity or SNR loss. The approach in [4] was developed for instantaneous feedback, but it can be extended [4] to long-term feedback. Either way, the receiver needs enough computational power to perform channel estimation, and vector quantization or principal component computation. The method in [4] further assumes that the beamforming codebook has already been communicated to the transmitter.

This article takes a fresh look at this problem and considers the case where the receiver has limited computational capabilities, and/or is paired up opportunistically with the transmitter. It explores how the transmitter can learn to beamform on-the-fly from very low-rate channel quality indicator bits fed back from the receiver, while simultaneously transmitting payload. Towards this end, the Analytic Center Cutting Plane Method (ACCPM) from optimization is leveraged to develop an online channel correlation matrix learning method based on one-bit Signal to Noise Ratio (SNR) feedback.

Consider a MISO link with a multiple antenna base station (BS) serving a single-antenna user. Time is split in transmission slots. For each transmission epoch, the BS designs a new transmit beamforming vector and uses it to send the data to its receiver which measures the average SNR, compares with a pre-determined threshold and sends a '1' or a '0' to the BS depending on whether the average SNR is above or below the threshold, respectively. The beamforming vectors, obtained from a novel optimization-based formulation,

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are designed so that they not only maximize a *transmitter-side estimate* of the average received SNR, but also are diverse enough to learn the channel correlation matrix accurately as time progresses. An analytic center based threshold selection algorithm is used in order to cut off a half space and reduce the current feasible region for the channel correlation matrix significantly at every time slot. Based on this algorithm, it is shown that in the absence of binary measurement or feedback communication errors, the feasible region of the channel estimate converges to within a ball of radius r from the true value within $\mathcal{O}\left(\frac{N_t^2}{r^2}\right)$ iterations (where N_t is the number of transmit antennas) and the average received SNR converges asymptotically to the maximum achievable SNR value (obtained with perfect knowledge of the channel correlation matrix). The proposed technique is then extended to the case where there are bit errors in the SNR measurement at the receiver or over the feedback link. In this case a maximum likelihood formulation is proposed to incorporate the bit flips occurring due to measurement noise or feedback channel noise. The conditions for identifiability of the maximum likelihood estimate are also studied in order to prove the convergence of the algorithm in the presence of errors.

2. SYSTEM MODEL

Consider a transmitter with N_t antennas and a receiver with a single antenna. Let the channel from the transmitter to the receiver be modeled as a $N_t \times 1$ complex random vector $\mathbf{h}$, with zero mean and correlation matrix $\mathbf{R_h} = E\left(\mathbf{h}\mathbf{h}^H\right)$. Time is divided into transmission rounds or *slots* of length T seconds, with each slot comprising enough symbols for the receiver to perform relatively accurate power estimation. At time $tT+\tau$, where t is a slot index and τ is 'fast time', the transmitter sends the complex zero-mean unit-variance symbol $x(tT+\tau)$ times a complex beamforming vector $\mathbf{w}_t$, and the receiver measures

$$y(tT+\tau) = \left[\mathbf{w}_t^H \mathbf{h}\right] x(tT+\tau) + z(tT+\tau), \tag{1}$$

where the additive noise $z(\cdot)$ has zero mean, variance σ^2, and is independent of $x(\cdot)$. The average received signal to noise ratio (SNR) for slot t is given by $E\left(\frac{|\mathbf{w}_t^H\mathbf{h}|^2}{\sigma^2}\right) = \frac{\mathbf{w}_t^H\mathbf{R_h}\mathbf{w}_t}{\sigma^2}$. If the transmitter has perfect knowledge of $\mathbf{R_h}$, then the beamforming vector that maximizes the average received SNR is the principal eigenvector of $\mathbf{R_h}$ scaled by the available transmit power. The transmitter initially has no channel state information (CSI), and its objective is to learn $\mathbf{R_h}$ and maximize the average received SNR, based on binary SNR feedback. More specifically, in each time slot t, the receiver estimates the average SNR and compares it with a threshold γ_t. A '1' is fed back to the transmitter if the average SNR is $\geq \gamma_t$ and a '0' is fed back otherwise. It is initially assumed that there is no measurement or feedback communication error. Based on the single-bit feedback at time t, the transmitter learns that

$$\begin{cases} \mathbf{w}_t^H\mathbf{R_h}\mathbf{w}_t \geq \gamma_t, & s_t = 1; \quad \text{or} \\ \mathbf{w}_t^H\mathbf{R_h}\mathbf{w}_t < \gamma_t, & s_t = 0, \end{cases} \tag{2}$$

where s_t is the 1-bit feedback at time t. For every feedback bit, the transmitter learns an additional inequality for $\mathbf{R_h}$. Therefore, if $\mathbf{w}_t$ and γ_t are chosen intelligently, the feasible region of $\mathbf{R_h}$ can be reduced significantly for each time slot and the estimate of the channel correlation matrix $\hat{\mathbf{R}}_\mathbf{h}$ can approach, as we will show, $\mathbf{R_h}$ as time passes. In particular, we will show how to choose $\mathbf{w}_t$ and γ_t to ensure that the channel autocorrelation matrix estimate at the transmitter $\hat{\mathbf{R}}_\mathbf{h}$ converges to $\mathbf{R_h}$, and the average received SNR converges to that attained by the principal eigenvector of $\mathbf{R_h}$.

3. PROBLEM FORMULATION

For every slot t, the transmitter has to choose $\mathbf{w}_t$ in such a way that it not only gathers a significant amount of information about $\mathbf{R_h}$ (from the 1-bit feedback), but also tries to deliver a high average received SNR to enable payload transmission in parallel with channel learning. To accomplish the former objective, the beamforming vectors chosen at each instant should be as diverse as possible to the previously chosen weight vectors so that over a period of time, the transmitter will learn about $\mathbf{R_h}$ from as many different directions as possible. For the latter, the best that the transmitter can do to deliver a high average received SNR is to assume that $\hat{\mathbf{R}}_\mathbf{h}$ is close to $\mathbf{R_h}$ and choose the beamforming weight vector along the direction of the principal eigenvector of $\hat{\mathbf{R}}_\mathbf{h}$. Since the transmitter does not have any CSI to start with, initially it has to give preference towards choosing weight vectors that can explore the channel correlation space efficiently, to improve the accuracy of $\hat{\mathbf{R}}_\mathbf{h}$; and then, as time passes, slowly shift its priority towards beamforming vectors in the direction of the principal eigenvector of $\hat{\mathbf{R}}_\mathbf{h}$. This ensures that as $\hat{\mathbf{R}}_\mathbf{h}$ approaches $\mathbf{R_h}$ (as will be shown later), $\mathbf{w}_t$ approaches the direction of principal eigenvector of $\mathbf{R_h}$, thus attaining the maximum average received SNR achieved with perfect knowledge of $\mathbf{R_h}$.

At the end of slot t, the transmitter has learned the following inequalities about $\mathbf{R_h}$ from the t feedback bits received

$$\mathbf{w}_i{}^H\mathbf{R_h}\mathbf{w}_i \geq \gamma_i, \quad \forall i \in \mathcal{G}_1 \tag{3}$$

$$\mathbf{w}_i{}^H\mathbf{R_h}\mathbf{w}_i < \gamma_i, \quad \forall i \in \mathcal{G}_2 \tag{4}$$

where $\mathcal{G}_1 = \{i : 1 \leq i \leq t, s_i = 1\}$, $\mathcal{G}_2 = \{i : 1 \leq i \leq t, s_i = 0\}$, $\mathcal{G}_1 \bigcup \mathcal{G}_2 = \{1, 2, \ldots, t\}$ and t is the number of elapsed time slots. We propose to update $\hat{\mathbf{R}}_\mathbf{h}$ as follows.

$$\mathbf{\Pi_1} \quad \hat{\mathbf{R}}_\mathbf{h} = \arg\max_{\mathbf{R_h}} \sum_{i\in\mathcal{G}_1} \log\left(\mathrm{Tr}\left(\mathbf{W}_i\mathbf{R_h}\right) - \gamma_i\right) + \sum_{j\in\mathcal{G}_2} \log\left(\gamma_j - \mathrm{Tr}\left(\mathbf{W}_i\mathbf{R_h}\right)\right) + \log\det\mathbf{R_h} \tag{5}$$

where $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H$ and the term $\mathbf{w}_i{}^H\mathbf{R_h}\mathbf{w}_i$ has been rewritten as $\mathrm{Tr}\left(\mathbf{W}_i\mathbf{R_h}\right)$. $\mathbf{\Pi_1}$ is a convex optimization problem which obtains the analytic center of the feasible region at time slot t formed by the linear inequalities (3)-(4) and the positive semi-definite cone [5] [6]. It can be solved efficiently using interior point methods with worst case complexity $\mathcal{O}(N_t^7)$.

3.1. Design of beamforming vector $\mathbf{w}_{t+1}$ and threshold γ_{t+1} : Analytic Center Cutting Plane Method (ACCPM)

Design of beamforming vector $\mathbf{w}_{t+1}$

After updating $\hat{\mathbf{R}}_\mathbf{h}$, the beamforming vector for time slot $t+1$, $\mathbf{w}_{t+1}$ is designed as follows.

$$\mathbf{\Pi_2} \quad \mathbf{w}_{t+1} = \arg\max_{\|\mathbf{w}\|=1} \mathbf{w}^H\hat{\mathbf{R}}_\mathbf{h}\mathbf{w} - \lambda_t\mathbf{w}^H\mathbf{V}_{w,t}\mathbf{V}_{w,t}^H\mathbf{w}$$

where $\mathbf{V}_{w,t} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_t]$, and λ_t is a decreasing function of t e.g., $\lambda_t = \frac{\lambda}{\lceil 0.1t \rceil}$. The solution of $\mathbf{\Pi_2}$ can be obtained in closed form i.e., $\mathbf{w}_{t+1}$ is the unit vector along the direction of principal eigenvector of the matrix $(\hat{\mathbf{R}}_\mathbf{h} - \lambda_t\mathbf{V}_{w,t}\mathbf{V}_{w,t}^H)$.

The objective function in $\mathbf{\Pi_2}$ consists of two terms, the first one is proportional to the transmitter side estimate of average received SNR (which is close to the actual average received SNR if the transmitter has estimated $\hat{\mathbf{R}}_\mathbf{h}$ close to $\mathbf{R_h}$), and the second one is the squared norm of the vector $(\mathbf{V}_{w,t}^H\mathbf{w})$, whose i^{th} entry is the dot-product of $\mathbf{w}$ with $\mathbf{w}_i$. Maximization of this objective function gives a weight vector that strikes a balance between maximizing the estimated average received SNR and minimizing similarity to the weight vectors chosen in previous time slots. The scalar λ_t is chosen as a decreasing function of t with $\lambda \gg 1$, where λ is the value of λ_t at $t = 1$. The rate of decrease of λ_t w.r.t. t is chosen based on how quickly the Tx can learn the channel.

In $\mathbf{\Pi_2}$, for small t, since $\lambda_t \gg 1$, the choice of weight vector is dictated by $\left(\mathbf{w}^H \mathbf{V}_{w,t} \mathbf{V}_{w,t}^H \mathbf{w}\right)$ resulting in diverse weight vectors that explore different directions, gathering information about $\mathbf{R_h}$ to help the Tx form an accurate estimate of $\mathbf{R_h}$. Then for large t, $\lambda_t \ll 1$ and the preference shifts to the first term $\mathbf{w}^H \hat{\mathbf{R}}_\mathbf{h} \mathbf{w}$, resulting in weight vectors aligned along the principal eigenvector of $\hat{\mathbf{R}}_\mathbf{h}$. Therefore, as $\hat{\mathbf{R}}_\mathbf{h} \to \mathbf{R_h}$ as $t \to \infty$, the beamforming vector chosen by the Tx asymptotically aligns itself with the direction of the principal eigenvector of $\mathbf{R_h}$, thus attaining the maximum average received SNR.

Design of threshold γ_{t+1}

After choosing $\mathbf{w}_{t+1}$, the transmitter selects an appropriate SNR threshold γ_{t+1} such that the subsequent inequality constraint for $\mathbf{R_h}$ obtained from the 1-bit feedback at time slot $t+1$ reduces considerably the feasible region at time t given by $\mathcal{P}_t$, where $\mathcal{P}_t = \{\mathbf{R} : \mathbf{R} \succeq 0, {\mathbf{w}_i}^H \mathbf{R} \mathbf{w}_i \geq \gamma_i,\ \forall i \in \mathcal{G}_1, {\mathbf{w}_i}^H \mathbf{R} \mathbf{w}_i < \gamma_i,\ \forall i \in \mathcal{G}_2,\ \mathcal{G}_1 \bigcup \mathcal{G}_2 = \{1, 2, \ldots, t\}\}$. This is crucial for the convergence of $\hat{\mathbf{R}}_\mathbf{h}$ to $\mathbf{R_h}$. Since the transmitter already communicates payload information to the receiver in parallel to learning to beamform, the new threshold can 'piggyback' on the payload transmission at limited overhead - unlike the receiver feedback on the reverse link, which is more severely limited in terms of rate. The basic method still works without having the transmitter dictate thresholds to the receiver, albeit convergence to the true channel correlation matrix cannot be guaranteed in this case.

One way to ensure that the feasible region is reduced at each time step is to choose $\mathbf{w}_{t+1}$ and γ_{t+1}, such that the resulting hyperplane ${\mathbf{w}_{t+1}}^H \mathbf{R} \mathbf{w}_{t+1} = \gamma_{t+1}$ passes through an interior point of $\mathcal{P}_t$. Here, we propose to design the beamforming vector $\mathbf{w}_{t+1}$ and the threshold γ_{t+1} such that the resulting hyperplane passes through the analytic center of $\mathcal{P}_t$ (Analytic Center Cutting Plane Method - ACCPM) which is $\hat{\mathbf{R}}_\mathbf{h}$. Since the analytic center is the point that maximizes the product of distances to the defining hyperplanes and the positive semi-definite cone, it gives the deepest interior point of $\mathcal{P}_t$. Hence for a given $\mathbf{w}_{t+1}$, γ_{t+1} is chosen as follows.

$$\gamma_{t+1} = {\mathbf{w}_{t+1}}^H \hat{\mathbf{R}}_\mathbf{h} \mathbf{w}_{t+1} \tag{6}$$

This ensures that the resultant hyperplane ${\mathbf{w}_{t+1}}^H \mathbf{R} \mathbf{w}_{t+1} = \gamma_{t+1}$, where $\mathbf{R} \in \mathcal{C}^{N_t x N_t}$ will pass through $\hat{\mathbf{R}}_\mathbf{h}$ and cut off a significant part of the current feasible region $\mathcal{P}_t$.

It has been shown that using ACCPM, the analytic center of the polyhedron $\mathcal{P}_t$ is restricted to a ball of radius r around the true value within $\mathcal{O}\left(\frac{N_t^2}{r^2}\right)$ iterations [7]. It follows that $\hat{\mathbf{R}}_\mathbf{h}$ (updated as the analytic center of the current feasible region) is restricted to a ball of radius r around $\mathbf{R_h}$ within $\mathcal{O}\left(\frac{N_t^2}{r^2}\right)$ iterations. Therefore, if λ_t is designed so that it becomes negligible by $\lceil \frac{N_t^2}{r^2} \rceil$ iterations (i.e. $\lambda_t \ll 1$), then the objective function of $\mathbf{\Pi_2}$ can be approximated as $\mathbf{w}^H (\hat{\mathbf{R}}_\mathbf{h}) \mathbf{w}$. Hence asymptotically, as $\hat{\mathbf{R}}_\mathbf{h} \to \mathbf{R_h}$, the beamforming weight vector will converge to the principal eigenvector of $\mathbf{R_h}$ and the average received SNR will approach the maximum achievable average SNR (obtained with perfect knowledge of $\mathbf{R_h}$).

4. MAXIMUM LIKELIHOOD FORMULATION

In practice certain bits may be flipped due to inaccurate SNR estimation at the receiver, or communication errors on the reverse link. Assuming a memoryless feedback link, these errors are independent from slot to slot. We model both using zero mean Gaussian random variables with variance σ_n^2. With the inclusion of this noise, the inequalities derived at the transmitter from the one bit feedback at time t are as follows.

$$\begin{cases} \mathbf{w}_t^H \mathbf{R_h} \mathbf{w}_t + n_t \geq \gamma_t, & s_t = 1; \\ \mathbf{w}_t^H \mathbf{R_h} \mathbf{w}_t + n_t < \gamma_t, & s_t = 0. \end{cases} \tag{7}$$

where $n_t \sim \mathcal{N}(0, \sigma_n^2)$ is the *equivalent* measurement noise at time t. The likelihood function of $\mathbf{R_h}$ given the received feedback bits $s_1, s_2, \ldots, s_t$ is as follows.

$$\begin{aligned} f(\mathbf{R_h}|\mathbf{s}_t) &= \prod_{i \in \mathcal{G}_1} \Pr\left[\mathrm{Tr}(\mathbf{W}_i \mathbf{R_h}) + n_t \geq \gamma_i\right] \\ &\qquad \prod_{i \in \mathcal{G}_2} \Pr\left[\mathrm{Tr}(\mathbf{W}_i \mathbf{R_h}) + n_t < \gamma_i\right] \\ &= \prod_{i \in \mathcal{G}_1} \Phi\left(\frac{\mathrm{Tr}(\mathbf{W}_i \mathbf{R_h}) - \gamma_i}{\sigma_n}\right) \prod_{i \in \mathcal{G}_2} \Phi\left(\frac{\gamma_i - \mathrm{Tr}(\mathbf{W}_i \mathbf{R_h})}{\sigma_n}\right) \end{aligned} \tag{8}$$

where $\mathbf{s}_t = [s_1, s_2, \ldots, s_t]$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx$ is the standard Gaussian c.d.f. At every time $t+1$, $\hat{\mathbf{R}}_\mathbf{h}$ is updated as the maximum likelihood estimate $\hat{\mathbf{R}}_\mathbf{h}^{\mathbf{MLE}}$ (as compared to the analytic center in the error-free case) obtained from the convex optimization problem $\mathbf{\Pi_3}$ which maximizes the log-likelihood function $\log\left(f(\mathbf{R_h}|\mathbf{s}_t)\right)$ with a positive semi-definite constraint.

$$\begin{aligned} \mathbf{\Pi_3} \quad \hat{\mathbf{R}}_\mathbf{h}^{\mathbf{MLE}} &= \arg\max_{\mathbf{R_h} \succeq 0} \sum_{i \in \mathcal{G}_1} \log \Phi\left(\frac{\mathrm{Tr}(\mathbf{W_i} \mathbf{R_h}) - \gamma_i}{\sigma_n}\right) \\ &\qquad + \sum_{i \in \mathcal{G}_2} \log \Phi\left(\frac{\gamma_i - \mathrm{Tr}(\mathbf{W_i} \mathbf{R_h})}{\sigma_n}\right) \end{aligned} \tag{9}$$

$\mathbf{\Pi_3}$ is a convex optimization problem since it involves the maximization of the logarithm of the c.d.f. of a Gaussian distribution which is concave, with a positive a semi-definite constraint which is convex. Once the channel correlation matrix estimate is updated, $\mathbf{w}_{t+1}$ is chosen as the principal eigenvector of $\left(\hat{\mathbf{R}}_\mathbf{h}^{\mathbf{MLE}} - \lambda_t \mathbf{V}_{w,t} \mathbf{V}_{w,t}^H\right)$ and $\gamma_{t+1} = \mathrm{Tr}\ (\mathbf{W}_{t+1} \hat{\mathbf{R}}_\mathbf{h}^{\mathbf{MLE}})$, where $\mathbf{V}_{w,t} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_t]$ and λ_t is a decreasing function of t. If the the autocorrelation matrix is statistically identifiable from the measurements, then the MLE will asymptotically approach the true value as the number of feedback bits increases. In this case, the average received SNR will approach the maximum SNR that can be achieved at the user side (obtained with perfect CSI). Now s_t can be written as follows

$$s_t = \mathrm{sign}\left[\mathrm{Tr}(\mathbf{W}_t \mathbf{R_h}) + c_t\right] = \mathrm{sign}\left[\mathrm{vec}\left(\mathbf{W}_t\right)^H \mathrm{vec}\left(\mathbf{R_h}\right) + c_t\right]$$

where $c_t = n_t - \gamma_t$. For identifiability, it is required that $\lim_{S \to \infty} \frac{1}{S} \sum_{t=1}^{S} \mathrm{vec}\left(\mathbf{W}_t\right) \mathrm{vec}\left(\mathbf{W}_t\right)^H$ exists and is non-singular [8]. This in turn requires the weight vectors to be diverse so that the log-likelihood function has a unique maximum in the limit. In our case, if we choose a high value of λ and design the decay rate of λ_t appropriately, it is possible to satisfy this condition.

5. SIMULATION RESULTS

The average received SNR and the estimation error $\|\mathbf{R_h} - \hat{\mathbf{R}}_\mathbf{h}\|_F$ using the ACCPM for threshold update and beamforming vector design are plotted in Figures 1 and 2 for $N_t = 5$ and $N_t = 10$ respectively. For simulation purposes, the channel vector $\mathbf{h}$ is drawn from $\mathcal{CN}(\mathbf{0}, \mathbf{R_h})$. It can be seen that, the average received SNR (solid line) converges to the maximum achievable average SNR value with perfect knowledge of $\mathbf{R_h}$ at the transmitter (dotted line), as time increases. It takes approximately 80 time slots for the algorithm to converge to the maximum achievable SNR for $N_t = 5$ which means that approximately 6 (80/(5(5+1)/2)) feedback bits are required to reconstruct a complex entry of $\mathbf{R_h}$ accurately. The time taken by the algorithm to converge to the maximum achievable SNR increases with the number of transmit antennas. The average received SNR and the $\mathbf{R_h}$ estimation error using the MLE algorithm is plotted in Figure 3 for $N_t = 5$ and $\sigma_n = 0.01$. There were 48 bit flips due to measurement and feedback noise in this case. It can be seen that $\hat{\mathbf{R}}_\mathbf{h}$ approaches $\mathbf{R_h}$ and the average received SNR approaches the maximum achievable SNR with perfect knowledge of $\mathbf{R_h}$. However, the time taken for convergence is higher than in the case without errors (Figure 1), i.e., 150 with errors versus 80 without errors. The reason for the slower convergence rate is that the maximum likelihood estimate is not necessarily the deepest interior point of $\mathcal{P}_t$, the feasible region in the absence of noise, which results in a slower rate of decrease of the feasible region as compared to ACCPM.

6. CONCLUSIONS

In this paper, we have proposed an efficient way to accurately estimate the channel correlation matrix at the transmitter of a MISO link based only on one bit feedback from the receiver, obtained by comparing its average received SNR with a threshold that is varied adaptively by the transmitter and communicated to the receiver. This algorithm is used for designing transmit beamforming vectors. The proposed technique is shown to be promising because the transmitter starts with no CSI, and, as time progresses, not only does it obtain an accurate estimate of the correlation matrix and the maximum-SNR beamformer, but it does so while transmitting payload in parallel with the learning process. A maximum likelihood formulation was also proposed to accommodate measurement/feedback communication errors and the identifiability conditions required for the asymptotic convergence of the ML estimate were discussed. Pertinent extensions to cognitive radio settings are currently under investigation, and will be reported in the journal version.

After acceptance of this paper, it has been brought to our attention that a very similar idea appears in a parallel submission by Xu and Zhang [9] that will be presented at the same conference. While the application focus of [9] is on transmit beamforming for wireless energy transfer, and several design choices are naturally different, these being independent pieces of work, the core idea is the same in both papers.

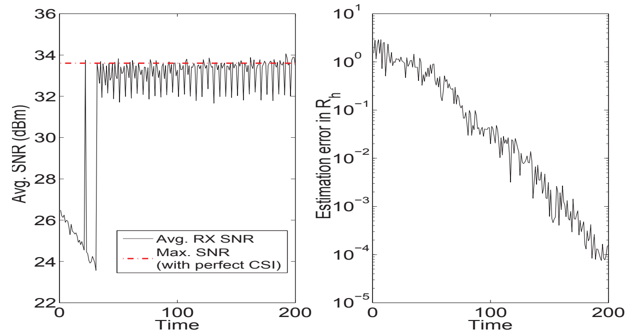

Fig. 1. **Average SNR and estimation error for N_t = 5**

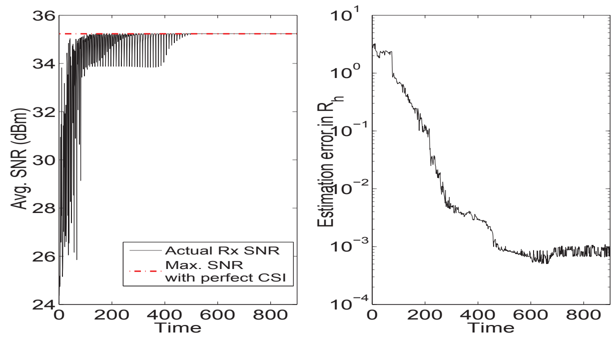

Fig. 2. **Average SNR and estimation error for N_t = 10**

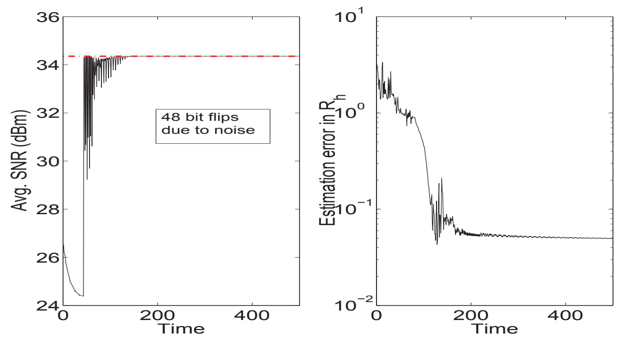

Fig. 3. **Average SNR and estimation error of MLE for N_t = 5** $\sigma_n = 0.01$

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