GERSCHGORIN DISK-BASED ROBUST SPECTRUM SENSING FOR COGNITIVE RADIO

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ABSTRACT

Spectrum sensing is a fundamental problem in cognitive radio. In this paper, we introduce two spectrum sensing methods based on Gerschgorin disk. The Gerschgorin radii contain the information of signal subspace, whereas the Gerschgorin centers capture the signal energy. The first proposal only relies on the Gerschgorin radii and thereby is robust against nonuniform noise. The second one, utilizing both the Gerschgorin radii and centers, can significantly improve the detection performance. Simulation results are included to illustrate the superiority of the proposed methods.

Index Terms— Spectrum sensing, cognitive radio, Gerschgorin disk

1. INTRODUCTION

Recently, the increasing usage of wireless communication devices as well as urgent demand for high data transformation rates have intensified the scarcity of radio spectrum. Cognitive radio (CR) [1] is one of the most promising methods to alleviate this problem. In the CR networks, secondary (unlicensed) users (SUs) opportunistically occupy the vacant spectrum authorized to primary (licensed) users (PUs), and vacate the spectrum immediately when the PUs are active again. To this end, the SUs are required to frequently and reliably sense the spectrum, thus achieving high opportunistic throughput capacity while causing little interference to the PUs [2]. As a result, spectrum sensing becomes an important issue in the CR networks.

Numerous spectrum sensing methods have been proposed for CR, such as energy detection (ED) [3],[4], coherent detection (matched filtering) [5],[6],[7], feature detection (cyclostationary detection) [8],[9], and other novel methods based on the volumn of the sample covariance matrix (SCM) [10]. Given the noise variance, the energy detector is proved to be optimal for independent and identically distributed (IID) noise [6]. However, its detection performance considerably degrades in practice due to the uncertainty in the estimated noise variance. The coherent detection and feature detection schemes usually suffer from synchronization errors and frequency offsets. Thus, the methods mentioned above may not be attractive in practical CR systems. In contrast, the eigenvalue-based detectors [11]-[14] have drawn considerable attention. For instance, the arithmetic-to-geometric mean (AGM) detector [13] and the ratio of maximum-minimum (MME) eigenvalue detector [14] need no *a priori* information about the observed data. However, they suffer from performance degradation for the case of nonuniform noise, which results from the uncalibrated receivers. To handle the spectrum sensing issue in nonuniform noise, we propose two novel spectrum sensing methods based on Gerschgorin disk [15], [16]. The first method only utilizes the Gerschgorin radii for spectrum sensing, thereby it is robust against the nonuniform noise. The second approach employs the information of both Gerschgorin radii and centers. Thus, this approach is able to outperform the former under the uniform-noise scenario.

The following notations are used throughout the paper. Superscripts T, H, and * denote transpose, conjugate transpose and conjugate, respectively. E$[\cdot]$ and $|\cdot|$ stand for mathematical expectation and absolute value, respectively.

2. PROBLEM FORMULATION

2.1. Signal Model

Consider a CR network where a multi-antenna SU tries to detect the signal of a PU. The observation at time n ($n = 1, \cdots, N$) under binary hypotheses ($\mathcal{H}_0$: the signal-absence hypothesis; $\mathcal{H}_1$: the signal-presence hypothesis) can be expressed as

$$\mathcal{H}_0 : \boldsymbol{x}(n) \quad = \quad \boldsymbol{w}(n) \tag{1}$$
$$\mathcal{H}_1 : \boldsymbol{x}(n) \quad = \quad \boldsymbol{h}s(n) + \boldsymbol{w}(n) \tag{2}$$

Here, $\boldsymbol{x}(n) = [x_1(n), \cdots, x_M(n)]^T$ denotes the complex received observation vector, $x_i(n)$ ($i = 1, \cdots, M$) is the

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output of the ith antenna, M is the number of antennas of the SU, and $\boldsymbol{h} \in \mathbb{C}^{M\times 1}$ denotes the vector of channel coefficients between the PU and SU. The channel is assumed to be flat fading and remains fixed during the sensing interval. Moreover, $s(n)$ is the primary signal, which is assumed to be zero-mean circularly symmetric Gaussian distributed, $\boldsymbol{w}(n) = [w_1(n), \cdots, w_M(n)]^T$ denotes the zero-mean additive white Gaussian noise (AWGN) vector. Without loss of generality, $s(n)$ and $\boldsymbol{w}(n)$ are assumed to be independent of each other.

2.2. Gerschgorin Disk

The covariance matrix of the observed data $\boldsymbol{x}(n)$ is $\boldsymbol{R} = E\left[\boldsymbol{x}(n)\boldsymbol{x}(n)^H\right]$, which can be partitioned as

$$\boldsymbol{R} = \begin{bmatrix} r_{11} & \cdots & r_{1,M} \\ \vdots & \ddots & \vdots \\ r_{M,1} & \cdots & r_{M,M} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R_1} & \boldsymbol{r} \\ \boldsymbol{r}^H & \mathrm{r}_{M,M} \end{bmatrix} \tag{3}$$

where $\boldsymbol{R_1}$ is an $(M-1)\times(M-1)$ leading principal submatrix obtained by deleting the last row and column of $\boldsymbol{R}$. The vector $\boldsymbol{r} = [r_{1,M}, \cdots, r_{M-1,M}]^T$ is obtained from the last column of the matrix $\boldsymbol{R}$ without its last element. Calculating the eigendecomposition of $\boldsymbol{R_1}$ gives

$$\boldsymbol{R_1} = \boldsymbol{V}\boldsymbol{\Sigma}\boldsymbol{V}^H \tag{4}$$

where $\boldsymbol{\Sigma} = \mathrm{diag}(\lambda_1, \cdots, \lambda_{M-1})$ is the diagonal matrix composed by the eigenvalues of $\boldsymbol{R_1}$ in descending order, and $\boldsymbol{V}$ is an $(M-1)\times(M-1)$ unitary matrix formed by the eigenvectors of $\boldsymbol{R_1}$, i.e.,

$$\boldsymbol{V} = [\boldsymbol{v}_1, \cdots, \boldsymbol{v}_{M-1}]. \tag{5}$$

In the sequel, a new $M\times M$ unitary transformation matrix $\boldsymbol{U}$ is formed as

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{V} & \boldsymbol{0} \\ \boldsymbol{0}^T & 1 \end{bmatrix}. \tag{6}$$

Accordingly, the transformed covariance matrix can be calculated as

$$\boldsymbol{G} = \boldsymbol{U}^H\boldsymbol{R}\boldsymbol{U} = \begin{bmatrix} \boldsymbol{V}^H\boldsymbol{R}_1\boldsymbol{V} & \boldsymbol{V}^H\boldsymbol{r} \\ \boldsymbol{r}^H\boldsymbol{V} & \mathrm{r}_{MM} \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} \lambda_1 & \cdots & 0 & \tau_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \lambda_{M-1} & \tau_{M-1} \\ \tau_1^* & \cdots & \tau_{M-1}^* & \mathrm{r}_{MM} \end{bmatrix} \tag{8}$$

where

$$\tau_i = \boldsymbol{v}_i^H\boldsymbol{r}, \ \ i = 1, \cdots, M-1. \tag{9}$$

Note that the $(M-1)\times(M-1)$ leading principle submatrix of $\boldsymbol{G}$ is exactly the eigenvalue matrix $\boldsymbol{\Sigma}$. The ith eigenvalue of $\boldsymbol{R}_1$, i.e., λ_i, is known as the ith Gerschgorin center of $\boldsymbol{G}$. The corresponding Gerschorin radius of $\boldsymbol{G}$ is calculated as

$$\rho_i = |\tau_i| = \left|\boldsymbol{v}_i^H\boldsymbol{r}\right|, \ \ i = 1, \cdots, M-1. \tag{10}$$

The ith disk is then defined as the collection of points in the complex plane whose distance to λ_i is less than or equal to ρ_i.

Under hypothesis $\mathcal{H}_1$, $\boldsymbol{r}$ can be expressed as

$$\boldsymbol{r} = \boldsymbol{h}_1\sigma_s^2 h_M^*, \tag{11}$$

where $\boldsymbol{h}_1$ is the vector composed of the first $(M-1)$ elements of $\boldsymbol{h}$, h_M is the last element of $\boldsymbol{h}$, and $\sigma_s^2 = E[|s(t)|^2]$ denotes the power of primary signal. Equation (11) indicates that the existence of primary signal makes $\boldsymbol{r}$ lie in the signal subspace. Under $\mathcal{H}_1$, the eigenvector $\boldsymbol{v}_1$ lies in the signal subspace, whereas the other eigenvectors $\boldsymbol{v}_2, \cdots, \boldsymbol{v}_{M-1}$ lie in the noise subspace. Meanwhile, ρ_i $(i = 1, \cdots, M-1)$ can be interpreted as the magnitude of projection of $\boldsymbol{r}$ onto the ith eigenvector $\boldsymbol{v}_i$. We denote ρ_1 as the signal Gerschgorin radii and ρ_i, $i = 2, \cdots, M-1$, as the noise Gerschgorin radii, respectively. Note that the Gerschgorin disks have been employed in [15] for source number detection. Later on, a variant of Gerschgorin disk has been used in [16] for low-complexity source enumeration.

3. GERSCHGORIN DISK-BASED DETECTORS

3.1. Gerschgorin Radius-based Detection

Practically, only the SCM is available, which is calculated as

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}(n)\boldsymbol{x}(n)^H. \tag{12}$$

In order to use the information of Gerschgorin disks, we partition $\hat{\boldsymbol{R}}$ as (3) and get the estimated submatrix $\hat{\boldsymbol{R}}_1$ and vector $\hat{\boldsymbol{r}}$. The eigendecomposition of $\hat{\boldsymbol{R}}_1$ is performed to obtain the eigenvalues $\hat{\lambda}_1, \cdots, \hat{\lambda}_{M-1}$, and corresponding eigenvectors $\hat{\boldsymbol{v}}_1, \cdots, \hat{\boldsymbol{v}}_{M-1}$. Thus, the Gerschgorin radii of the transformed SCM are expressed as

$$\hat{\rho}_i = \left|\hat{\boldsymbol{v}}_i^H\hat{\boldsymbol{r}}\right|, \ \ \ i = 1, \cdots, M-1. \tag{13}$$

As we only consider the scenario of single PU, in which $\hat{\rho}_1$ is much larger than $\hat{\rho}_i$ $(i = 2, \cdots, M-1)$ under $\mathcal{H}_1$, it is reasonable to take the first Gerschgorin radius (GR) $\hat{\rho}_1$ as the test statistic, i.e.,

$$\xi_{GR} \triangleq \hat{\rho}_1 = \left|\hat{\boldsymbol{v}}_1^H\hat{\boldsymbol{r}}\right|. \tag{14}$$

We denote this spectrum sensing method as GR method, since it is based on the Gerschgorin radius. According to (12), $\hat{\boldsymbol{r}}$ is expressed as $\hat{\boldsymbol{r}} = \frac{1}{N}\sum_{n=1}^{N}\boldsymbol{x}_1(n)x_M^*(n)$, where $\boldsymbol{x}_1(n)$ is obtained by deleting the last element of $\boldsymbol{x}(n)$. Under the signal-absence hypothesis $\mathcal{H}_0$, $\hat{\boldsymbol{r}}$ asymptotically approaches to a zero vector due to the independence between noises. Moreover,

$\hat{\boldsymbol{v}}_1$ is the noise eigenvector, which is uncorrelated with $\hat{\boldsymbol{r}}$, thus making $\hat{\rho}_1$ to be approximately equivalent to zero. Under hypothesis $\mathcal{H}_1$, however, $\hat{\boldsymbol{v}}_1$ lies in the signal subspace, which is spanned by the channel vector $\boldsymbol{h}_1$. It is implied in (11) that $\hat{\boldsymbol{r}}$ lies close to the signal subspace. Consequently, the projection of $\hat{\boldsymbol{r}}$ onto $\hat{\boldsymbol{v}}_1$ under $\mathcal{H}_1$ is much larger than that of under $\mathcal{H}_0$.

Because no information for noise variance is needed, the GR detector can achieve the constant false alarm rate property. Moreover, as the Gerschgorin radii are independent of the noise variance, the proposed GR approach is also robust against the nonuniform noise due to the uncalibrated receiver. The proposed GR method for spectrum sensing is summarized in Table 1.

Table 1: GR algorithm for spectrum sensing

Step 1: Calculate the SCM using (12).

Step 2: Partition $\hat{\boldsymbol{R}}$ as $\hat{\boldsymbol{R}} = \begin{bmatrix} \hat{\boldsymbol{R}}_1 & \hat{\boldsymbol{r}} \\ \hat{\boldsymbol{r}}^H & \hat{r}_{MM} \end{bmatrix}$.

Step 3: Perform the eigendecomposition of $\hat{\boldsymbol{R}}_1$: $\hat{\boldsymbol{R}}_1 = \hat{\boldsymbol{V}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{V}}^H$, and then get the eigenvector matrix of $\hat{\boldsymbol{R}}_1$ as $\hat{\boldsymbol{V}} = [\hat{\boldsymbol{v}}_1, \cdots, \hat{\boldsymbol{v}}_{M-1}]$.

Step 4: Calculate the GR test statistic as $\xi_{GR} = |\hat{\boldsymbol{v}}_1^H \hat{\boldsymbol{r}}|$.

Step 5: Decide the presence of primary signal or not using

$$\xi_{GR} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{GR},$$

where γ_{GR} is the predetermined threshold.

3.2. Gerschgorin Disk-based Detection

The Gerschgorin centers give us information about the primary signal energy. Fig. 1 shows the Gerschgorin disks generated by the transformed SCM $\hat{\boldsymbol{G}}$ under $\mathcal{H}_1$, where the number of antennas and snapshots are set as $M = 6$ and $N = 100$. The largest disk is the source Gerschgorin disk and the smaller four are the noise Gerschgorin disk collection. Fig. 1 explicitly shows that the source Gerschgorin disk has a much larger center (also the signal eigenvalue $\hat{\lambda}_1$) than those of the noise Gerschgorin disk collection (also the noise eigenvalues). The eigenvalue dispersion under hypothesis $\mathcal{H}_1$ can be exploited for spectrum sensing.

The authors in [13] have developed an eigenvalue-based detector under the generalized likelihood ratio test (GLRT) framework, resulting in the AGM detector. The AGM method is available to detect single PU without the *a priori* knowledge of PU and with robustness against noise uncertainty. In order to utilize both the Gerschgorin centers (i.e., the eigenvalues)

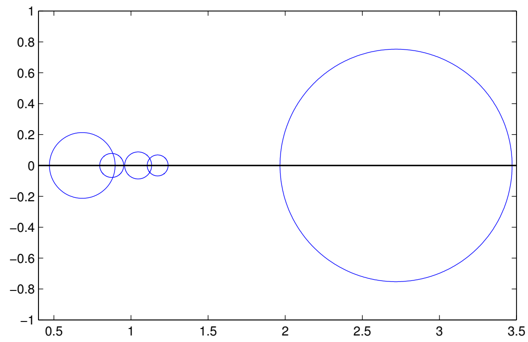


Fig. 1: Gerschgorin disks of matrix $\hat{\boldsymbol{G}}$, $M = 6, N = 100$

and the radii to detect signals, a Gerschgorin disk-based (GD) method is given as

$$\xi_{GD} \triangleq \frac{\frac{1}{M-1}\sum_{i=1}^{M-1}\hat{\lambda}_i}{\left(\prod_{i=1}^{M-1}\hat{\lambda}_i\right)^{\frac{1}{M-1}}} \times \hat{\rho}_1. \tag{15}$$

The proposed GD detector employs the information of subspace projection and signal energy, thus achieving better detection performance than the AGM and GR methods, as will be illustrated in Section 4. The proposed GD method for spectrum sensing is summarized in Table 2.

Table 2: GD algorithm for spectrum sensing

Step 1: Calculate the SCM using (12).

Step 2: Partition $\hat{\boldsymbol{R}}$ as: $\hat{\boldsymbol{R}} = \begin{bmatrix} \hat{\boldsymbol{R}}_1 & \hat{\boldsymbol{r}} \\ \hat{\boldsymbol{r}}^H & \hat{r}_{MM} \end{bmatrix}$.

Step 3: Perform the eigendecomposition of $\hat{\boldsymbol{R}}_1$: $\hat{\boldsymbol{R}}_1 = \hat{\boldsymbol{V}}\hat{\boldsymbol{\Sigma}}\hat{\boldsymbol{V}}^H$, where $\hat{\boldsymbol{\Sigma}} = [\hat{\lambda}_1, \cdots, \hat{\lambda}_{M-1}]$ and $\hat{\boldsymbol{V}} = [\hat{\boldsymbol{v}}_1, \cdots, \hat{\boldsymbol{v}}_{M-1}]$ are the eigenvalue matrix and eigenvector matrix of $\hat{\boldsymbol{R}}_1$, respectively.

Step 4: Calculate the GD test statistic as (15).

Step 5: Decide the presence of primary signal or not according to

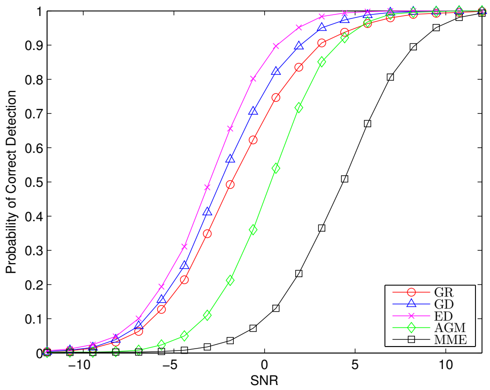
$$\xi_{GD} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma_{GD},$$

where γ_{GD} is the predetermined threshold.

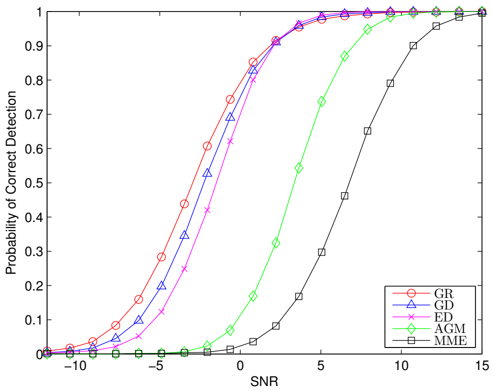
4. SIMULATION RESULTS

In this section, we present simulation results to evaluate the performance of the proposed detectors. Our proposals are compared with several representative methods, that is, the

AGM, MME, and ED methods. All the results are calculated based on 200,000 independent Monte-Carlo trials. In the simulation, the channel coefficients are randomly generated from zero-mean complex Gaussian variables, and the signal and noise are generated according to (1) and (2).



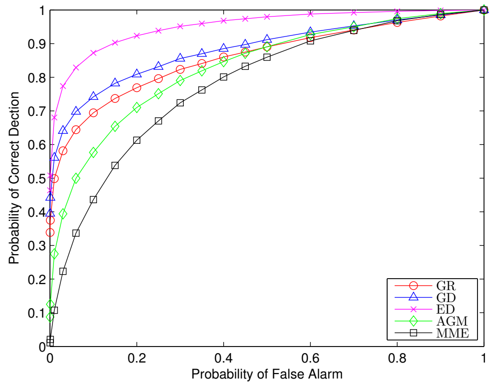
(a) Uniform noise



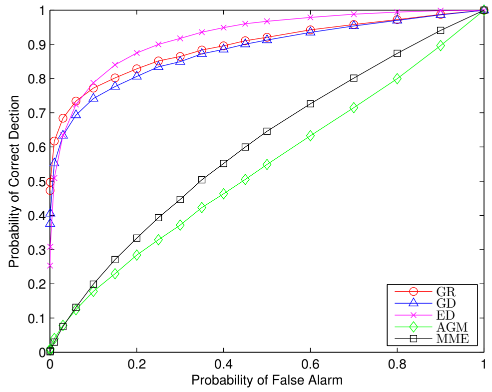
(b) Nonuniform noise

Fig. 2: Probability of detection versus SNR for different noise types. $P_{fa} = 10^{-3}, M = 6, N = 10$

Fig. 2 shows the probability of detection versus SNR for both uniform and nonuniform noises, where $M = 6$, $N = 10$ and $P_{fa} = 10^{-3}$. The SNR is defined as the ratio of the signal power to the averaged noise power $\overline{\sigma_w^2}$, i.e., SNR $= 10\log(\sigma_s^2/\overline{\sigma_w^2})$. Fig. 2(a) depicts the uniform noise case where noise variance $\sigma_w^2 = 1$ for all the antennas. The proposed GR method is superior to AGM and MME methods, because the gap between the test statistics of the former one under $\mathcal{H}_1$ and $\mathcal{H}_0$ is much larger than those of the latters. The GD method outperforms the GR method since it uses more information from the Gerschgorin centers than the latter. Fig. 2(b) illustrates the detection performance under the nonuniform noise case, where noise variance from different antennas are set as $[3.0200, 0.3162, 4.3652, 0.2291, 2.2387, 0.5370]$. The GR



(a) Uniform noise



(b) Nonuniform noise

Fig. 3: ROC for different noise types. SNR=-3dB, $M = 6, N = 10$

method has better detection performance than the others, indicating that it is robust against the nonuniform noise. The performance of the GD method degrades rationally, since it utilizes the Gerschgorin centers which contain the noise variance.

The corresponding receiver operating characteristic (ROC) curves are shown in Fig. 3 for fixed SNR=-3dB. It is seen that the GR method is optimal for low SNR and relatively small P_{fa} in the case of nonuniform noise, which is practically attractive.

5. CONCLUSION

Two spectrum sensing methods based on the Gerschgorin disks have been proposed. The GR method has good performance especially when there is non-uniform noise, while the GD method is optimal provided that there is uniform noise. Due to space limitations, we did not include derivation of the theoretic thresholds and theoretic performance analyses of the two approaches, which will be our future works.

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