INFORMATION ALIGNMENT FOR CONSENSUS WITH INTERFERENCE

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ABSTRACT

This paper studies distributed averaging of arbitrary vectors in the presence of network interference by casting an algebraic structure over the interference. While communicating locally with its neighbors for consensus, each agent causes an additive interference, lying on a low-dimensional subspace, in other communication links. We consider a particular case when this interference subspace depends only on the interferer, referred to as *uniform outgoing interference*. We show that consensus is possible in a low-dimensional subspace of the initial conditions whose dimension is complimentary to the largest interference subspace across all of the agents. In this context, we derive a global *information alignment* and a local pre-conditioning, followed by local consensus iterations to ensure subspace consensus. We further provide the conditions under which this subspace consensus recovers the exact average. The analytical results are illustrated graphically to describe the setup and the information alignment scheme.

Index Terms— Average-consensus, Information alignment, Interference subspaces, Signal recovery

1. INTRODUCTION

Distributed averaging of information at geographically dispersed agents has been a significant of recent work, see e.g. see [1–9]. When the inter-agent communication is noiseless and interference-free, the protocol and related results can be found in [10]. A number of papers, including [11–13], subsequently consider average-consensus in a setting where the inter-agent communication is assumed to be imperfect. Reference [14] considers consensus with link failures and channel noise. Subsequently, [15] considers consensus with asymmetric links and with asymmetry in packet losses. Consensus under stochastic disturbances is considered in [16], while [17] studies a natural superposition property of the communication medium and uses computation codes to achieve energy efficient consensus. In [18], a similar interference scenario is considered for average-consensus. Related work also includes [19,20], which exploits full duplex communication for *group consensus* where two-way communication is enabled at the same time and faster convergence is reported.

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1.1. RELATION TO PRIOR WORK

In contrast to the past work outlined above, we focus on an algebraic model for network interference. While communicating locally with its neighbors, each agent causes an additive interference, belonging to a low-dimensional subspace, in other communication links. This (low-dimensional) interference subspace, in general, depends on both the communication link and the interfering agent. A fortiori, it is clear that if the interference by an agent is persistent in all dimensions, then there is no way to recover perfect consensus unless schemes similar to interference alignment [21] are used. In such interference alignment schemes, the data is projected onto higher dimensions such that the interferences and the data lie in different low-dimensional subspaces; clearly, requiring an increase in the communication resources.

On the other hand, if the interference from each agent lies in (possibly different) low-dimensional subspaces, the problem we address is whether one can exploit this low-dimensionality for consensus. Subsequently, we address how much information can be recovered when the collection of local interferences span the entire $\mathbb{R}^n$? In this paper, we consider *uniform outgoing interference*, where the interference caused by an agent lies in a subspace that only depends on the interfering agent. In this context, we show that interference-free consensus is possible with a global *information alignment* and a local pre-conditioning scheme.

The rest of the paper is organized as follows: Notation is set in Section 1.2. Section 2 formulates the problem and Section 3 describes our approach. We provide a graphical illustration in Section 4, and Section 5 concludes the paper.

1.2. Notation and Preliminaries

We use lowercase bold letters to denote vectors and uppercase italic letters for matrices. We use the symbols $\mathbf{1}_n$ and $\mathbf{0}_n$ to denote the n-dimensional column vectors of all 1's and all 0's, respectively. The identity and zero matrices of size n are denoted by I_n and $\mathbf{0}_{n\times n}$, respectively. We assume that a network of N agents is given by a communication graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of agents, and $\mathcal{E}$ is the collection of ordered pairs, $(i,j), i,j \in \mathcal{V}$, such that agent j can send information to agent i. From this graph, we denote the set of agents that can send information to agent i as $\mathcal{N}_i$.

2. PROBLEM FORMULATION

We consider average-consensus in a multi-agent network, $\mathcal{G}$, when the interagent communication is subject to unwanted interference. Unwanted interference is where the desired communication, $\mathbf{q}^j \in \mathbb{R}^n$, from agent, $j \in \mathcal{V}$, to agent, $i \in \mathcal{V}$, has an additive unwanted component, $\mathbf{z}^{ij} \in \mathbb{R}^n$, coming arbitrarily from the agents in $\mathcal{V}$. In particular, every communication link, $j \rightarrow i$ or $(i, j) \in \mathcal{E}$, incurs the following interference:

$$\mathbf{z}^{ij} = \sum_{m \in \mathcal{V}} b_{ij}^m \Gamma_{ij}^m \mathbf{q}^m, \tag{1}$$

where: $b_{ij}^m = 1$, if agent $m \in \mathcal{V}$ interferes with $j \rightarrow i$ communication, and 0 otherwise; and $\Gamma_{ij}^m \in \mathbb{R}^{n \times n}$ is the interference gain. What agent i actually receives from agent j is thus: $\mathbf{q}_k^j + \sum_{m \in \mathcal{V}} b_{ij}^m \Gamma_{ij}^m \mathbf{q}_k^m$, at time k. This interference model can be studied in the following three special cases:
Uniform Interference: $\Gamma_{ij}^m = \Gamma$, i.e., the interference is uniform across all agents and links, see our related work in [22];
Uniform Outgoing Interference: $\Gamma_{ij}^m = \Gamma_m$, i.e., the interference subspaces only depend on interferers;
Uniform Incoming Interference: $\Gamma_{ij}^m = \Gamma_i$, i.e., the interference subspaces only depend on the receivers.

In this paper, we consider the case of uniform outgoing interference; some extensions are considered in [23]. In the rest of the paper, we explicitly assume the following:

(a) *No agent, $i \in \mathcal{V}$, knows its interfering neighbors over any $j \rightarrow i$ channel, $j \in \mathcal{N}_i$.*

(b) *Each interferer, $m \in \mathcal{V}$, knows a set of basis vectors that spans the null space, Θ_{Γ_m}, of its interference gain matrix, $\Gamma_m \in \mathbb{R}^{n \times n}$. We assume that* $\dim(\Theta_{\Gamma_m}) = \gamma_0 \leq n$.

This interference setup is shown in Fig. 1: Agent, j, transmits, $\mathbf{q}_k^j$, at time k to agent i. This transmission, $\mathbf{q}_k^j$, reaches the intended receiver, i, as the unaltered intended transmission, $\mathbf{q}_k^j$, plus projected interfering transmissions, $\Gamma_m \mathbf{q}_k^m$.

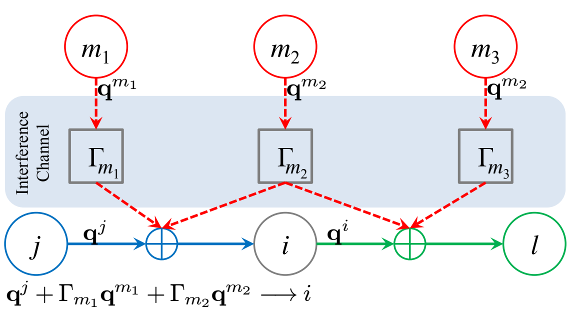

Fig. 1. Uniform outgoing interference: Note that interference on $j \rightarrow i$ link may also be caused by the transmitting agent, j, i.e., agent m_1 can be agent j.

Given the above interference model, the *standard* protocol for average-consensus, [10, 24], implemented on the multi-agent network is given by, $(k \geq 0, i \in \mathcal{V})$:

$$\mathbf{q}_{k+1}^i = \sum_{j \in \mathcal{N}_i} w_{ij} \left(\mathbf{q}_k^j + \sum_{m \in \mathcal{V}} b_{ij}^m \Gamma_m \mathbf{q}_k^m \right), \tag{2}$$

where $\mathbf{q}_0^i \in \mathbb{R}^n$ is the agent i's initial condition.It is evident that interference is only incurred on the links allowed by the underlying communication graph, $\mathcal{G}$, i.e., when $w_{ij} \neq 0$, which, in general, is non-zero for each $j \in \mathcal{N}_i$ for average-consensus to be successful [10]. The above protocol, Eq. (2), reduces to the standard case [10], when there is no interference, i.e., $b_{ij}^m = 0$, for all i, j, m, and converges to the average of the initial conditions:

$$\lim_{k \rightarrow \infty} \mathbf{q}_k^i = \frac{1}{N} \sum_{j=1}^{N} \mathbf{q}_0^j, \qquad \mathbf{x}_0^j \in \mathbb{R}^n, \tag{3}$$

under the typical assumptions [10, 24] on the weight matrix, $W = \{w_{ij}\} \in \mathbb{R}^{N \times N}$, and the communication graph, $\mathcal{G}$. However, when there is interference, i.e., $b_{ij}^m \neq 0$, it can be verified that, in general, the standard protocol in Eq. (2) diverges. This is because the spectral norm of W is 1, and any non-zero input, in general, makes Eq. (2) diverge.

The goal of this paper is to consider consensus in the presence of interference, Eq. (2), not only to establish convergence, but further to explicitly characterize the steady-state.

3. UNIFORM OUTGOING INTERFERENCE

To establish consensus in the presence of interference, our first step is *global information alignment*. To this aim, each initial condition, $\mathbf{q}_0^i \in \mathbb{R}^n$, is projected on a subspace, $\mathcal{S}_0$, with projection matrix, $I_{\mathcal{S}_0}$, i.e., $\mathbf{x}_0^i = I_{\mathcal{S}_0} \mathbf{q}_0^i$. The goal of this paper is to characterize this signal subspace, $\mathcal{S}_0$, under the given structure on the interference subspaces, Γ_i's (recall Assumption (b)). We will show that there exists an $I_{\mathcal{S}_0}$ on which interference-free consensus can be achieved. In the following, we implement consensus on the projected initial conditions, $\mathbf{x}_0^i$'s, and subsequently, characterize $I_{\mathcal{S}_0}$ to establish the relationship with the original initial conditions, $\mathbf{q}_0^i$'s.

Given our interference model, Fig. 1 and Assumptions (a)-(b), we note that each receiving agent, i, receives the interference as a linear combination, $\sum_{m \in \mathcal{V}} b_{ij}^m \Gamma_m \mathbf{x}_k^m$, of the interferers. In this setting, agent i is unable to perform an operation to cancel the interference signal (recall Assumption (a)). Interference cancelation thus has to be enabled within the transmission, i.e., influenced by the interferers themselves. With this insight, we modify each agent's transmission by $T_j \in \mathbb{R}^{n \times n}$, for all $j \in \mathcal{V}$; agent i thus receives

$$T_j \mathbf{x}_k^j + \sum_{m \in \mathcal{V}} b_{ij}^m \Gamma_m T_m \mathbf{x}_k^m, \tag{4}$$

on the link (i, j) at time k. These local transformations, T_i's, are subsequently referred to as *pre-conditioners*. The structure of these (local) transformations is illustrated in Fig. 2.

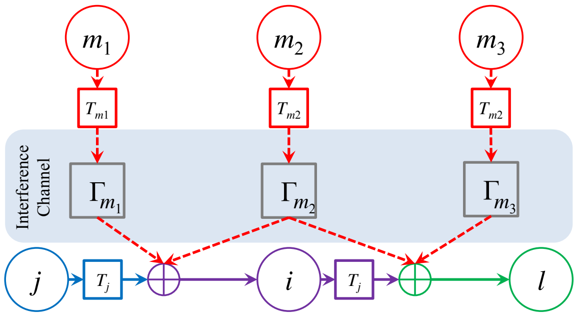

Fig. 2. Uniform Outgoing Interference, only depends on the interfering agent; the blocks, T_m, are the (local) pre-conditioners at each agent to enable interference cancellation.

To establish the main results, consider the following (locally transformed) consensus protocol:

$$\mathbf{x}_{k+1}^i = \sum_{j\in\mathcal{N}_i} W_{ij}\left(T_j\mathbf{x}_k^j + \sum_{m\in\mathcal{V}} b_{ij}^m \Gamma_m T_m \mathbf{x}_k^m\right), \quad (5)$$

where $W_{ij} \in \mathbb{R}^{n\times n}$ is the coefficient (matrix) that agent i assigns to agent j's state, $\mathbf{x}_k^j$; recall that in the standard form, Eq. (2): $W_{ij} = w_{ij}I_n$. From Eq. (5), we now have

$$\mathbf{x}_{k+1}^i = \sum_{j\in\mathcal{N}_i} W_{ij}T_j\mathbf{x}_k^j + \sum_{m\in\mathcal{V}} B_{im}\Gamma_m T_m \mathbf{x}_k^m, \quad (6)$$

where $B_{im} = \sum_{j\in\mathcal{N}_i} W_{ij}b_{ij}^m$. Note that in this protocol, each agent $i \in \mathcal{V}$ preconditions its information using the matrix T_i, such that it may pass unharmed through the interference subspace, Γ_i. Let us now recall Assumption (b) where we assumed that each agent, i, knows the γ_0-dimensional null space, Θ_{Γ_i}, of its corresponding interference gain, Γ_i. We have the following result:

Lemma 1. *For any matrix, $I_{\mathcal{S}_0}$, with rank γ_0. There exists a full rank matrix, T_i, such that*

$$\Gamma_i T_i I_{\mathcal{S}_0} = \mathbf{0}_{n\times n}. \quad (7)$$

Proof. Without loss of generality, denote the SVD of $\Gamma_i = U_i S_i V_i^\top$, where the first $n-\gamma_0$ diagonals of S_i are non-zero. Similarly, let the SVD of $I_{\mathcal{S}_0} = U_{\mathcal{S}_0} S_{\mathcal{S}_0} V_{\mathcal{S}_0}^\top$, where the non-zero singular values of $I_{\mathcal{S}_0}$ are the last γ_0 diagonal elements of $S_{\mathcal{S}_0}$. Define

$$T_i = \left[\begin{array}{c|c} \widetilde{V}_i & \widehat{V}_i \end{array}\right] U_{\mathcal{S}_0}^\top, \quad (8)$$

where $\oplus\widehat{V}_i = \Theta_{\Gamma_i}$, and $\widetilde{V}_i$ is chosen such that T_i is full rank; the symbol '$\oplus$' denotes the column span. That a full rank, T_i, exists is always guaranteed because all of the γ_0 columns of $\widehat{V}_i$ are linearly independent. One such choice is $T_i = V_i U_{\mathcal{S}_0}^\top$; however, knowing only the null space, $\widehat{V}_i$, suffices, and the lemma follows. □

The following lemma characterizes the dynamics of the information-aligned consensus.

Lemma 2. *For some $0 \le \gamma_0 \le n$, let each outgoing interference matrix, $\Gamma_i \in \mathbb{R}^{n\times n}$, have rank $n-\gamma_0$. Let $I_{\mathcal{S}_0} \in \mathbb{R}^{n\times n}$ be the projection matrix that projects the initial conditions in $\mathbb{R}^n$, on $\mathcal{S}_0$, where $\dim(\mathcal{S}_0) = \gamma_0$. There exist T_i at each i, and W_{ij}'s for all $(i,j) \in \mathcal{E}$ such that Eq.* (6) *becomes*

$$\mathbf{x}_{k+1}^i = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_k^j,$$

at each $i \in \mathcal{V}$, when $\mathbf{x}_0^i \in \mathcal{S}_0$.

Proof. Without loss of generality, we assume that $\mathcal{S}_0 = \mathcal{R}_A$, where $\mathcal{R}_A$ denotes the range space of some $A \in \mathbb{R}^{n\times n}$, such that $\dim(\mathcal{R}_A) = \gamma_0$. Let

$$I_{\mathcal{S}_0} = A^\dagger A, \quad (9)$$

where $I_{\mathcal{S}_0}$ is the orthogonal projection that projects any arbitrary vector in $\mathbb{R}^n$ on $\mathcal{S}_0$. Let the projection of the initial conditions, $\mathbf{q}_0^i \in \mathbb{R}^n$, on $\mathcal{S}_0$ be $\mathbf{x}_0^i = I_{\mathcal{S}_0}\mathbf{q}_0^i$ and choose T_i to be the *local* invertible preconditioning, as given by Lemma 1; we thus have $\Gamma_i T_i \mathbf{x}_0^i = \Gamma_i T_i I_{\mathcal{S}_0}\mathbf{q}_0^i = \mathbf{0}_n, \forall i$. Choose

$$W_{ij} = w_{ij}T_j^{-1}. \quad (10)$$

From Eq. (6), we have

$$\mathbf{x}_{k+1}^i = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_k^j + \sum_{m\in\mathcal{V}} B_{im}\Gamma_m T_m \mathbf{x}_k^m.$$

We now show that $\mathbf{x}_k^i \in \mathcal{S}_0$ for all i,k, by induction. Consider $k=0$, then

$$\mathbf{x}_1^i = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_0^j + \sum_{m\in\mathcal{V}} B_{im}\Gamma_m T_m \mathbf{x}_0^m = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_0^j,$$

which is a linear combination of vectors in $\mathcal{S}_0$ and thus is in $\mathcal{S}_0$. Assume that $\mathbf{x}_k^i \in \mathcal{S}_0, \forall i$, and some k, leading to $\Gamma_i T_i \mathbf{x}_k^i = \Gamma_i T_i I_{\mathcal{S}_0}\mathbf{x}_k^i = \mathbf{0}_n$. Then for $k+1$:

$$\mathbf{x}_{k+1}^i = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_k^j + \sum_{m\in\mathcal{V}} B_{im}\Gamma_m T_m \mathbf{x}_k^m = \sum_{j\in\mathcal{N}_i} w_{ij}\mathbf{x}_k^j,$$

$\forall i \in \mathcal{V}$, which is a linear combination of vectors in $\mathcal{S}_0$ at each agent, and the lemma follows. □

With this lemma, the following theorem describes the main result on uniform outgoing interference.

Theorem 1. *Let Θ_{Γ_i} denote the null space of Γ_i, and let $\dim(\Theta_{\Gamma_i}) = \gamma_0, \forall i \in \mathcal{V}$. In the presence of uniform outgoing interference, the protocol in Eq.* (5) *recovers the average-consensus in a γ_0-dimensional subspace, $\mathcal{S}_0$, of $\mathbb{R}^n$, when we choose T_i according to Lemma 1, and $W_{ij} = w_{ij}T_j^{-1}$, at each agent, $i \in \mathcal{V}$.*

The proof[1] follows from Lemma 2 and consensus in the presence of uniform outgoing interference, Eq. (5), converges to

$$\mathbf{x}_\infty^i = \frac{1}{N}\sum_{j=1}^{N}\mathbf{x}_0^j = \frac{1}{N}\sum_{j=1}^{N} I_{\mathcal{S}_0}\mathbf{q}_0^j, \tag{11}$$

with $\mathbf{q}_0^i \in \mathbb{R}^n, \forall i \in \mathcal{V}$. Note that the application of Theorem 1 in a completely distributed manner requires that each agent, $i \in \mathcal{V}$, knows the pre-conditioners, $\{T_j\}_{j\in\mathcal{N}_i}$, *only* in its neighborhood; and thus is *completely local*. In addition, all agents are required to agree on the signal subspace, $I_{\mathcal{S}_0}$, which implies a *global information alignment*. This information can be safely assumed to be known at each agent.

The protocol described in Theorem 1 can be illustrated with the help of Fig. 2. Notice that a transmission from any agent, $i \in \mathcal{V}$, passes through agent i's dedicated preconditioning matrix, T_i. The network (both non-interference and interference) sees only $T_i\mathbf{x}_k^i$ at each k. Since the interference is only a function of the transmitter (uniform outgoing), all of the agents ensure that a particular signal subspace, $I_{\mathcal{S}_0}$, is not corrupted by the interference channel. The significance here is that even when the interferences are misaligned such that $\oplus_{i\in\mathcal{V}}\Gamma_i = \mathbb{R}^n$, the protocol in Eq. (5) recovers the average in a $\gamma_0 = \dim(\Theta_{\Gamma_i})$ dimensional subspace. On the other hand, the null space of $\oplus_{i\in\mathcal{V}}\Gamma_i$ may very well be 0-dimensional. For example, if each Γ_i is rank 1, Eq. (5) recovers the average in an $n-1$ dimensional signal subspace. We now briefly capture some straightforward generalizations:

(1) Perfect Consensus: If the agent initial conditions, $\mathbf{q}_0^i$, lie in a γ_0-dimensional subspace of $\mathbb{R}^n$, then Theorem 1 recovers perfect consensus.

(2) Principal/Selective Consensus: If the agent initial conditions, $\mathbf{q}_0^i$, lie in the range space of some matrix with (ordered) singular values, $\lambda_1, \lambda_2, \ldots, \lambda_n$. Then the initial condition subspace, $\mathcal{S}_0$, can be chosen to recover consensus in any arbitrary γ_0 non-zero singular values.

(3) Time-varying scenario: The interference model can be easily adjusted for time-varying interference subspaces, $\Gamma_{i,k}$'s, and Theorem 1 can be accordingly adjusted via time-varying pre-conditioners, $T_{i,k}$'s.

4. GRAPHICAL ILLUSTRATION

This section illustrates Theorem 1. Consider a network of $N = 10$ agents, each with a randomly chosen initial condition (i.c.) in $\mathbb{R}^3$ such that each i.c. belongs to the range space of a rank 2 matrix, A. From Lemma 2, the projection, $I_{\mathcal{S}_0}$, is given by $A^\dagger A$, with $\mathbf{x}_0^i = I_{\mathcal{S}_0}\mathbf{q}_0^i = \mathbf{q}_0^i$; the i.c.'s and their average are shown as blue squares and a white diamond, respectively, in Fig. 3 (a). Uniform outgoing interference at each agent is chosen as one of the three 1-dimensional subspaces (with 2-dimensional null-spaces) such that each subspace appears at some agent in the network, see Fig. 3 (b). From Lemma 1, each agent chooses a pre-conditioner, T_i, to project its transmission on the null space of its own interference, see Fig. 3 (c) where the transmissions over k are shown. Subsequently, each receiver, $i \in \mathcal{V}$, receives misaligned data, $T_j\mathbf{x}^j$, from each of its neighbors, $j \in \mathcal{N}_i$, and translates back to the i.c. subspace, $I_{\mathcal{S}_0}$, via T_j^{-1} that is incorporated in the consensus weights, $W_{ij} = w_{ij}T_j^{-1}$. Resulting consensus iterates in $I_{\mathcal{S}_0}$ are shown in Fig. 3 (d) where the average is locally computed as described in Theorem 1.

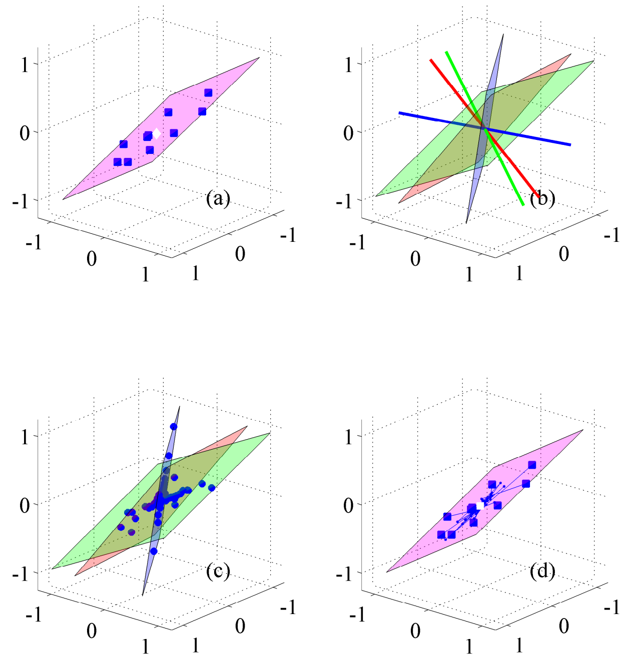

Fig. 3. Illustration of Theorem 1: (a) Two-dimensional signal space, $I_{\mathcal{S}_0}$, in $\mathbb{R}^3$; (b) One-dimensional interference subspaces with $\gamma_0 = 2$-dimensional null spaces; (c) Agent transmissions aligned in the corresponding null spaces over time, k; (d) Consensus iterates after the translations by T_j^{-1}.

5. CONCLUSIONS

In this paper, we cast an algebraic structure over the interference incurred in a multi-agent collaborative network. Each agent while implementing consensus with its neighbors simultaneously interferes with other agents implementing the same consensus iterations. Under *uniform outgoing interference*, we provide an innovative *information alignment* scheme that utilizes local preconditioning to reach consensus on a low-dimensional subspace, even if the collection of local interferences span the entire $\mathbb{R}^n$. That only a local protocol achieves this is somewhat surprising.

[1] Theorem 1 can be easily adjusted for arbitrary rank interferences by choosing $\gamma_0 = \min_{i\in\mathcal{V}}\{\dim(\Theta_{\Gamma_i})\}$.

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